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Asymptotically safe inflation from quadratic gravity

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ABSTRACT

Asymptotically Safe theories of gravity have recently received much attention. In this work we discuss a class of inflationary models derived from quantum-gravity modification of quadratic gravity according to the induced scaling around the non-Gaussian fixed point at very high energies. It is argued that the presence of a three dimensional ultraviolet critical surface generates operators of non-integer power of the type $R^{2-\theta/2}$ in the effective Lagrangian, where $\theta > 0$ is a critical exponent. The requirement of a successful inflationary model in agreement with the recent *Planck* 2015 data puts important constraints on the strength of this new type of couplings.

1. Introduction

If the inflationary scenario is correct, then it is possible that quantum gravitational phenomena could be detected in anisotropy experiments of the cosmic microwave background radiation and in observations of the large scale structure of the Universe [1].

Although a consistent quantization of the gravitational field is still lacking, it is assumed that quantizing the matter fields and the gravitational field on a classical background should be sufficient to explain the generation of the initial spectrum of perturbations.

In recent times a promising theory of quantum gravity has appeared in the framework of the *asymptotic safety* (AS) proposal. Its central idea, as first discussed by Weinberg [2], is to define the continuum limit around a non-Gaussian fixed point (NGFP) characterized by an ultraviolet (UV) critical manifold of finite dimensionality [3–24].

One of the most striking consequences of this approach is the fact that the fundamental theory seems to be rather different from the classical gravity based on the Einstein–Hilbert action. In the infinite cutoff limit, it is characterized by a vanishing Newton's constant which is therefore *antiscreened* at high energies. Classical gravity can only be recovered as a low-energy effective theory at some intermediate scale and it does not define a fundamental theory. Although we do not know yet the structure of the fundamental Lagrangian, all the investigations performed so far, considering general $f(R)$ truncations or more complicated tensorial structure like $R_{\mu\nu}R^{\mu\nu}$, have confirmed the finite dimensionality of the UV critical surface (see [25–27] for reviews and also [28] for a recent investigation within the $f(R)$ truncation).

In the simple case of two relevant directions, as in the case of *quantum* Einstein–Hilbert gravity, several investigations have focused on the implications of the running of the Newton's constant in models of the Early Universe. In [29–33] it has been shown that the renormalization-group induced evolution of the Newton's constant and cosmological constant can provide a consistent cosmic history of the Universe from the initial singularity to the superaccelerated expansion.

In particular in [32] it has been argued that the scaling properties of the 2-points correlation function of the graviton near the NGFP induce a scale invariant spectrum of the primordial perturbations, characterized by a spectral index n_s which, to a very good approximation, must satisfy $n_s \approx 1$.

Subsequent investigations have tried to produce successful models of inflation by considering an extended structure of the effective Lagrangian near the NGFP [34], but it turns out that the use of an "optimal cutoff" might result in a fine tuning problem [35,36]. In [37] the running of the gravitational and cosmological constants has been described in terms of a Jordan–Brans–Dicke model with a vanishing Brans–Dicke parameter, while the viability of the AS scenario in models where the inflaton is the Higgs field has been discussed in [38,39].

An effective Lagrangian encoding the leading quantum gravitational effect near the NGFP has been proposed in [40] where a RG-improvement of the linearized β-functions has been performed using the idea that the relevant cutoff in this situation is provided

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by the local curvature, i.e. $k^2 \sim R$ (see also [41]). In a recent investigation a similar approach has been successfully applied to study the quantum gravity modifications of the Starobinsky model [42] (see also [43–48] for other examples of quantum deformed quadratic gravity inflationary models).

In this work we extend the analysis of [40] by including the additional relevant direction produced by the R^2 operator on the UV critical surface.

It will be shown that the requirement of a successful inflationary model in agreement with the recent *Planck* 2015 data release of CMB anisotropy puts significant constraints on the renormalized flow generated by the R^2 term. In particular the dynamics of the inflaton after the inflationary phase can be rather different from the well known $R + R^2$ Starobinsky model.

The rest of the paper is organized as follows: In Section 2 we introduce the basic formalism and obtain the effective field theory description in terms of a $f(R)$ theory. In Section 3 we compute the spectral index n_s for the primordial perturbations and the tensor-to-scalar ratio r using a conformal mapping to the Einstein frame. In Section 4 the oscillatory phase after inflation is discussed, and Section 5 is devoted to the conclusions.

2. Basic formalism

Let us consider the quadratic gravity Lagrangian

$$L_k = \frac{k^2}{16\pi g_k}(R - 2\lambda_k k^2) - \beta_k R^2 \tag{1}$$

where k is a running energy scale and g_k, λ_k and β_k are dimensionless running coupling constants whose infinite momentum limit is controlled by a NGFP, i.e. $\lim_{k\to\infty}(g_k, \lambda_k, \beta_k) = (g_*, \lambda_*, \beta_*) \neq (0,0,0)$ [6,49]: does this running show up at the level of predictions for a specific inflationary model?

In the case at hand the qualitative behavior of the UV critical manifold of the *quantum* theory defined by (1) at the NGFP is rather simple, as its continuum limit can be described only by three relevant couplings. In particular there exist trajectories that emanate from the NGFP and possess a long classical regime where the effective action is approximated by the standard Einstein–Hilbert action [49]. On the contrary the quantitative details of the renormalized flow around the NGFP are still rather uncertain. In fact, not only the precise location of the NGFP is regulator dependent (as expected), but also the value of the critical exponents depends on the truncation strategy employed to solve the flow equation [12,50]. Moreover recent investigations based on unimodular gravity [51], and general arguments [52] suggest that the critical exponents are indeed real [53,54].

For these reasons we would like to find an approximation to the renormalized flow which encodes the general qualitative feature of the scaling around the NGFP and assumes real critical exponents. We thus approximate the running of λ_k with its tree-level scaling $\lambda_k \sim c_0 k^{-2}$ where c_0 is a dimensionful constant, and decouple the running of g_k from the running of β_k. This latter approximation is justified by the impressive stability of the critical exponents for the Newton's constant against the inclusion of higher order truncations as shown in several investigations [12], but it has also the important advantage to allow for an analytic expression of the flow in our model. In fact, under these assumptions the renormalized flow thus reads [10]

$$g_k = \frac{6\pi c_1 k^2}{6\pi\mu^2 + 23c_1(k^2 - \mu^2)} \tag{2}$$

$$\beta_k = \beta_* + b_0\left(\frac{k^2}{\mu^2}\right)^{-\frac{\theta_3}{2}} \tag{3}$$

where μ is an infrared renormalization point, $c_1 = g_k(k = \mu)$ (see [10] for details). According to [6] $\beta_* = \beta_k(k \to \infty) \simeq 0.002$, while b_0 is a free parameter obtained by the linearization of the RG flow around the NGFP, and θ_3 is the critical exponent for the R^2 coupling.

It is important to stress that, as long as $c_1 < 6\pi/23$ the running described by Eq. (2) smoothly interpolates between the Gaussian fixed point (GFP) and the NGFP, $g_* = g_k(k \to \infty) = 6\pi/23$ and therefore it captures the qualitative features of the flow described in [49]. The constants b_0, c_0 and c_1 depend on the physical situation at hand and must be fixed by confronting the model with observations: in principle these are the only free parameters of our theory corresponding to the three relevant directions of the UV critical surface of the action (1). In other words, by changing c_1, c_0 and b_0 it is possible to explore various RG trajectories all ending at the NGFP. The relevant question is whether it is possible to actually constraint these numbers, in particular the value of b_0.

As argued in [40,42,55] by substituting (2), (3) and $\lambda_k = c_0/k^2$ in (1), a renormalization group improved effective Lagrangian can be obtained by the scale identification $k^2 \to \xi R$, where ξ is positive number. It must be stressed that the general structure of the resulting RG-improved effective Lagrangian agrees very well with the high-curvature solution of the fixed point equation for a generic asymptotically safe $f(R)$ theory, as it emerges from the analysis of [20] and [40].

At last, we obtain the following RG-improved action

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R + \alpha R^{2-\frac{\theta_3}{2}} + \frac{R^2}{6m^2} - \Lambda\right] \tag{4}$$

where μ is chosen so that $\kappa^2 = 8\pi G_N = 48\pi^2 c_1/(6\pi\mu^2 - 23(\mu^2 + 2\xi c_0)c_1) \equiv 1/M_{\rm pl}^2$. Moreover $\Lambda = \mu^2 c_0(6\pi - 23c_1)/(6\pi\mu^2 - 23(\mu^2 + 2\xi c_0)c_1)$ and $M_{\rm pl}^2/m^2 = 12(23\xi/(96\pi^2 - \beta_*))$; in particular if $\xi > 96\pi^2\beta_*/23$ and $c_0 < \mu^2(6\pi - 23c_1)/46\xi c_1$, then $1/m^2$ and Λ are positive definite.

On the other hand

$$\alpha = -2\mu^{\theta_3} b_0/M_{\rm pl}^2 \tag{5}$$

which only depends on θ_3 and b_0, but not on the fixed point values. Concerning θ_3 the numerical evidence accumulated so far has shown that its value is rather stable against the introduction of higher order truncation in the flow equation [50], as it should be expected for a critical exponent. On the contrary b_0 is by construction a non-universal quantity whose value cannot be determined by the RG group. It labels a specific trajectory emanating from the fixed point and its actual value should be determined by matching with a low energy observable.

Let us now introduce an auxiliary field φ defined via

$$\varphi(\chi) \equiv 1 + \alpha\left(2 - \frac{\theta_3}{2}\right)\chi^{1-\frac{\theta_3}{2}} + \frac{\chi}{3m^2} \tag{6}$$

In principle this relation can be always inverted at least locally, provided $\varphi_{,\chi} \neq 0$ so that (4) is equivalent to

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[\varphi R - (\varphi - 1)\,\chi(\varphi) - \alpha\left(2 - \frac{\theta_3}{2}\right)\chi(\varphi)^{1-\frac{\theta_3}{2}} + \frac{\chi(\varphi)}{3m^2}\Big] \tag{7}$$

In practice, due to its non-linearity, the task of inverting Eq. (6) can be very difficult and one must often resort to numerical methods. In our case, according to the analysis of [12], as θ_3 is rather close to unity we can safely set $\theta_3 = 1$ for any practical calculation (as it can be numerically checked). In this case we explicitly obtain the two branches

$$\chi_\pm = \frac{3}{8}\Big(27\alpha^2 m^4 + 8m^2\varphi - 8m^2 \pm 3\sqrt{3}\sqrt{27\alpha^4 m^8 + 16\alpha^2 m^6(\varphi-1)}\Big) \tag{8}$$

with the reality condition $\chi \geq 1 - 27m^2\alpha^2/16$.

Using these solutions we can obtain a canonically coupled scalar field by introducing a conformally related metric $g^E_{\mu\nu}$ by means of $g^E_{\mu\nu} = \varphi g_{\mu\nu}$ with $\varphi = e^{\sqrt{2/3}\kappa\phi}$. In the Einstein frame[1] [57,58] the action thus reads

$$S = \int d^4x\sqrt{-g_E}\left[\frac{1}{2\kappa^2}R_E - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_\pm(\phi)\right] \tag{9}$$

where

$$\begin{aligned} V_\pm(\phi) = \frac{m^2 \mathrm{e}^{-2\sqrt{\frac{2}{3}}\kappa\phi}}{256\kappa^2}\Bigg\{ & 192\left(\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi} - 1\right)^2 - 3\alpha^4 + 128\Lambda \\ & - 3\alpha^2\left(\alpha^2 + 16\,\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi} - 16\right) \mp 6\alpha^3\sqrt{\alpha^2 + 16\,\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi} - 16} \\ & - \sqrt{32}\alpha\left[\left(\alpha^2 + 8\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi} - 8\right) \pm \alpha\sqrt{\alpha^2 + 16\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi} - 16}\right]^{\frac{3}{2}}\Bigg\} \end{aligned} \tag{10}$$

and we have measured α and Λ in units of the scalaron mass m by means of the rescaling $\alpha \to \alpha/3\sqrt{3}m$ and $\Lambda \to \Lambda m^2$, so that both α and Λ are dimensionless numbers.

3. Inflation dynamics and primordial spectrum

For $\Lambda \gg 1$ the potential behaves as $V_\pm(\phi) \sim \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\kappa\phi}m^2\Lambda/\kappa^2$ while for $\alpha \gg 1$ we have $V_\pm(\phi) \sim \pm\alpha^4 e^{-2\sqrt{\frac{2}{3}}\kappa\phi}m^2/\kappa^2$. In both cases this implies $a_E(t) \sim t^{3/8}$ which does not provide inflation.

In order to study the inflation scenario in slow-roll approximation, we need to know the shape of the potentials $V_\pm(\phi)$. First we notice that, for all values (α, Λ), the potential has the plateau $V_\pm(\phi) = \frac{3m^2}{4\kappa^2}$ for $\phi \to \infty$. To verify slow-roll conditions inflation must start from $\phi > M_{\rm pl}$ and then proceed from the right to left. The behavior of the potential for $\phi \ll M_{\rm pl}$, and thus the inflation scenario, strongly depends on the values (α, Λ). In particular $V_\pm(\phi)$ can either develop a minimum, or $V_\pm(\phi) \to -\infty$ for $\phi \to -\infty$. In this work we study the class of potentials such that the inflation ends after a finite number N of e-folds, with a phase of parametric oscillations of the field ϕ. In order to have a well defined reheating phase it is clear that the potential must have a minimum; furthermore it can be proved that, in our case, a well defined exit from inflation occurs only for potentials with a minimum $\phi_{\rm min}$ such that $V(\phi_{\rm min}) \leq 0$. These conditions are verified only for $V_+(\phi)$ if $\alpha \in [1, 3]$ and $\Lambda \in [0, 1.5]$ (the potential $V_-(\phi)$ can have a minimum for some special values of (α, Λ), but it is always $V(\phi_{\rm min}) > 0$). In these cases the potential is depicted in Fig. 1 for various values of α.

In other words, although for α and Λ very close to zero (10) is only a small modification of the classical Starobinsky $R + R^2$ model, for $\alpha \in [1, 3]$ and $\Lambda \in [0, 1.5]$ the potential $V_+(\phi)$ develops a non-trivial minimum at negative values of the potential which makes our model significantly different from the original $R + R^2$ Starobinsky inflation. In particular, as we shall see, after exit from inflation, the dynamics of the preheating phase is characterized by a *lower limit* to $|\dot\phi|$ as it can be seen in Fig. 3 in the ϕ–$\dot\phi$ plane (see [59] for a general study of inflationary potentials with a negative value of the minimum).

[1] Note that, as discussed by [56] the field redefinition employed in going from the Jordan to the Einstein frame involves new additional contributions in the path-integral arising from the Jacobians. In investigations including the presence of matter field in the starting Lagrangian, these new terms cannot be neglected.

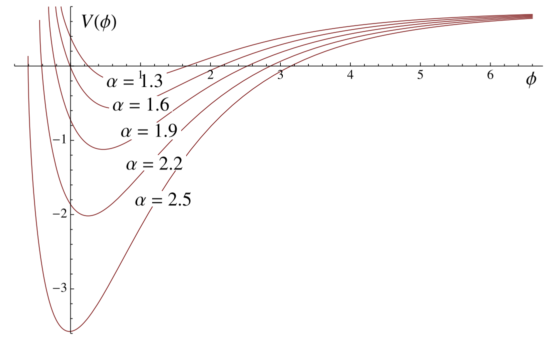

Fig. 1. Inflation potential for various values α and $\Lambda = 1.4$.

Let us introduce the slow-roll parameters ϵ and η,

$$\epsilon(\phi) \equiv \frac{1}{2\kappa^2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \quad \eta(\phi) = \frac{1}{\kappa^2}\left(\frac{V''(\phi)}{V(\phi)}\right) \tag{11}$$

and define the number of e-folds before the end of inflation

$$N = \int_{t_N}^{t_{\rm end}} H(t)\,dt. \tag{12}$$

The amplitude of the primordial scalar power spectrum reads

$$\Delta^2_{\mathcal{R}} = \frac{\kappa^4 V(\phi)}{24\pi^2\epsilon(\phi)} \tag{13}$$

and, in this approximation, the spectral index n_s and the tensor-to-scalar ratio r are given by

$$\begin{aligned} n_s &= 1 - 6\epsilon(\phi_i) + 2\eta(\phi_i) \\ r_T &= 16\epsilon(\phi_i) \end{aligned} \tag{14}$$

where $\phi_i = \phi(t_N)$ is the value of the inflaton field $\phi(t)$ at the beginning of the inflation.

In our case it is not possible to analytically evaluate these expressions, and we must resort to a numerical estimation of the initial value of the field which determines the final number of e-folds at the end of inflation. As usual inflation is supposed to stop when the slow-roll condition is violated, $\epsilon(\phi) = 1$. This condition determines the final value $\phi_f = \phi(t_{\rm end})$ of the inflaton field, while the initial value of the field $\phi(t)$ is obtained by inverting Eq. (12), once a number of e-folds is fixed. Our results are displayed in the following table:

Cases		$N = 50$		$N = 55$		$N = 60$	
Λ	α	n_s	r	n_s	r	n_s	r
	1.0	0.965	0.0069	0.968	0.0058	0.971	0.0050
0	1.8	0.966	0.0074	0.969	0.0063	0.972	0.0055
	2.6	0.967	0.0076	0.969	0.0065	0.972	0.0056
	1.0	0.965	0.0070	0.968	0.0059	0.971	0.0051
1	1.8	0.966	0.0074	0.969	0.0063	0.972	0.0055
	2.6	0.967	0.0076	0.969	0.0065	0.972	0.0056

Confronting n_s and r with the ’15 *Planck* data, our model agrees very well, as displayed in Fig. 2.

According to Eq. (13) the normalization of the scalar power spectrum at the pivot scale $k_* = 0.05$ Mpc^{-1} provides us with $m \sim (1.5 \div 7) \cdot 10^{14}$ GeV, depending on the value of α.

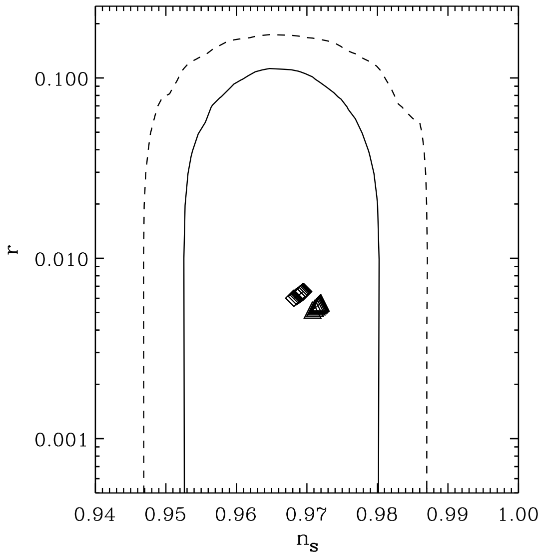

Fig. 2. We compare the theoretical predictions in the r–n_s plane for different values of α for the *Planck* Collaboration 2015 data release for the TT correlation assuming $\Lambda\text{CDM} + r$ [60]. Triangles are for $N = 55$ and squares for $N = 60$ e-folds. Solid and dashed lines are the 1σ and 2σ confidence levels, respectively.

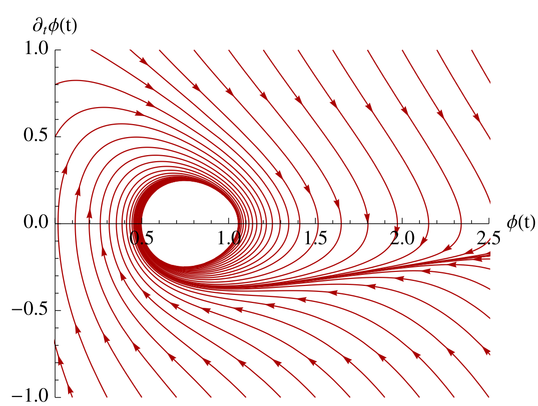

Fig. 3. Phase space evolution in the ϕ–$\dot\phi$ space for $\alpha = 1$ and $\Lambda = 1$. Note the presence of a limit cycle.

It should be stressed that, in our model, if $\Lambda < 0$ there would be no exit from inflation with the standard reheating phase. As argued in [61] the presence of matter field could change the sign of Λ depending on the number of Dirac, scalar and vector fields. We hope to address this issue in a future investigation.

4. Oscillatory phase after inflation

After the end of inflation, the inflaton field ϕ begins to oscillate around the minimum $\phi_{\min}$ of $V_+(\phi)$. To study this phase, we can do the following approximation

$$V_+(\phi) \sim V(\phi) = \frac{a}{2}\left[(\phi - \phi_{\min})^2 - b\right] \tag{15}$$

where $\phi_{\min}(\alpha, \Lambda)$, $a(\alpha, \Lambda) = V_+''(\phi_{\min})$ and $b(\alpha, \Lambda) = -2\,V_+(\phi_{\min})/V_+''(\phi_{\min})$ depend on the values of α and Λ, and are given by the following expressions

$$\phi_{\min}(\alpha, \Lambda) = \frac{\sqrt{\frac{3}{2}}\left(3\alpha^3\left(\alpha^2 - 4\right) - 32\alpha\Lambda + 4\left(\alpha^2 - 6\right)|\alpha|^3\right)}{6\alpha\left(\alpha^2 - 8\right)\left(\alpha^2 + 2\right) - 64\alpha\Lambda + 8\left(\alpha^2 - 9\right)|\alpha|^3}$$

$$a(\alpha, \Lambda) = \frac{48 + 18\,\alpha^2 - 3\,\alpha^4 + 32\,\Lambda - 4\,\alpha^3\,|\alpha| + 36\,\alpha\,|\alpha|}{24}$$

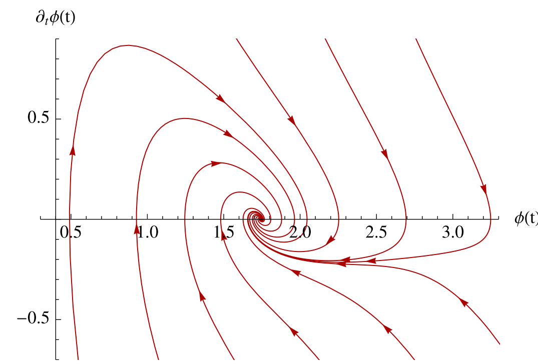

Fig. 4. Phase space evolution in the ϕ–$\dot\phi$ space for $\alpha = 1.5$ and $\Lambda = 10$. This case arises when $V_+(\phi_{\min}) > 0$.

$$\begin{aligned} b(\alpha, \Lambda) = &\frac{8\alpha\left(15\alpha^4 - 3\alpha^6 - 96\Lambda + 8\alpha^2(15 + 4\Lambda)\right)|\alpha|}{\frac{8}{3}\left(48 + 18\alpha^2 - 3\alpha^4 + 32\Lambda - 4\alpha\left(\alpha^2 - 9\right)|\alpha|\right)^2} \\ &+ \frac{-25\alpha^8 + 132\alpha^6 - 384\alpha^2\Lambda}{\frac{8}{3}\left(48 + 18\alpha^2 - 3\alpha^4 + 32\Lambda - 4\alpha\left(\alpha^2 - 9\right)|\alpha|\right)^2} \\ &+ \frac{48\alpha^4(21 + 4\Lambda) - 1024\Lambda(3 + \Lambda)}{\frac{8}{3}\left(48 + 18\alpha^2 - 3\alpha^4 + 32\Lambda - 4\alpha\left(\alpha^2 - 9\right)|\alpha|\right)^2} \end{aligned}$$

The time evolution of the field $\phi(t)$ is given by the Friedmann equation

$$\ddot\phi(t) + 3\,H(t)\,\dot\phi(t) + V'(\phi(t)) = 0 \tag{16}$$

where $H(t)$ is the Hubble constant

$$\begin{aligned} 3\,H(t) &= 3\left[\frac{1}{3}\left(\frac{1}{2}\dot\phi(t)^2 + V(\phi(t))\right)\right]^{1/2} \\ &= \sqrt{\frac{3}{2}}\,\left[\dot\phi(t)^2 + a\,(\phi(t) - \phi_{\min})^{\,2} - \mathrm{ab}\right]^{1/2} \end{aligned} \tag{17}$$

Putting $x(t) = \sqrt{a}\,(\phi(t) - \phi_{\min})$ and $y(t) = \dot\phi(t)$, equation (16) reads

$$\begin{cases} \dot y = -\left[\frac{3}{2}\left(y^2 + x^2 - \mathrm{ab}\right)\right]^{\frac{1}{2}} y - \sqrt{a}\,x \\ \dot x = \sqrt{a}\,y \end{cases} \tag{18}$$

The long time behavior of this dynamical system is mainly determined by the sign of $ab = -2\,V(\phi_{\min})$. If $ab \le 0$ (i.e. $V(\phi_{\min}) \ge 0$) then the point $(0, 0)$, that is the minimum point $(\phi_{\min}, V(\phi_{\min}))$, is an attractive node. If $ab > 0$ (i.e. $V(\phi_{\min}) < 0$) a limit cycle $(y^2 + x^2 = ab)$ appears. Note that, in this sense, $ab = 0$ is an Hopf bifurcation point. A numerical study of the original Friedman equation (16) has confirmed our analytical findings. In particular we have obtained the phase diagrams relative to the cases $V_+(\phi_{\min}) < 0$ (our case, Fig. 3) and $V_+(\phi_{\min}) > 0$ (Fig. 4). The dynamical system analysis can be useful to determine the scale factor $a(t)$ after the inflation era, that is related to the Hubble constant by $H(t) = \frac{\dot a(t)}{a(t)}$. In particular, using the polar coordinates and then performing the method of averaging, we obtain

$$a(t) = \begin{cases} \left[\sin\left(\sqrt{\frac{3}{8}\,|ab|}\;t\right)\right]^{2/3} & ab > 0 \\ t^{2/3} & ab = 0 \\ \left[\sinh\left(\sqrt{\frac{3}{8}\,|ab|}\;t\right)\right]^{2/3} & ab < 0 \end{cases} \tag{19}$$

The scale factor $a(t)$, in the case $ab = 0$ ($V(\phi_{\min}) = 0$, Starobinsky model), describes the usual matter dominated era, while the solutions with $V(\phi_{\min}) \neq 0$ are compatible with a matter dominated era only at the beginning of the oscillatory phase. On the other hand a consistent treatment of the following reheating phase must include the contribution of the matter fields, an investigation which is beyond the scope of this work.

5. Conclusions

In this work we have extended the idea of [40] by including in the renormalized flow the presence of the additional relevant direction associated to the R^2 operator. We have approximated the flow of this operator around the NGFP with its linear expression, an approximation which should capture the essential qualitative features of the flow and allow an analytical treatment of the resulting non-linear $f(R)$ Lagrangian.

The most important point of our investigation is that our inflation model should significantly differ from the well known Starobinsky model because it predicts a tensor-to-scalar ratio which is significantly higher, and an inflationary dynamics which is characterized by a limit-cycle behavior at the inflation exit. More important, our predictions are in agreement with the latest *Planck* data which put important constraints on our model.

An important limitation of this study is the simple tensorial structure of the effective Lagrangian which assumes a functional dependence of the $f(R)$ type. Quadratic operators like $R_{\mu\nu}R^{\mu\nu}$ are also associated to relevant directions around the NGFP and in principle their presence could dramatically change the inflation dynamics and the generation of the primordial spectrum of fluctuations. On the other hand, our approach can be easily extended in order to include the contribution of these additional operators, whose critical exponents have been calculated in [13]. We hope to address this issue in a future work.

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