Models for the Active Optics system of the ASTRI SST-2M prototype proposed for the Cherenkov Telescope Array

Daniele Gardiol*[a], Enrico Giro[b], Luigi Lessio[b], Davide Loreggia[a], Gabriele Rodeghiero[b], Federico Russo[a], for the ASTRI collaboration[c] and the CTA Consortium[d]

[a]INAF-Osservatorio Astronomico di Torino – Strada Osservatorio 20, I-10025 Pino Torinese - Italy;
[b]INAF-Osservatorio Astronomico di Padova – Vicolo Osservatorio 5, I-35122 Padova – Italy;
[c]http://www.brera.inaf.it/astri; [d]http://www.cta-observatory.org/

ABSTRACT

ASTRI SST-2M is an end-to-end prototype of Small Size class of Telescope proposed for the Cherenkov Telescope Array (CTA). Currently under completion at the Serra La Nave observing station (Mt. Etna, Catania, Italy), the ASTRI SST-2M telescope is the first imaging dual-mirror telescope ever realized for Cherenkov telescopes. A mini-array of nine such telescopes will form the ASTRI mini-array proposed as a precursor and initial seed of CTA to be installed at the final CTA southern site. ASTRI SST-2M is equipped with an active optics system, controlling both the segmented primary mirror and the monolithic secondary mirror, which allows optical re-alignment during telescope slew. We describe the hardware and software solution that have been implemented for optics control and the models we developed for the system.

Keywords: Imaging Atmospheric Cherenkov Telescope, CTA, ASTRI, gamma-rays, active optics

1. INTRODUCTION

The dual-mirror ASTRI (Astrofisica con Specchi a Tecnologia Replicante Italiana) Small Size Telescope prototype, named ASTRI SST-2M, is a wide field of view Schwarzschild-Couder Cherenkov telescope developed within the Cherenkov Telescope Array (CTA) framework[1]. An end-to-end telescope prototype is currently under completion, that will apply a dual mirror configuration to an Imaging Atmospheric Cherenkov Telescope[2]. It is equipped with a 4.3 meter segmented primary, composed of eighteen hexagonal pieces, and a 1.8 meter monolithic secondary[3]. A photograph of the prototype is shown in Figure 1 – left panel. Optical first light of the telescope was performed end of May 2015 using an optical camera with a field of view of approximately 1° and rough alignement of the optics (Figure 1 – right bottom panel). The mirrors of the prototype are actively controlled to allow for fine alignment during telescope operations. In paragraph 2 we give a brief description of the Active Optics System, while in paragraph 3 we discuss in detail the models. In paragraph 4 we summarise our conclusions.

2. THE ASTRI SST-2M ACTIVE OPTICS SYSTEM

The Active Optics system of the ASTRI SST-2M prototype is described in a dedicated paper[4]. Here we just recall it briefly for the reader. The Primary Mirror (M1) is segmented into 18 hexagonal panels distributed on three concentric coronae (see Figure 3 – left panel). The optical profile of the mirror follows an aspheric function so that the segments belonging to each corona share the same sag. The segment support triangles have been designed to support and actively

*gardiol@oato.inaf.it; phone +39 011 810 1986; fax +39 011 810 1930

Integrated Modeling of Complex Optomechanical Systems II, edited by Marco Riva,
Proc. of SPIE Vol. 10012, 1001207 · © 2016 SPIE · CCC code: 0277-786X/16/$18
doi: 10.1117/12.2214548

tilt the primary mirror segments in order to maintain the optics alignment and preserve the telescope Point Spread Function (PSF) specification. Each segment of the primary mirror is held by three supports in triangle configuration. Two of them are active so that it is possible to tune the two tilts of the mirror, moving the optical axis in the desired direction. The total stroke of each actuator is more than 10 mm, with an axial resolution better than 5 µm and an accuracy of about 3 µm. The Secondary Mirror (M2) is a 1.8 meters monolithic mirror. The support and driving system of M2 is realised by three active actuators allowing tilts and focus correction. The M2 system is designed to have a total stroke of 15 mm with an axial resolution of 10 µm and accuracy of ± 20 µm.

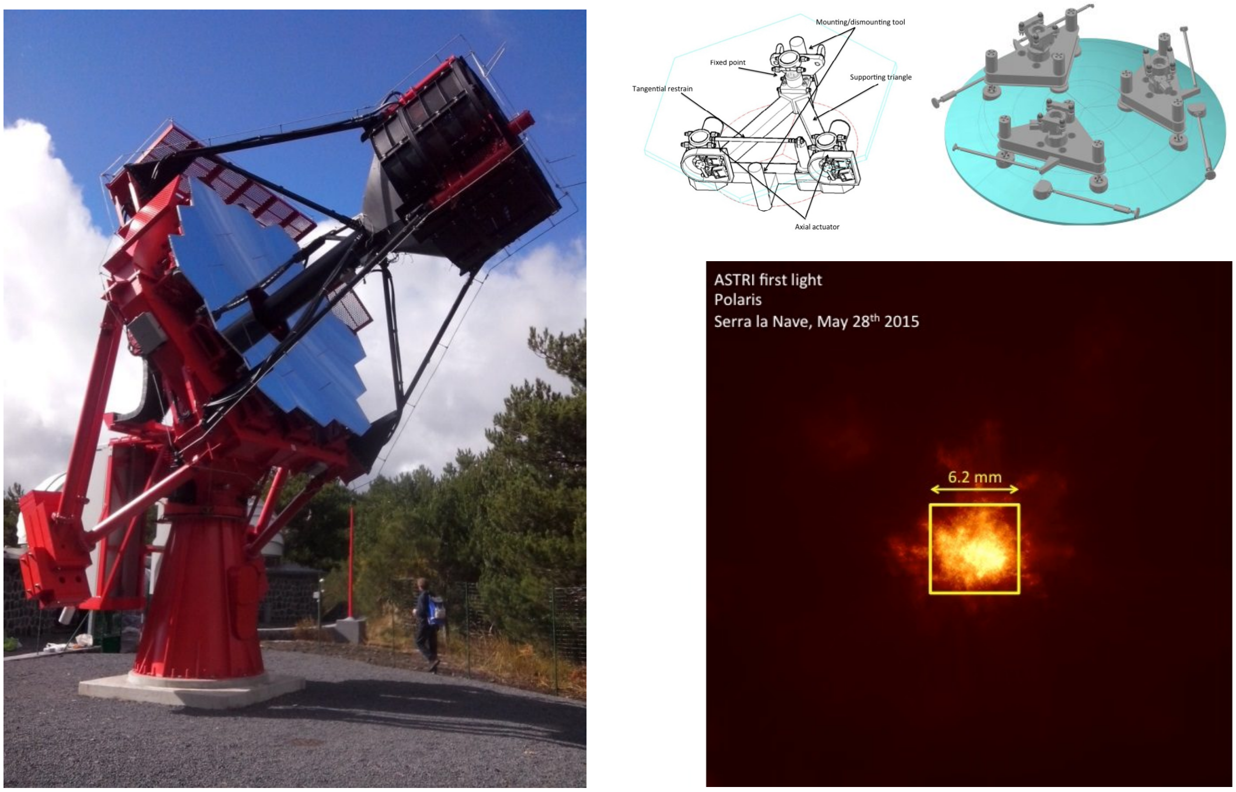

Figure 1. (left) The ASTRI SST-2M telescope prototype in Serra La Nave; (upper right) Sketch of the M1 and M2 support; (bottom right) The telescope optical first light.

3. KINEMATIC MODEL

As seen in the previous paragraph, each of the eighteen segments of the primary mirror is held in place by three supports. One support is fixed while the remaining two are active, allowing the mirror segment to be tilted in order to maintain proper alignment during telescope operations. Specifically designed triangles hold the supports connecting the mirror segments to the M1 dish through spacers that provide the gross position of the mirror. The mirror segment control is performed *via* axial movement of two piston rods by means of eccentric shafts. In the M2 system all three supports are active. Therefore, the kinematic model for M1 can be expressed in terms of eighteen functions $f\colon \boldsymbol{R}^2 \rightarrow \boldsymbol{R}^2$ (one for each segment) that provide the image position *(x,y)* onto the focal plane given the step number of the two motors *(p,q)* driving each the active support. In a similar way a function $g\colon \boldsymbol{R}^3 \rightarrow \boldsymbol{R}^3$ will describe the kinematic model for M2. Once determined, this functions can be used for two main purposes:

- to perform analysis and characterisation of the mirrors behaviour against a simulated motion of each segment actuator, providing a link between the single actuator control and the Cherenkov image shift onto the focal plane and

- to be the baseline for the algorithmic coding of the primary mirror segments and the secondary mirror control within the Active Mirror Control software

The kinematic model is logically divided into three steps:

1) an Actuator model for each mirror axial support that feeds
2) a Geometrical model of the mirrors accounting for nominal and modified positions and orientations of mirrors used by
3) an Optical model that computes the image displacements onto the focal plane.

The kinematic model is implemented in IDL (Interactive Data Language) and Zemax.

3.1 Actuator model

Each axial actuator generates a vertical movement applied to the segment by an eccentric mechanism that moves a piston rod acting on the back of the mirror through a suitable interface. The eccentric mechanism is responsible for a non-linearity response of the piston rod movement vs. the input motor step value. Laboratory measurements performed on a complete prototype of the segment show (see Figure 2) that it is possible to describe the non-linearity with a fifth order polynomial in the form

$$t = \sum_{n=0}^{5} a_n p^n$$

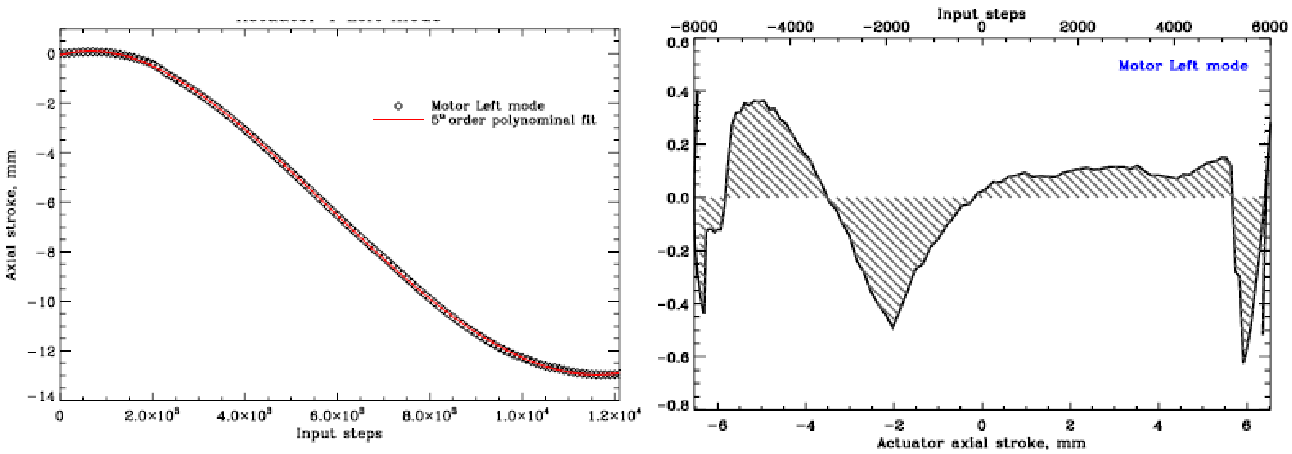

Figure 2. Kinematic model results for axial actuator (Actuator model) derived from laboratory measurements. Left panel: laboratory measurements and polynomial fit; right panel: fit residuals expressed in microns

3.2 Geometrical model

From an analytical point of view the triangle holding the mirror segment can be characterised by three points in space

$$\begin{cases} \boldsymbol{P}_{1,R} = (x_1, y_1, z_1) \\ \boldsymbol{P}_{2,R} = (x_2, y_2, z_2) \\ \boldsymbol{P}_{3,R} = (x_3, y_3, z_3) \end{cases}$$

that define the reference plane π of equation $\pi: a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ where

$$a = \begin{vmatrix} y_2 - y_1 & z_2 - z_1 \\ y_3 - y_1 & z_3 - z_1 \end{vmatrix}; \; b = -\begin{vmatrix} x_2 - x_1 & z_2 - z_1 \\ x_3 - x_1 & z_3 - z_1 \end{vmatrix}; \; c = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}$$

while the versor normal to plane π is given by

$$\boldsymbol{n} = \frac{\nabla\pi}{\|\nabla\pi\|} = \pm\frac{a\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k}}{\sqrt{a^2 + b^2 + c^2}}$$

where the sign is chosen in order to orientate the versor $\boldsymbol{k}$ in the desired direction. Note that the orientation of versor $\vec{n}$ (and of plane π) is different for each segment. Let's assume that support #1 is fixed while supports #2 and #3 are active. The actuators of each segment will therefore move in a direction perpendicular to the reference plane π along two straight lines that can be easily written in parametric form as

$$\begin{cases} \boldsymbol{P}_2 = \boldsymbol{P}_{2,R} + t_2\boldsymbol{n} \\ \boldsymbol{P}_3 = \boldsymbol{P}_{3,R} + t_3\boldsymbol{n} \end{cases}$$

being t_2 and t_3 parameters that express the position of the two actuators. As a consequence of actuator movement the mirror will rotate around a fixed point given by

$$\boldsymbol{P}_1 = \boldsymbol{P}_{1,R} + l\boldsymbol{n}$$

where l is fixed. A generic position of the actuators is therefore given by the couple (t_2, t_3). This couple is such that $t_2, t_3 \in [l - \Delta l, l + \Delta l]$, where $2\Delta l$ is the total range of the actuators . If we redefine now the three points providing the segment position as

$$\begin{cases} \boldsymbol{P}_1 = \boldsymbol{P}_{1,R} + l\boldsymbol{n} \\ \boldsymbol{P}_2 = \boldsymbol{P}_{2,R} + (l + t_2)\boldsymbol{n} \\ \boldsymbol{P}_3 = \boldsymbol{P}_{3,R} + (l + t_3)\boldsymbol{n} \end{cases}$$

then $(t_2, t_3) \to (t_2 - l, t_3 - l)$, so that $t_2, t_3 \in [-\Delta l, \Delta l]$ in the ideal situation where the nominal position of the segment is attained for the actuator position $(t_2, t_3) \to (0{,}0)$. A generic position (t_2, t_3) of the mirror can also be expressed, within the total range, as the composition of a *concordant* motion and a *discordant* motion (u, v), according to equations

$$\begin{cases} t_2 = u + v \\ t_3 = u - v \end{cases}; \quad \begin{cases} u = (t_2 + t_3)/2 \\ v = (t_2 - t_3)/2 \end{cases}$$

If we now define the position of the mirror (considered as a rigid body) using a right-handed Cartesian reference system (x', y', z') (see Figure 3), it can be seen that a movement of type u is a rotation about x' while a movement of type v is a rotation about y'. Let's now suppose that we start the movement from a position of the segment defined by the couple $(t_{2,i}, t_{3,i})$ and move to a new position $(t_{2,f}, t_{3,f})$. The versors defining the reference planes π_i and π_f are

$$\boldsymbol{n}_{i,f} \equiv \boldsymbol{k}_{i,f} = \frac{\nabla\pi_{i,f}}{\|\nabla\pi_{i,f}\|} = \pm\frac{a_{i,f}\boldsymbol{i} + b_{i,f}\boldsymbol{j} + c_{i,f}\boldsymbol{k}}{\sqrt{a_{i,f}^2 + b_{i,f}^2 + c_{i,f}^2}}$$

The other two versors of the Cartesian frames can be computed as

$$\boldsymbol{j}_{i,f} = \frac{\boldsymbol{P}_{i,f}^{BAR} - \boldsymbol{P}_{1;i,f}}{\left\|\boldsymbol{P}_{i,f}^{BAR} - \boldsymbol{P}_{1;i,f}\right\|} \quad ; \quad \boldsymbol{i}_{i,f} = \boldsymbol{j}_{i,f} \times \boldsymbol{k}_{i,f} \;\; ; \;\; \boldsymbol{P}_{i,f}^{BAR} = (\boldsymbol{P}_{1;i,f} + \boldsymbol{P}_{2;i,f} + \boldsymbol{P}_{3;i,f})/3$$

The rotation matrix of the movement is defined as

$$R = \begin{bmatrix} \boldsymbol{i}_f \cdot \boldsymbol{i}_i & \boldsymbol{j}_f \cdot \boldsymbol{i}_i & \boldsymbol{k}_f \cdot \boldsymbol{i}_i \\ \boldsymbol{i}_f \cdot \boldsymbol{j}_i & \boldsymbol{j}_f \cdot \boldsymbol{j}_i & \boldsymbol{k}_f \cdot \boldsymbol{j}_i \\ \boldsymbol{i}_f \cdot \boldsymbol{k}_i & \boldsymbol{j}_f \cdot \boldsymbol{k}_i & \boldsymbol{k}_f \cdot \boldsymbol{k}_i \end{bmatrix}$$

while the orientations of the initial and final position with respect to the global reference frame will be given by

$$R_i = \begin{bmatrix} \boldsymbol{i}_i \cdot \boldsymbol{i} & \boldsymbol{j}_i \cdot \boldsymbol{i} & \boldsymbol{k}_i \cdot \boldsymbol{i} \\ \boldsymbol{i}_i \cdot \boldsymbol{j} & \boldsymbol{j}_i \cdot \boldsymbol{j} & \boldsymbol{k}_i \cdot \boldsymbol{j} \\ \boldsymbol{i}_i \cdot \boldsymbol{k} & \boldsymbol{j}_i \cdot \boldsymbol{k} & \boldsymbol{k}_i \cdot \boldsymbol{k} \end{bmatrix}; \quad R_f = \begin{bmatrix} \boldsymbol{i}_f \cdot \boldsymbol{i} & \boldsymbol{j}_f \cdot \boldsymbol{i} & \boldsymbol{k}_f \cdot \boldsymbol{i} \\ \boldsymbol{i}_f \cdot \boldsymbol{j} & \boldsymbol{j}_f \cdot \boldsymbol{j} & \boldsymbol{k}_f \cdot \boldsymbol{j} \\ \boldsymbol{i}_f \cdot \boldsymbol{k} & \boldsymbol{j}_f \cdot \boldsymbol{k} & \boldsymbol{k}_f \cdot \boldsymbol{k} \end{bmatrix}$$

Using the Euler representation, where the rotations are performed in the current reference frame, it is possible to define three subsequent rotations (α, β, γ)about respectively axes (x, y, z) as

$$R(\alpha, \beta, \gamma) \equiv R_x(\alpha) R_y(\beta) R_z(\gamma) = \begin{bmatrix} cos\beta cos\gamma & -cos\beta sin\gamma & sin\beta \\ sin\alpha sin\beta cos\gamma + cos\alpha sin\gamma & -sin\alpha sin\beta sin\gamma + cos\alpha cos\gamma & -sin\alpha cos\beta \\ -cos\alpha sin\beta cos\gamma + sin\alpha sin\gamma & cos\alpha sin\beta sin\gamma + sin\alpha cos\gamma & cos\alpha cos\beta \end{bmatrix}$$

The value of the angles (α, β, γ) can be computed starting from the rotation matrix as

$$\begin{cases} \alpha = -atg(r_{23}, r_{33}) \\ \beta = atg\left(r_{13}, \sqrt{r_{11}^2 + r_{12}^2}\right) = atg\left(r_{13}, \sqrt{r_{23}^2 + r_{33}^2}\right) \\ \gamma = -atg(r_{12}, r_{11}) \end{cases}$$

where $atg(\blacksquare, \blacksquare)$ is the four-quadrants inverse tangent function.

Using RPY (Roll/Pitch/Yaw) representation instead, then the rotation angles are all defined with respect to the original reference frame, and the compound rotation matrix can be written as

$$R_{RPY}(\alpha, \beta, \gamma) \equiv R_z(\gamma) R_y(\beta) R_x(\alpha) = \begin{bmatrix} cos\beta cos\gamma & sin\alpha sin\beta cos\gamma - cos\alpha sin\gamma & -cos\alpha cos\beta cos\gamma + sin\alpha sin\gamma \\ cos\beta sin\gamma & sin\alpha sin\beta sin\gamma - cos\alpha cos\gamma & cos\alpha sin\beta sin\gamma - sin\alpha cos\gamma \\ -sin\beta & sin\alpha cos\beta & cos\alpha cos\beta \end{bmatrix}$$

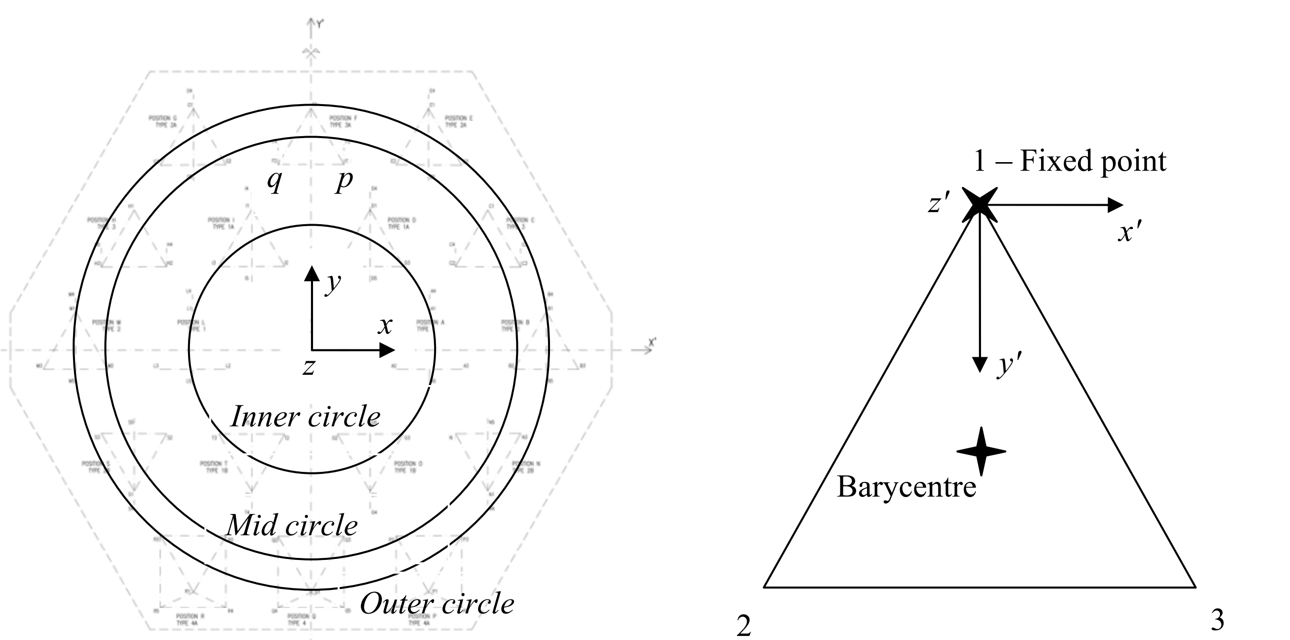

Figure 3. Geometrical configuration of all the M1 actuators (left) and of a single segment support (right)

and

$$\begin{cases} \alpha = atg(r_{32}, r_{33}) \\ \beta = -atg\left(r_{31}, \sqrt{r_{11}^2 + r_{21}^2}\right) = atg\left(r_{31}, \sqrt{r_{32}^2 + r_{33}^2}\right) \\ \gamma = atg(r_{21}, r_{11}) \end{cases}$$

Therefore once known the coordinates of each segment actuator $(\boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_3)$ in a global reference system (see Figure 3 left panel that shows the position of the eighteen triangle onto the primary mirror support) it is possible to compute, using the geometrical model, the six parameters $(x, y, z, \alpha, \beta, \gamma)$ (again for each segment) to be used as an input for the optical model. The same applies, in a simpler way, to M2.

3.3 Optical model

The optical model is used to take into account the optical properties of the telescope. It is based on the ZEMAX optical design of the telescope. The three different groups of six hexagonal segments (from the point of view of the mirror sag) are properly arranged in order to build the whole primary mirror exploiting the non-sequential mode offered by the ray tracing software. Each M1 segment and the secondary mirror can therefore be moved independently according to the results of the geometrical model. Ray tracing of selected rays is performed to mimic the image position onto the telescope focal plane. Figure 4 and Figure 5 show the cumulative results of the geometrical and optical models applied to the three circles of M1 segments respectively for *u* type and *v* type movements. In both figures the upper row shows the *x* position of the image onto the focal plane, while the lower row shows the *y* position. Note the different scales on the ordinates and the non-linearity due to geometry and optics. Finally Figure 6 shows the result of the whole kinematic model applied to one single M1 segment, *i.e.* the computed values for the *x* position (left) and *y* position (right) of the image onto the focal plane as a function of the motor step value couple (p_2,p_3). The cumulative non-linearity behaviour is largely dominated by the effect due to the axial actuators. Figure 7 shows the computed values for *x* position (left) and *y* position (right) of the image onto the focal plane as a function of the motor step values, imposing no defocus condition and assuming linearity of the motors' gain, as we do not have a laboratory prototype of the support system to make measurements.

4. CONCLUSIONS

We have described in this paper the Active Optics system of the ASTRI SST-2M telescope prototype. Active positioning of the eighteen segments of M1 and of the monolithic M2 is performed by means of axial actuators driven by stepper motors, two for each M1 segment, the third being fixed, and three for M2, allowing for telescope focusing. We developed and implemented a completely custom kinematic model, that can be used to predict the system performance, divided into three steps: an axial actuator model, a geometrical model, an optical model. The actuator model is derived from laboratory measurements on an available segment support prototype and can be analytically described by the coefficients of a fifth order polynomial. The geometrical model combines all the thirtynine actuators in eighteen couples geometrically disposed onto the primary mirror support plus the three actuators holding the secondary mirror. The optical model converts the output of the geometrical model into the desired final quantities, *i.e.* the position of an image onto the focal plane assuming a given system configuration. This has been used to make prediction on the telescope performance.

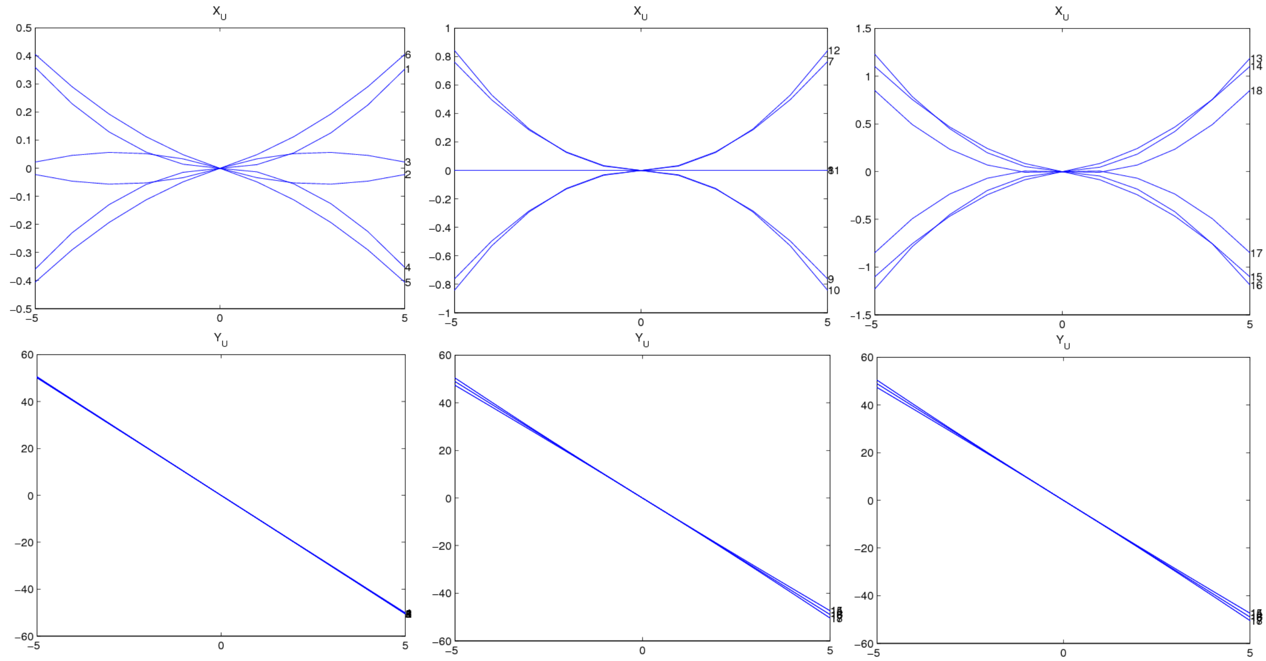

Figure 4. Results of the Geometric and Optical models on M1 for *u* type movements.
From left to right: inner, median, outer circle. Upper row: *x* FoV coordinate; lower row: y Fov coordinate

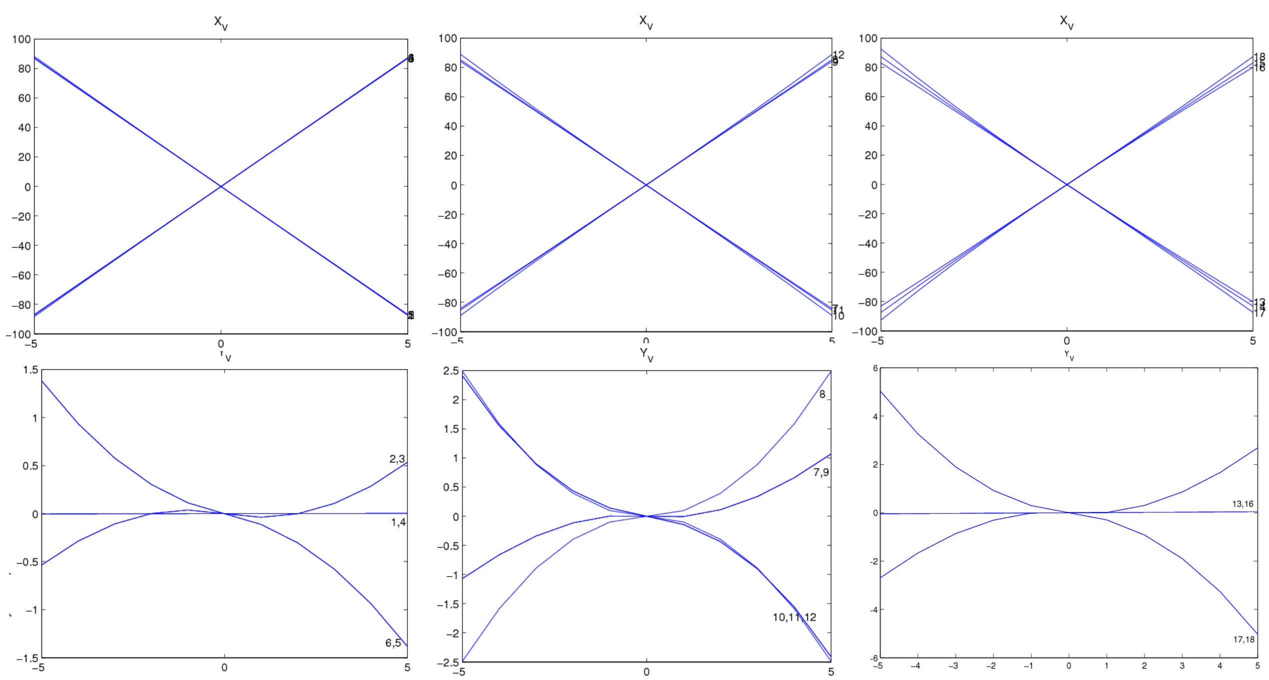

Figure 5. Results of the Geometric and Optical models on M1 for *v* type movements.
From left to right: inner, median, outer circle. Upper row: *x* FoV coordinate; lower row: y Fov coordinate

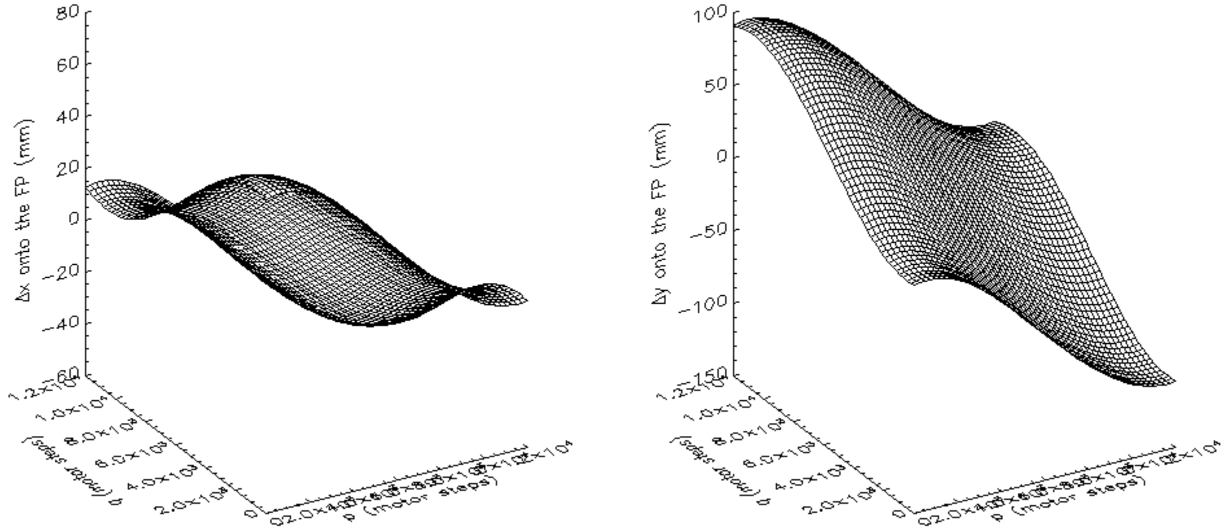

Figure 6 - Displacement of the image onto the focal plane as computed by the kinematic model for one M1 segment

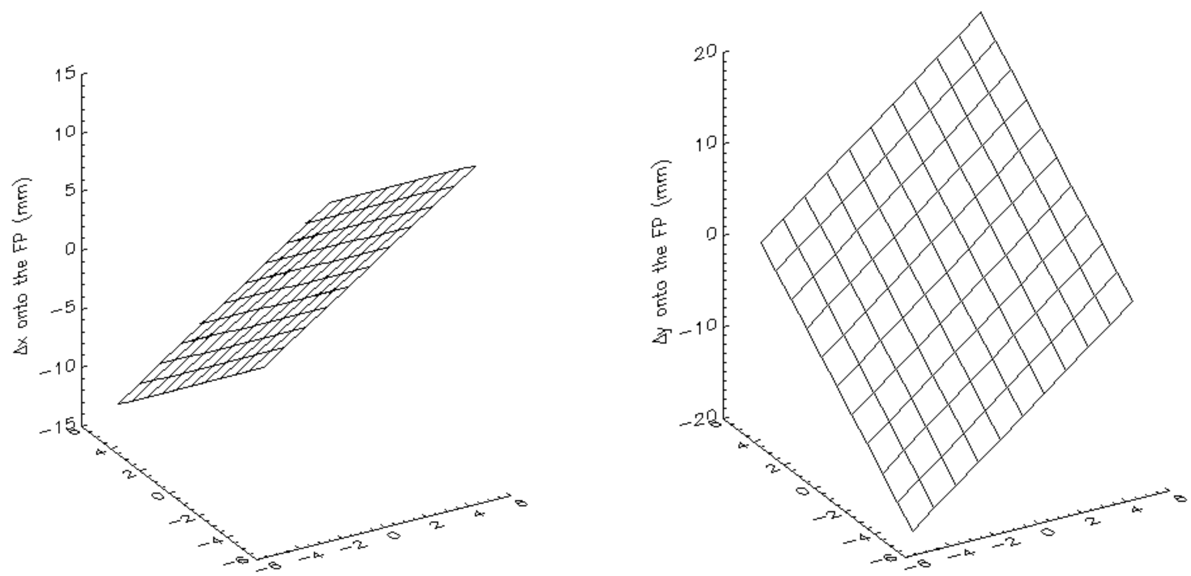

Figure 7 - Displacement of the image onto the focal plane as computed by the model for M2, assuming no defocus

ACKNOWLEDGMENTS

This work was partially supported by the ASTRI “Flagship Project" financed by the Italian Ministry of Education, University, and Research (MIUR) and led by the Italian National Institute of Astrophysics (INAF). We also acknowledge partial support by the MIUR Bando PRIN 2009. We gratefully acknowledge support from the agencies and organizations listed in this page: http://www.cta-observatory.org/?q=node/22.

REFERENCES

[1] Acharya, B. S., et al., "Introducing the CTA concept", Astroparticle Physics 43, 3-18, (2013).
[2] Pareschi, G., et al., "The dual-mirror Small Size Telescope for the Cherenkov Telescope Array", Procs. 33rd ICRC, arXiv:1307.4962, (2013)
[3] Canestrari, R., et al., "The ASTRI SST-2M prototype for the Cherenkov Telescope Array: manufacturing of the structure and of the mirrors", Proc. SPIE 9145, (2014).
[4] Gardiol, D., et al, "Tests characterization and alignment for the optics of the ASTRI SST-2M telescope prototype for the Cherenkov Telescope Array", Proc. SPIE 9151, (2014).