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Inversion of sparse photometric data of asteroids using triaxial ellipsoid shape models and a Lommel-Seeliger scattering law

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Abstract

The inversion of sparse photometric data of asteroids to derive from them information about the spin and shape properties of the objects is a hot topic in the era of the Gaia space mission. We have used a rigorous analytical treatment of the Lommel-Seeliger light-scattering law computed for the particular case of bodies having the shapes of ideal triaxial ellipsoids, and we have implemented this in the software developed for the treatment of Gaia photometric data for asteroids. In a set of numerical simulations, the performances of the photometry inversion code improve significantly with respect to the case in which purely geometric scattering is taken into account. When applied to real photometric data of asteroids obtained in the past by the Hipparcos satellite, however, we do not see any relevant improvement of the performances, due to the poor accuracy of these measurements. These results suggest that the role played by the light-scattering properties of asteroid surfaces is indeed relevant. On the other hand, any refined treatment of light-scattering effects cannot improve the reliability of photometric inversion when the quantity and quality of available data are much worse than what we expect to obtain from Gaia.

Keywords: Asteroids, Photometry, Spin, Shape, Light Scattering

1. Introduction

The inversion of sparse photometric data of asteroids, aiming at deriving from them extensive information about the spin properties and the overall shapes of these objects, is currently a hot topic in asteroid science. The reason

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is that we are now in the beginning of a new era characterized by extensive sky surveys from the ground and from space. In this paper, we are mostly interested in Gaia, the cornerstone mission of the European Space Agency (ESA) successfully launched on December 19, 2013. After reaching its final orbital configuration in $L2$ and carrying out the first phase of verification for the functioning of the on-board instruments, Gaia is now in its operational phase, which is planned to last for at least five years. In addition to the observation of stellar and extragalactic sources, during its continuous scanning of the whole celestial sphere, Gaia will also detect and measure the position, the apparent luminosity and the reflectance spectrum at visible wavelengths of all Solar system objects brighter than magnitude 20.7 (with the exception of the brightest planets) that will repeatedly cross its field of view[1]. The expect number of Solar system objects that will be detected is of the order of $300,000$, including primarily main-belt asteroids, as well as lesser numbers of near-Earth objects, Jupiter Trojans, and planetary satellites, a few comets and a handful of transneptunian objects [2]. In the case of main-belt asteroids, which will constitute the major part of the Gaia data set of Solar system objects, each single body will be observed many times during the operational lifetime of Gaia, the number of recorded transits being expected to be of the order of about 70 per object. These data will represent a sort of "forbidden dream" for asteroid remote observers. Gaia data will make it possible to obtain an improvement of the order of two orders of magnitude in the accuracy of the orbital elements, exploiting the unprecedented astrometric performances of the mission [3]. In addition, it will be possible to derive new and accurate masses for about 100 objects, sizes for about $1,000$ objects, spin periods, spin-axis orientations, and overall shapes for tens of thousands of objects, and a new taxonomic classification for of the order of $100,000$ asteroids [4, 5, 6, 7].

In this paper, we focus our attention on the topic of photometric inversion of Gaia asteroid data. This is a difficult task, taking into account that Gaia will not obtain lightcurves, but only sparse photometric data. The very high number of observed objects, each one being observed about 70 times, will produce a huge data base. This will make it impossible, at least in the framework of the Gaia Data Processing and Analysis Consortium (DPAC), which is responsible of the production of the results that will be published in the Gaia catalogue at the end of the mission, to use the most refined methods of photometric inversion that have been developed by several authors in recent years. The reason is that we cannot imagine to use numerical methods which are so detailed in the treatment of the possible asteroid shapes to require significant CPU time for each single object. The DPAC treatment of asteroid photometric data is based on the assumption that the shapes of the objects will be approximated by means of ideal triaxial ellipsoids. A genetic algorithm will be used to find the best combination of spin period, spin axis orientation, triaxial shape, and linear phase-magnitude relation (see below), which produces the best fit for a given set of data. This approach reduces to a minimum the requirements in terms of CPU time, because the computation of the visible and illuminated area of a triaxial ellipsoid asteroid observed in any possible observational circumstance can be computed quickly using analytical formulas. The effectiveness and the limitations of this approach have been already discussed in a number of published papers, including Torppa et al. [8, and references therein], in which it was shown that a triaxial ellipsoid approximation tends to be quite reasonable in a large variety of realistic cases.

In what follows, we refer mostly to Cellino et al. [9], in which a very exhaustive test of the genetic algorithm for photometric inversion was presented. In particular, it was shown that the inversion algorithm gives overall acceptable solutions when applied to simulated data of asteroids having a large variety of simulated shapes, spin-axis orientations, and orbits, as well as a variety of simulated photometric errors and numbers of available data [9, 7]. More recently, these results have also been essentially confirmed by Santana-Ros et al. [10]. The assessment of the performances of adopted inversion algorithm presented in Cellino et al. [9], included also an extensive analysis of the whole data-set of asteroid photometric data obtained by the Hipparcos mission. This sample of data obtained from the space for 48 real asteroids, is the closest thing we have at disposal to evaluate the expected performances of the inversion algorithm when applied to the forthcoming Gaia data. There is, however, an important difference. Hipparcos data are much worse, both in terms of quantity and quality of the available measurements, with respect to what we expect to obtain from Gaia. As discussed in Cellino et al. [9], among the 48 objects included in the Hipparcos catalogue, a large number of them have only a few observations, and for nearly all the observed objects the error bars of the photometric measurements are so big as to make *a priori* any attempt of inversion particularly difficult. In spite of these difficulties, it was shown that the inversion algorithm was able to give decent results, in terms of ability to derive the correct spin period, in about 50% of the cases. In terms of spin-axis determination the results were similar, although this can be more difficult to prove, because in several cases the determination of the pole given in available catalogs is not exempt from uncertainties.

The reason why we have done a new assessment of the performances of the inversion algorithm, that we present in this paper, is that so far all the attempts of inversion of both real and simulated photometric data were carried out without making any realistic assumption about the light-scattering properties of asteroid surfaces. In other terms, the computation of the magnitude of an object seen in any given observing circumstance has been done by Cellino et al. [9] and in other papers, by assuming that the luminosity flux that we receive is simply determined by the computed area of the illuminated surface as seen by the observer (geometric scattering assumption: strictly speaking, violating the reciprocity principle of radiative transfer), plus a correction taking into account a simple linear relation for the dependence of the magnitude upon the phase angle[2].

Recently, however, some of us (Muinonen and Lumme 2015, Muinonen et al. 2015) have made rigorous analytical computations of the disk-integrated luminosity of an asteroid having a triaxial ellipsoid shape, and having a surface which scatters the incident sunlight according to the Lommel-Seeliger relation. It is evident that this is an important step forward, and one which is particularly important from the point of view of the inversion of the Gaia photometric data of asteroids, due to the fact that, as explained above, the photometric inversion algorithm developed to treat Gaia asteroid data is based on the assumption of triaxial ellipsoid shapes.

We have therefore performed a new set of inversion tests of both simulated and real photometric data. The results are given in the following Sections.

2. Implementation of the Lommel-Seeliger scattering law

The problem of asteroid photometric inversion consists of being able to determine a set of unknown physical parameters, namely the spin period, the spin-axis direction, the axial ratios, and the slope of a supposedly linear magnitude-phase relation, having at disposal a set of measured magnitudes obtained at different epochs. The idea is that, according to some simple hypotheses (see below), the above-mentioned physical parameters determine the values of magnitude that an object displays at different epochs, corresponding to varying observational circumstances. The problem is therefore to find a set of parameters which minimizes the residuals between the expected magnitude values and the actual observations.

The genetic algorithm of inversion works by considering large numbers of possible solutions, each one consisting of a set of the unknown parameters, limiting progressively the range of possible solutions, until convergence to a unique set of parameters minimizing the residuals is found. We also note that the genetic algorithm is run some tens of times for any given set of observed data, and it is required that the same best-fit solution must be found more than once. Moreover, the best-fit solution, to be accepted, must be better, in terms of residuals, than other possible alternative solutions resulting in different genetic attempts. Here, "alternative solutions" means solutions having rotation period which differ by 0.1 hours or more from the best solution, and/or have longitude of the pole differing by more than 90°. Two solutions are considered equivalent when they have average RMS residuals less than 0.0015 mag. In other words, the program of photometric inversion tries and finds the triaxial ellipsoid shape that gives the smallest residuals with respect to a given set of magnitude measurements. In the case that the processed object has really the shape of an ideal triaxial ellipsoid, the derived solution tends to give residuals very close to zero, as verified in many numerical experiments. When the shape of the object is not an ideal ellipsoid, the residuals increase, but the tests performed so far have shown that an acceptable solution can be often found, even if the solution residuals are found to be not so good.

We have now to better specify which are the "simple hypotheses" mentioned above, namely the relation between the physical parameters and the corresponding magnitudes predicted by the model at any given epoch. Intuitively, it is evident that the larger the area of illuminated surface visible by the observer, the brighter the object. This means that a computation of the extent of the portion of illuminated surface area seen by the observer, according to the observational circumstances, must be computed, corresponding to a purely geometric scattering approach. In principle, however, one has to take into account more realistic properties of light scattering of the surface.

A detailed computation of the luminosity of a triaxial ellipsoid-shaped body scattering sunlight according to the Lommel-Seeliger law is explained in separate papers [11, 12]. For the purposes of the present work, it is sufficient to

[2]The phase angle is the angle between the directions to the Sun and to the observer, as seen from the target body. All asteroids tend to be brighter when observed at smaller phase angles, that is, closer to solar opposition. The observed relation between magnitude and phase angle is known to be about linear over a large interval of phase angles.

note that, if we call $L(\alpha)$ the luminosity of a triaxial ellipsoid asteroid observed at an epoch t when the phase angle is α, the apparent magnitude $m(\alpha) = -2.5 \log L(\alpha) + C$ turns out to be given by the following expression:

$$m(\alpha) = -2.5 \log (1 + k\alpha) + \Psi(t) + C' \tag{1}$$

where $\Psi(t)$ is a function of the integral over the body's surface of an expression involving the angles of incidence and emergence of the light. In turn, this expression depends upon the axial ratios and pole orientation of the triaxial ellipsoid shape, and on the particular observational circumstances at time t. In Equation 1, k is a parameter which describes the change of luminosity for varying phase angle. This parameter, as well as the other unknowns, namely the axial ratios, the spin period and the orientation of the pole, must be determined by a least-squares procedure.

We note that, being given a set of photometric measurements of a given asteroid, the inversion algorithm works in terms of differences between each observed magnitude and a reference magnitude measurement, normally chosen to be simply the first recorded magnitude of the given set of data. Working in terms of magnitude differences, the C' constant in Equation 1 disappears in the computations.

With respect to the computation of the magnitude carried out in previous papers, including Cellino et al. [9], in our present analysis the difference is in the computation of the term $\Psi(t)$, which takes now a form including the role played by the Lommel-Seeliger scattering law. In previous papers, the magnitudes were computed by simply computing the extent of visible and illuminated surface at time t, using a purely geometric scattering approach. Both in the old and the new treatment, however, $\Psi(t)$ is given by an analytical function of a few time-dependent parameters, and can be quickly computed.

3. Results

We have implemented in the genetic algorithm for photometric inversion the new formula (see Eq.1) which, according to the Lommel-Seeliger light scattering model, gives the predicted magnitudes of a triaxial ellipsoid observed at different epochs. We have then run the inversion algorithm, first on a set of numerical simulations, and subsequently on the same set of Hipparcos data considered in Cellino et al. [9].

As for numerical simulations, we have used a set of data produced by one of us using the Runvisual software [7]. This includes 8 simulated asteroids published in the Carbognani et al. [7] paper, plus two additional simulated asteroids, (5) Astraea and (234) Barbara, which were not included in the above-mentioned paper, having been generated later, using the same software. For each simulated object, we used two different shape models, one corresponding to an assumed complex shape, and one corresponding to the ideal triaxial ellipsoid shape that best fits the assumed complex shape of the object. As for the complex shapes, as explained in Carbognani et al. [7], the choice was to use the convex models obtained by Kaasalainen et al. [13], Durech et al. [14], by means of numerical inversion of ground-based lightcurves. Moreover, for each object of our sample we used two sets of simulated photometric data generated for each of the two shape models: one set consisted of photometric data computed according to purely geometric scattering. The other set included the light scattering model adopted by Kaasalainen et al. [15], namely a combination of a 10% of Lambert and 90% of Lommel-Seeliger. Note that this is not identical, but quite similar, to the pure Lommel-Seeliger scattering law. As for the number and the epochs of the simulated observations, they correspond to a simulated Gaia survey, using the software developed at the Nice Observatory for the simulation of observations of asteroids during the nominal lifetime of Gaia operations.

We carried out our photometric inversions using two versions of our genetic algorithm: the one that we used in the past, based on purely geometric scattering, and a brand new version, in which the computation of the magnitudes includes the new analytical treatment of the Lommel-Seeliger law for triaxial ellipsoid shapes. Note that we ran this second version of the inversion algorithm using, for each simulated object, only the set of data including light-scattering effects, whereas when using the older version of the algorithm, we used both sets of data, including also those produced using geometric scattering. Note also that, in the case of the simulated (234) Barbara object, the simulated shape had the property of being spinning about the second axis of inertia, and not around the axis of maximum moment of inertia, the stable configuration. This makes any attempt to invert the simulated Barbara object particularly difficult.

The results are given in Tables 1 and 2. In both Tables, we use the obtained value of the spin period as the main diagnostic parameter to distinguish between correct and wrong photometric inversions. The descriptor "OK" means

Table 1. Results of the inversion using the old algorithm (scattering effects omitted). The simulated data were computed by considering both ideal triaxial shapes and complex shapes, and, for both of them, the magnitude computation was carried out both including scattering effects as well as not including them. N is the number of simulated Gaia observations computed for each object. The P, λ_P and β_P parameters are the chosen values of rotation period and ecliptic longitude and latitude of the pole, respectively, adopted to compute the simulated photometric data. Note that the Barbara simulation refers to an object not spinning around the axis of maximum inertia. For the meaning of "OK" and "BAD" flags used to indicate the results of the inversion attempts, see the text.

Simulation		N	Simulated parameters P	λ_P	β_P	*Geom. Scattering* Elliptical	*Geom. Scattering* Complex	*Scattering* Elliptical	*Scattering* Complex
(3)	Juno	41	7.209531	103.0	27.0	OK	BAD	BAD	BAD
(5)	Astraea	47	16.80061	126.0	40.0	OK	OK	OK[1]	OK[1]
(9)	Metis	68	5.07918	180.0	22.0	OK	OK	BAD	BAD
(192)	Nausikaa	81	13.62170	326.0	33.0	OK	OK	OK[2]	OK[1]
(234)	Barbara	42	26.47434	150.0	-39.4	BAD	OK[2]	OK[2]	OK
(484)	Pittsburghia	98	10.64976	69.0	47.0	OK	OK	OK[2]	OK
(532)	Herculina	57	9.40495	288.0	11.0	OK	BAD[3]	BAD	BAD
(584)	Semiramis	52	5.06893	75.0	-69.0	OK	OK	OK	BAD
(1088)	Mitaka	48	3.03538	115.0	-46.0	OK	OK	OK[1]	BAD
(1270)	Datura	72	3.35810	69.0	76.0	OK	OK	OK[2]	OK

[1]: Pole longitude +180 deg
[2]: Bad pole
[3]: Good pole

that the correct spin period was found in the inversion solution, with an accuracy of at least 0.001 hours. It should be noted that such excellent accuracies are not only a proof of the good performances of the inversion algorithm, but they are also required by the problem. The reason is that the period must be found with such an accuracy as to avoid that, over a time interval of 5 years, corresponding to the expected operational lifetime of Gaia, a poor determination of the period produces unacceptable errors in the computation of the rotation phase, that can make it impossible to find a correct solution. Conversely, "BAD" in Tables 1 and 2 indicates either cases when a solution for the spin axis is found, but it is wrong (in these cases the solution residuals tend also to be quite large), or cases in which a unique spin period solution is not found by the inversion program. Some notes in the Tables identify cases in which the pole solution could be more or less acceptable. Normally, the pole coordinates are found to be within less than 30° from the correct pole solution. Of course, in many cases the obtained pole solutions are much better than that, mainly for the simulations of ideal triaxial shapes with geometric scattering, for which the pole is typically found within just a few degrees from the correct solution, and also the axial ratios are generally found with excellent accuracies, within 0.02. In the cases of complex shapes, the pole solutions become worse, but still acceptable, and the axial ratio solutions become only indications of the overall triaxial shapes which best represent the simulated shapes. In some cases, and particularly for simulations of ideal triaxial ellipsoid shapes, it can also happen that the longitude of the pole is found with an error of 180° with respect to the correct solution.

Table 1 refers to the inversion attempts done using the old inversion algorithm, assuming geometric scattering. It is easy to see that the inversion solution tends to be always correct for simulations of ideal triaxial ellipsoids and geometric scattering. The only one failure is given by the simulated Barbara asteroid which, as mentioned above, was simulated as a body which does not rotate around its stable inertia axis. In the case of geometric scattering, when we consider complex shapes the performances of the inversion algorithm tend to become slightly worse, with two bad solutions. Curiously, in this case the inversion solution of the tricky Barbara case turns out to be correct. In the case of the simulations performed by taking into account more realistic light-scattering effects, the results of the inversion tend to be still good (8 out of 10 successful solutions) in the case of simulated ellipsoidal shapes, whereas the performances tend to become much worse (4 out of 10) when we consider complex shapes. Note that, in all cases,

Table 2. Results of the inversion using the new algorithm (including Lommel-Seeliger scattering). The simulated data were computed by considering both ideal triaxial shapes and complex shapes. The magnitude conputation was done in all cases including scattering effects. Note that the Barbara simulation refers to an object not spinning around the axis of maximum inertia. For the meaning of "OK" and "BAD", see text.

Simulation		Elliptical	Complex
(3)	Juno	OK	BAD
(5)	Astraea	OK	OK[1]
(9)	Metis	OK	OK
(192)	Nausikaa	OK	OK
(234)	Barbara	BAD	OK
(484)	Pittsburghia	OK	OK
(532)	Herculina	OK	BAD
(584)	Semiramis	OK	OK
(1088)	Mitaka	OK	OK
(1270)	Datura	OK	OK

[1]: Pole longitude +180 deg

we find the correct spin period for the simulated Barbara, suggesting that deviations from the ideal situation assumed by the inversion algorithm (triaxial shapes and geometric scattering) tend to keep the periodicity of the shape features, with the overall effect of producing solutions characterized by worse residuals, but not necessarily bad determinations of the parameters.

In Table 2, we present the results for the inversion of simulated data using the algorithm which takes into account the theoretical form of Lommel-Seeliger scattering as applied to triaxial ellipsoid shapes. It is easy to see that the results tend to be good, and much better than in the case of the inversion done with the old version of the algorithm, as it can be easily checked by comparing the results shown in Table 2 with those displayed in Table 1. It seems therefore that the new version of the inversion algorithm, including a Lommel-Seeliger law, is much better. Note that, again, we do not obtain the right solution for the case of the purely elliptical simulation of Barbara, due to its anomalous rotation, as explained above. Note also that, in the case of complex shapes with scattering, Barbara turns out to be successfully inverted (at least, in terms of determination of the spin period), whereas two simulated cases give wrong period solutions. In other words, in some cases, shape effects can be responsible for wrong solutions. This is not surprising, because we already know that we cannot hope that a correct inversion solution can be always found. In the past, when analyzing the old Hipparcos photometric data of asteroids, we concluded that the correct solution is found in about 50% of the cases. In Table 2, this percentage increases up to 80%, a quite promising result, but of course we have here only a small number of simulated cases.

Next, we consider two cases of simulated Gaia observations for an irregular Gaussian-random-sphere sample asteroid in Torppa and Muinonen [18]. The two cases correspond to two surface scattering laws: in the first case, the observations are simulated using the Lommel-Seeliger surface-element scattering law; in the second case, they are simulated using an efficient, numerical scattering law developed for low-albedo close-packed particulate media with fractional-Brownian-motion surface roughness (particulate-medium scattering law; Wilkman et al. [16, 17]). Both data sets include 69 observations over a time interval of five years, and Gaussian random noise with a standard deviation of 0.01 mag has been added to the simulated disk-integrated brightnesses. The true rotation period and the ecliptic longitude and latitude of the pole are given in Table 3. For the particulate-medium scattering law, the parameters are as follows (see Wilkman et al. [see 17]): the relative standard deviation of surface heights $\sigma = 0.08$, the Hurst exponent related to the fractal characteristics $H = 0.40$, and the volume density $\rho = 0.45$ (packing fraction).

The genetic algorithm to invert Lommel-Seeliger ellipsoids was applied to the two simulated data sets. It is clear that this is just a first, and very preliminary test of application of the genetic algorithm to this new kind of simulations, and we plan to make a much more exhaustive analysis of a large number of cases in the future. Table 3 shows the parameter values retrieved. Overall, the genetic algorithm performs very well and in a promising way in both cases.

Table 3. The true and retrieved spin parameters for the Gaussian sample asteroid with Lommel-Seeliger and particulate-medium surface scattering laws. The number of simulated measurements was 69.

	True parameters	LS scattering law	Particulate-medium scattering law
Rotation period (h)	10.17395622	10.173952	10.173962
Pole longitude (°)	25.01849912	27.98	29.20
Pole latitude (°)	62.89427541	65.82	65.02
rms	-	0.046	0.048

For the pole longitude, a larger deviation is detected for the simulation with the particulate-medium scattering law. For the pole latitude, the deviation is slightly larger for the simulation with the Lommel-Seeliger law. We would like to emphasize that more simulations would be needed for firmer conclusions.

The last step of our analysis has been to run again, using the new version of the inversion algorithm, the whole data set of photometric observations of asteroids present in the Hipparcos catalogue. This is essentially a repetition of the work already published by Cellino et al. [9], the difference being that the new inversion software includes now the Lommel-Seeliger scattering law.

The results of this analysis can be synthetically summarized by saying that we found, in all cases, results which are identical to the old ones published by Cellino et al. [9]. We refer the readers to the above-mentioned paper and, in particular, to the Tables of the results therein included. This lack of any improvement with respect to the previous inversions of Hipparcos data can be explained by taking into account that Hipparcos data are intrinsically of quite bad quality, as already mentioned. The number of observations per object is in many cases very limited, and in all cases the photometric errors are probably even worse than the already large nominal error bars. In this situation, it seems that light-scattering effects tend to be in general too small to be really relevant. This contradicts the hope that we had when we began this new analysis of Hipparcos data, but we have to accept the results. On the other hand, in the case of Gaia data, we expect to obtain data that will be qualitatively and quantitatively much better than the old Hipparcos data. And the analysis of simulated data described above suggests that in this situation the role played by light-scattering effects will be important, and that these effects have to be taken into account in the photometry inversion software.

4. Conclusions and future work

According to the analysis presented in this paper, light-scattering effects can be relevant for the purposes of photometric inversions when one has at disposal sets of accurate photometric data of asteroids, like those that Gaia has begun to obtain since some months. Of course, the results of photometric inversion must be based on an analysis of all detections of each object during the operational lifetime of the mission, because it is essential that the data of each object refer to the widest possible variety of observing circumstances. The results of asteroid photometric inversion are therefore a typical end-of-mission task.

We have seen that simulations suggest that a number of the order of 20% of photometric inversion solutions might be wrong. It is reasonable to expect that the role played by light scattering mechanisms can be shape-dependent, being more important for more regular and less elongated shapes, for which the overall variation of brightness in different observing circumstances tends to be intrinsically weaker. To minimize the number of wrong inversion solutions, however, it is possible to set some filtering, at the cost of possibly rejecting some correct solutions. In particular, wrong solutions tend to give large solution residuals. The limited set of cases presented in this paper, together with previous results obtained by an analysis of the residuals of the solutions obtained using previous versions of the inversion software, confirm that wrong solutions are also those giving the worst residuals. The next step of this analysis will be to better quantify this point, through analyzes of more extensive simulated data sets, including also simulated photometric errors, in order to find some limiting value of the solution residuals, depending on the average quality of the available data for any given object, in order to find an optimal compromise between the need of minimizing the number of wrong inversion solutions, and the need of avoiding to discard exceedingly high numbers of acceptable solutions.

Of course, we are dealing here with the problem of maximizing the quality of the inversion of photometric data obtained by the Gaia survey, alone. In the future, other sky surveys from the ground, like the Large Synoptic Survey Telescope (LSST), will also produce huge amounts of sparse photometric data. Once any problem of calibration made necessary by the need of putting together data obtained by different detectors, using different filters, etc., will be solved, we can conceivably expect that the quality of inversion solutions, based on measurements made over a larger variety of observing circumstances, will improve significantly. Having at disposal larger photometric data sets, including also traditional lightcurves and/or photometric data obtained close to Solar opposition, when a non-linear brightness surge is measured in most cases (the so-called "opposition effect") will also require further investigations, to determine the optimal range of phase angle for which the adopted genetic approach can be suitably applied. In the case of complementing sparse photometric data with full lightcurves, for the moment we can only say that, according to Santana-Ros et al. [10], it seems that adding one single, full lightcurve to a set of sparse photometric data treated using the genetic algorithm does not produce major improvements in the inversion solution. Of course, further investigations are needed.

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