Review

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MEASURING COSMOLOGICAL PARAMETERS WITH GAMMA RAY BURSTS*

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In a few dozen seconds, gamma ray bursts (GRBs) emit up to $\sim 10^{54}$ erg in terms of an equivalent isotropically radiated energy $E_{\rm iso}$, so they can be observed up to $z \sim 10$. Thus, these phenomena appear to be very promising tools to describe the expansion rate history of the universe. Here, we review the use of the $E_{p,i}$–$E_{\rm iso}$ correlation of GRBs to measure the cosmological density parameter Ω_M. We show that the present data set of GRBs, coupled with the assumption that we live in a flat universe, can provide independent evidence, from other probes, that $\Omega_M \sim 0.3$. We show that current (e.g. Swift, Fermi/GBM, Konus-WIND) and forthcoming gamma ray burst (GRB) experiments (e.g. CALET/GBM, SVOM, Lomonosov/UFFO, LOFT/WFM) will allow us to constrain Ω_M with an accuracy comparable to that currently exhibited by Type Ia supernovae (SNe–Ia) and to study the properties of dark energy and their evolution with time.

Keywords: Cosmological parameters; gamma ray bursts; gamma rays; observations.

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1. Introduction

The Hubble diagram of Type Ia supernovae (SNe–Ia) observed at the end of 1990s was hard to reconcile with the decelerated trend implied by the Einstein–de Sitter

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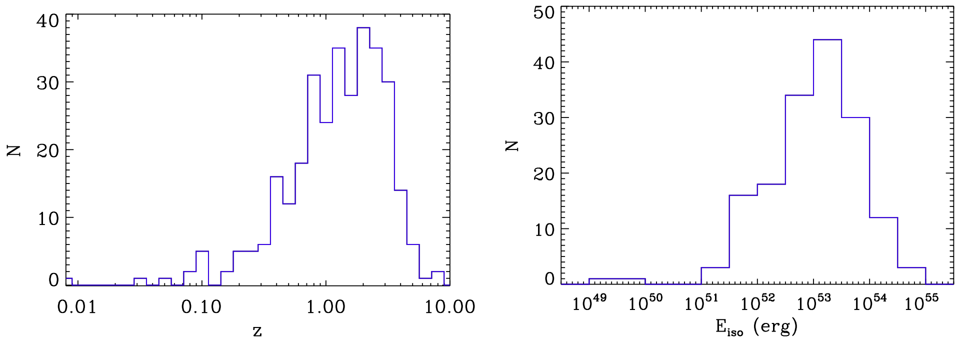

Fig. 1. Distributions of redshift (left panel) and of equivalent isotropically radiated energy $E_{\rm iso}$ (computed by assuming the standard flat Friedmann–Lemaitre–Robertson–Walker (FLRW) cosmology with $H_0 = 70$ and $\Omega_M = 0.3$) of GRBs as of end 2012.

model, then suggesting an accelerated expansion of the universe.[1–4] In the following decade, both SNe–Ia[5–9] and other cosmological probes, such as the cosmic microwave background (CMB),[10–14] galaxy clusters and baryonic acoustic oscillations (BAO)[15–17] lent further support to the existence of an unknown form of "dark energy" propelling the acceleration. The equation of state of such dark energy is often expressed as $P = w(z)\rho$, where $w(z) = w_0 + w_a z/(1+z)$.[18,19]

After combining SNe data with flat universe constraints from CMB measurements,[10,11] Riess *et al.*[20] found $w_0 \sim -1$ and $w_a \sim 0$. These results identified the "dark energy" with the cosmological constant. However, one main issue (among the others) remains still unsolved: a cosmological constant interpreted as a vacuum energy is about 120 orders of magnitude smaller than its "natural" value computed from quantum mechanics.[21] This fact justifies a lot of interest in models where the present energy density (or a dominant fraction of it) is slowly varying with time.

In this short review, we show that gamma ray bursts (GRBs) can significantly contribute to shedding some light on this last issue. Due to their huge energetic outputs, up to $\sim 10^{54}$ erg in terms of equivalent isotropically radiated energies, released in a few tens or hundreds of seconds (Fig. 1), GRBs are the brightest cosmological sources in the universe.[22–25] Therefore, they have been observed up to $z \sim 8$–9[26,27] (Fig. 1), well beyond the observing redshift range of SNe–Ia, limited to $z < 2$.[17,28,29] In addition, GRBs emit most of their radiation as hard X-rays, thus they are only marginally affected by uncertainties connected with correction for reddening.[30,31] In recent years, it has been shown[32,33] that the robust correlation between the photon energy at which the νF_ν spectrum peaks and the GRB radiated energy[34,35] can be successfully used to measure the cosmological density parameter Ω_M.

2. Spectra and Energetics of GRBs

The spectrum of GRBs is nonthermal and can be empirically described by the so-called Band function, i.e. a smoothly broken power law characterized by three parameters: the low-energy spectral index α, the high energy spectral index β and the "rollover" energy E_0.[36] As can be seen in Fig. 2, the νF_ν spectrum shows a peak; the photon energy at which this peak occurs is a characteristic quantity in GRB emission models and is called the "peak energy" E_p.

Every GRB for which it is possible to measure the redshift and the spectrum can be characterized by two key parameters: the total radiated energy, computed by integrating the spectrum in a standard 1–10,000 keV energy band and assuming isotropic emission, $E_{\rm iso}$, and the peak energy of the cosmological rest frame νF_ν spectra of GRBs: $E_{p,i} = E_p \times (1 + z)$. The updated distribution (as of the end of 2012) of $E_{p,i}$ for 156 events (Fig. 2) is approximately a Gaussian centered at a few

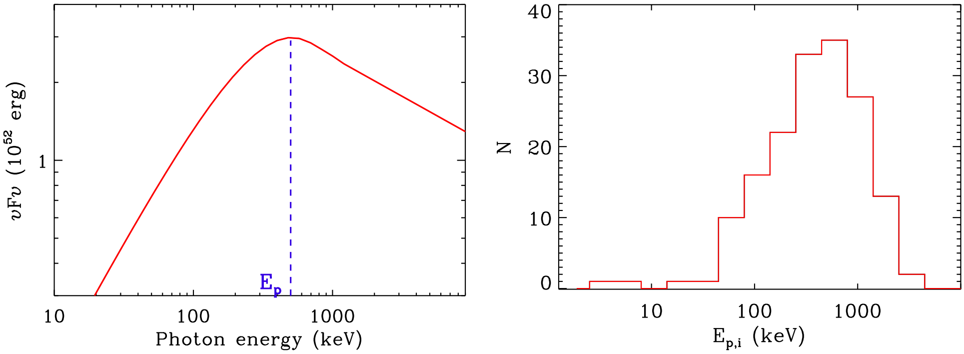

Fig. 2. Left: typical νF_ν (energy) spectrum of a GRB in its cosmological rest frame. Right: distribution of the photon energy $E_{p,i}$ at which the cosmological rest frame νF_ν spectrum of GRB reaches its maximum.

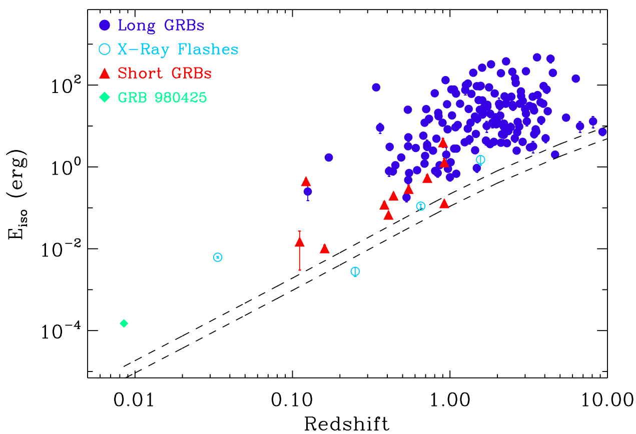

Fig. 3. (Color online) Distribution of isotropic-equivalent radiated energy, $E_{\rm iso}$, as a function of redshift for different classes of GRBs (as of end 2012). The dashed lines in the right panel show typical instrumental sensitivity limits as a function of redshift (updated from Amati and Della Valle[40]).

hundreds of keV and with a low energy extension down to a few keV, corresponding to the so-called X-ray flashes (XRFs) or X-ray rich (XRR) events. The distribution of $E_{\rm iso}$ is somewhat similar, extending from $\sim 10^{48}$ to more than $\sim 10^{54}$ erg, and peaking at $\sim 10^{52}$ erg (Fig. 1).

The distribution of $E_{\rm iso}$ as a function of redshift (Fig. 3) shows the lack of detection of weak events at high redshift (which is expected as result of an obvious bias due to instruments detection threshold) while the lack of bright events at low redshifts may be the result of two factors: an intrinsically low rate of events (less than 3% of SNe–Ibc are associated with long duration GRBs[37]) combined with a correlation between the jet opening angle and GRB intensity (the smaller the opening angle is, the brighter the GRB appears to be).[38,39]

3. The $E_{p,i}$–$E_{\rm iso}$ Correlation

In 2002, based on a small sample of BeppoSAX GRBs with known redshift and spectral parameters, it was discovered that $E_{p,i}$ is significantly correlated with $E_{\rm iso}$.[34,35] This correlation has the form

$$\log E_{p,i}(\text{keV}) = m \log E_{\rm iso}(10^{52}\text{erg}) + q\,,$$

with $m \sim 0.5$ and $q \sim 2$, and is characterized by an extra-Poissonian scatter normally distributed with a $\sigma_{\rm ext}$ of ~ 0.2 dex around the best fit law. Subsequent observations with various detectors and spectrometers confirmed and extended the $E_{p,i}$–$E_{\rm iso}$ correlation (Fig. 4), showing that it holds for all long GRBs and XRFs with well measured redshift and spectral parameters.[32,35] The data, together with the power law best fitting the long GRB points and their confidence regions, have been taken from Amati *et al.*[41]

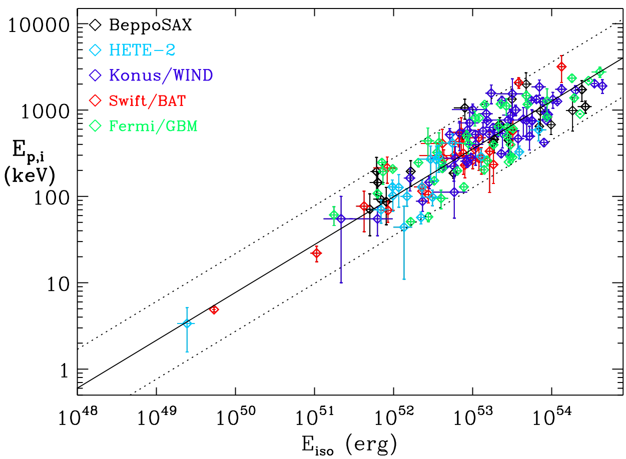

Fig. 4. (Color online) The $E_{p,i}$–$E_{\rm iso}$ correlation in long GRBs (as of the end of 2012). The black line shows the best fit power law. For each point, the color identifies the instrument which performed the spectral measurement.

As discussed by several authors, the existence of the $E_{p,i}$–$E_{\rm iso}$ relation, its extension over several orders of magnitude in both coordinates, its slope and its intrinsic dispersion are a challenging observational evidence for the current models of the physics and geometry of the prompt emission of GRBs.[35,42,43] In addition, the $E_{p,i}$–$E_{\rm iso}$ plane can be used to identify different classes of GRBs (short versus long) and get clues on their different nature. For instance, the consistency of XRFs with the $E_{p,i}$–$E_{\rm iso}$ correlation strongly supports the idea that they are not a different class of sources, but form the faint tail of the "cosmological GRB" population. There is also a clear evidence that short GRBs, for which redshift estimates became available only in the last few years, do not follow the correlation, indicating that their main emission mechanism, and/or the geometry of their emission, may be different from long ones. Finally, the weak long GRB 980425 (only 40 Mpc away) which is also the "prototype" event for the GRB–SNe connection, is inconsistent with the $E_{p,i}$–$E_{\rm iso}$ correlation, suggesting the possible existence of a "local" population of sub-energetic GRBs with different properties with respect to cosmological long GRBs.[37,38,44]

4. Reliability of the $E_{p,i}$–$E_{\rm iso}$ Correlation

Different GRB detectors are characterized by different thresholds and spectroscopic sensitivity, therefore, they can spread relevant selection effects/biases in the observed $E_{p,i}$–$E_{\rm iso}$ correlation. In the past, there were claims that a high fraction (70%–90%) of BATSE GRBs without redshift would be inconsistent with the correlation for any redshift.[45,46] However, this "peculiar" conclusion was refuted by other authors[47–50] who show that, in fact, most BATSE GRBs with unknown redshift were well consistent with the $E_{p,i}$–$E_{\rm iso}$ correlation. We also note that the inconsistency of such a high percentage of GRBs of unknown redshift would have implied that most GRBs with known redshift should also be inconsistent with the $E_{p,i}$–$E_{\rm iso}$ relation, and this fact was never observed. Moreover, Amati *et al.*[51] showed that the normalization of the correlation varies only marginally using GRBs measured by individual instruments with different sensitivities and energy bands, while Ghirlanda *et al.*[52] showed that the parameters of the correlations (m and q) are independent of redshift.

Furthermore, the Swift satellite, thanks to its capability of providing quick and accurate localization of GRBs, thus reducing the selection effects in the observational chain leading to the estimate of GRB redshift, has further confirmed the reliability of the $E_{p,i}$–$E_{\rm iso}$ correlation.[51–53]

Finally, based on time-resolved analysis of BATSE, BeppoSAX and Fermi GRBs, it was found that the $E_{p,i}$–$E_{\rm iso}$ correlation also holds within each single GRB with normalization and slope consistent with those obtained with time-averaged spectra and energetics/luminosity.[52,54–56] This ultimate test confirms the physical origin of the correlation, also providing clues to its explanation.

5. The Beginning of GRB Cosmology

The idea to use GRBs as cosmological rulers was proposed in 2004, when it was found that the $E_{p,i}$–$E_{\rm iso}$ correlation tightened when $E_{\rm iso}$ was replaced with the collimation-corrected radiated energy $E_\gamma = (1 - \cos\theta_{\rm jet}) \times E_{\rm iso}$.[57,58] This result was based on a small sub-sample of GRBs with known $E_{p,i}$ and $E_{\rm iso}$ for which it was possible to infer the jet opening angle $\theta_{\rm jet}$ from the "break time" t_b at which the decay of the light curve of the optical afterglow becomes steeper.[59] By exploiting the low scatter of the $E_{p,i}$–E_γ correlation and applying statistical methods accounting for the lack of calibration, it was possible to derive, within the "standard" Friedmann–Lemaitre–Robertson–Walker (FLRW) cosmological model, estimates of Ω_M and Ω_Λ consistent with the "concordance" values mostly coming from the analysis of Type Ia SNe and the CMB. A review of these methods and results is provided by Ghirlanda *et al.*[60] The results of these investigations were encouraging, and prompted other studies aimed at deriving a GRB Hubble diagram, based, e.g. on the joint use of the $E_{p,i}$–E_γ correlation together with other weaker correlations between luminosity and observed properties,[61] or the calibration of the $E_{p,i}$–E_γ correlation with Type Ia SNe.[62,63]

However, in the last years, the simple jet model assumed to compute $\theta_{\rm jet}$ from the break time of the optical afterglow light curve has been questioned on the basis of different behaviors exhibited by the X-ray afterglow light curves. In many cases, achromatic "jet-breaks" were not detected.[64–66] This fact makes the determination of E_γ, and thus the characterization and use of the $E_{p,i}$–E_γ correlation, less firm and, in any case, model dependent.

6. The GRB Cosmology Through the $E_{p,i}$–$E_{\rm iso}$ Correlation

In view of the previous drawbacks, the possibility to measure the cosmological parameters through the $E_{p,i}$–$E_{\rm iso}$ correlation was investigated,[32] which has the advantage, with respect to the $E_{p,i}$–E_γ correlation, of: (i) allowing for a GRB sample about four times larger because it is based on only two observables (the $E_{p,i}$–E_γ correlation requires, in addition to $E_{\rm iso}$ and $E_{p,i}$, also t_b) (ii) Unlike E_γ computation, the use of the $E_{p,i}$–$E_{\rm iso}$ correlation it is not model dependent. Indeed, no assumptions about the jet model, afterglow model, density and profile of the circumburst environment, or efficiency of conversion of fireball kinetic energy into radiated energy are needed.

The question is: how to measure Ω_M if we need Ω_M to compute the luminosity distance and in turn $E_{\rm iso}$? We can avoid this "circularity problem" after floating the value of Ω_M between 0 and 1 and assuming that the dispersion of the $E_{p,i}$–$E_{\rm iso}$ correlation will minimize corresponding to the choice of the "correct" value.

Based on a sample of 70 long GRBs with known $E_{p,i}$ and $E_{\rm iso}$, it was found that, after assuming a flat universe, the χ^2 obtained by fitting the correlation with a simple power law is a function of the value of Ω_M assumed in the computation of $E_{\rm iso}$.[32] However, given the significant extra-Poissonian scatter of the correlation,

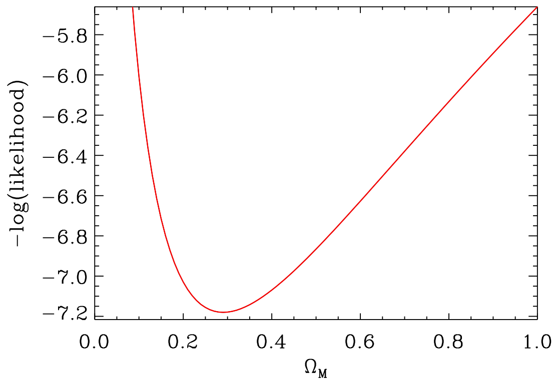

Fig. 5. Goodness of fit, in terms of normalized −log(likelihood), of the $E_{p,i}$–$E_{\rm iso}$ correlation of long GRBs as a function of the value of Ω_M used to compute the $E_{\rm iso}$ values in a flat FLRW universe (from Amati *et al.*[41]). See the text for a description of the maximum-likelihood method adopted.

the simple χ^2 method cannot be used to obtain reliable confidence levels (c.l.) on both the parameters of the correlation (normalization and slope) and the cosmological parameters. In order to get rid of this, Amati *et al.*[32] adopted a maximum likelihood method[67,68] that takes into account the uncertainties in both the X and Y quantities and the extra variance $\sigma_{\rm ext}$. Remarkably, as can be seen in Fig. 5, the −log(likelihood) shows a nice parabolic shape, with a minimum at $\Omega_M \sim 0.30$. This is a very simple but relevant result: (a) it shows that the $E_{p,i}$–$E_{\rm iso}$ correlation can indeed be used to extract clues on the values of cosmological parameters; (b) it provides evidence, independently of Type Ia SNe, that Ω_M is significantly smaller than 1 and around 0.3.

In Table 1, we show the 68% c.l. intervals for Ω_M and w_0 in a flat FLRW universe derived with 70 GRBs of Amati *et al.*, 156 GRBs available as of the end of 2012 (Amati *et al.*[41]), a sample of 250 GRBs (156 real + 94 simulated), expected to be available within a few years, and a sample of 500 GRBs (156 real + 344 simulated) which may be expected from future dedicated space missions (see next section). These values were obtained with the same approach as Amati *et al.* but using the likelihood function proposed by Reichart,[68] which has the advantage of not requiring the arbitrary choice of an independent variable among $E_{p,i}$ and $E_{\rm iso}$. Interestingly, after increasing the number of GRBs from 70 to 156, we note that the accuracy of the estimate of Ω_M improves by a factor of $\sim \sqrt{N_2/N_1}$, as expected for GRB samples which are not significantly affected by systematics. The accuracy of these measurements is still lower than obtained with supernova data, but promising in view of the increasing number of GRBs with measured redshift and spectra (see also Figs. 6 and 7 and the next section).

In the last three lines of Table 1, we report the estimates of Ω_M and w_0 derived from the present and expected future samples by assuming that the $E_{p,i}$–$E_{\rm iso}$ correlation is calibrated with a 10% accuracy by using, e.g. the luminosity distances

Table 1. Comparison of the 68% confidence intervals on Ω_M and w_0 ($\Omega_M = 0.3$, $w_a = 0.5$) for a flat FLRW universe obtained with the sample of 70 GRBs by Amati *et al.*, the sample of 156 GRBs available as of the end of 2012 (Amati *et al.*[41]) and simulated samples of 250 and 500 GRBs (see text). In the last three lines, we also show the results obtained for the same samples by assuming that the slope and normalization of the $E_{p,i}$–$E_{\rm iso}$ correlation are known with a 10% accuracy based, e.g. on calibration against SNe–Ia or self-calibration with a large enough number of GRBs at similar redshift.

GRB #	Ω_M (flat)	w_0 (flat, $\Omega_M = 0.3$, $w_a = 0.5$)
70 (real) GRBs (Amati + 08)	$0.27^{+0.38}_{-0.18}$	< -0.3 (90%)
156 (real) GRBs (Amati + 13)	$0.29^{+0.28}_{-0.15}$	$-0.9^{+0.4}_{-1.5}$
250 (156 real + 94 simulated) GRBs	$0.29^{+0.16}_{-0.12}$	$-0.9^{+0.3}_{-1.1}$
500 (156 real + 344 simulated) GRBs	$0.29^{+0.10}_{-0.09}$	$-0.9^{+0.2}_{-0.8}$
156 (real) GRBs, calibration	$0.30^{+0.06}_{-0.06}$	$-1.1^{+0.25}_{-0.30}$
250 (156 real + 94 simulated) GRBs, calibration	$0.30^{+0.04}_{-0.05}$	$-1.1^{+0.20}_{-0.20}$
500 (156 real + 344 simulated) GRBs, calibration	$0.30^{+0.03}_{-0.03}$	$-1.1^{+0.12}_{-0.15}$

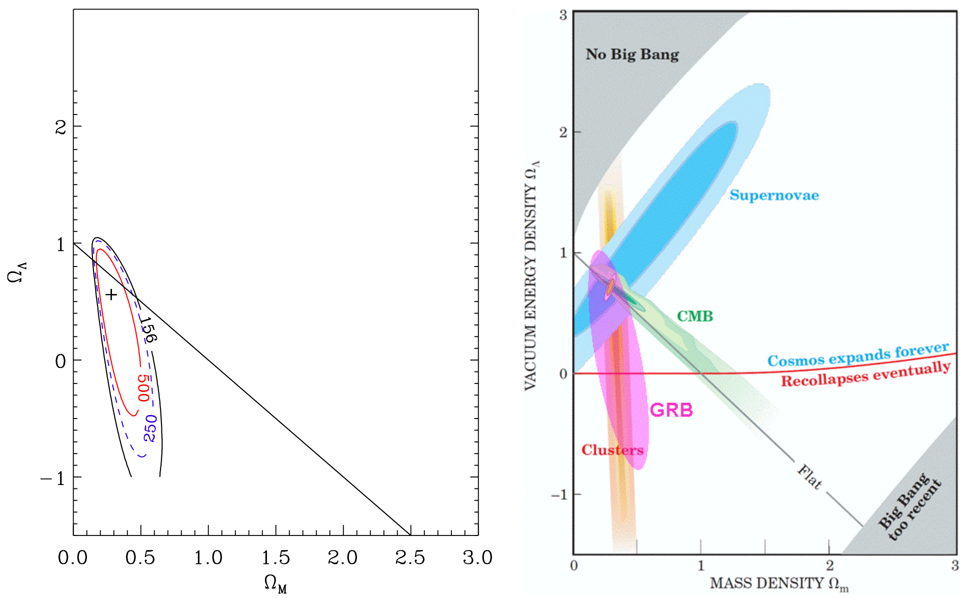

Fig. 6. (Color online) Left: 68% c.l. contour in the Ω_M–Ω_Λ plane obtained by releasing the flat universe assumption with the sample of 156 GRBs available at the end of 2012 (black) compared to those expected in the next years with the increasing of GRBs in the sample (blue and red).[41] Right: 68% c.l. contour in the Ω_M–Ω_Λ plane obtained by assuming a sample of 250 GRBs expected in the near future (pink) compared to those from other cosmological probes (adapted from a figure by the Supernova Cosmology Project[72]).

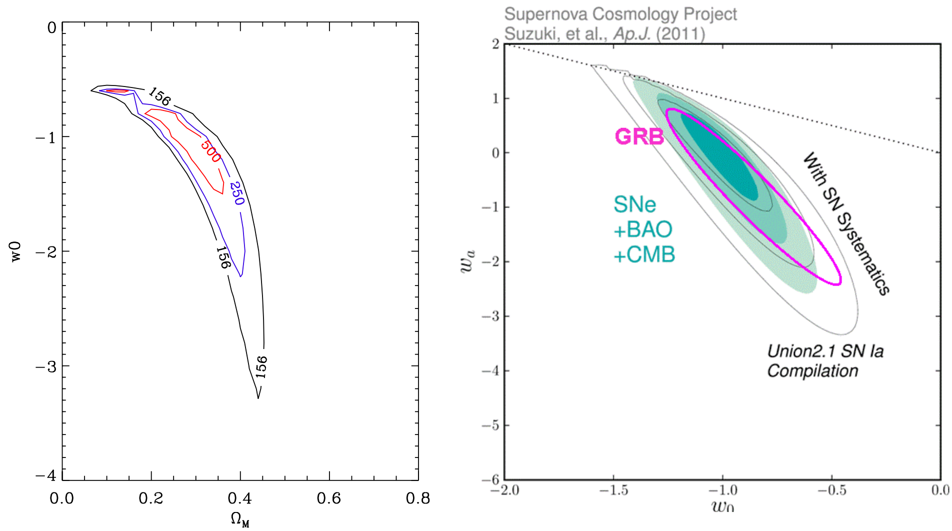

Fig. 7. (Color online) Left: 68% c.l. contours in the Ω_M–w_0 plane for a flat FLRW universe derived from the present and expected future samples by assuming that the $E_{p,i}$–$E_{\rm iso}$ correlation is calibrated with a 10% accuracy by using, e.g. the luminosity distances provided by SNe–Ia, self-calibration or the other methods described in the text. Right: 68% c.l. contour in the w_0–w_a plane for a flat FLRW universe with $\Omega_M = 0.3$ obtained, by assuming that the $E_{p,i}$–$E_{\rm iso}$ correlation is calibrated with a 10% accuracy, using the 500 GRBs expected form next generation experiments (pink) compared to those from other cosmological probes (adapted from a figure by the Supernova Cosmology Project[17]).

provided by SNe–Ia, GRBs self-calibration or the other methods shortly described below. The perspectives of this method, combined with the expected increase of the number of GRBs in the sample, for the investigation of the properties of "dark energy," are shown in Fig. 7.

It is important to note that, as the number of GRBs in each z-bin increases, the feasibility and accuracy of the self-calibration of the $E_{p,i}$–$E_{\rm iso}$ correlation will also improve. Thus, the expected results shown in the last part of Table 1 and in Fig. 7 may be obtained even without the need of calibrating GRBs against other cosmological probes.

7. Further Investigations and Perspectives

Table 1 shows a sharp increase of the accuracy attached to Ω_M as a consequence of the increasing number of GRBs in the $E_{p,i}$–$E_{\rm iso}$ plane. Currently, the main contribution to enlarge the GRB sample comes from joint detections by Swift, Fermi/GBM or Konus-WIND. Hopefully, these missions will continue to operate in the next years, then providing us with an "actual" rate of ∼15–20 GRB/year. However, a real breakthrough in this field should come from next generation (>2017) missions capable of promptly pinpointing the GRB localization and of carrying out broadband spectroscopy. We build our hopes, e.g. on the Japanese-led CALET/GBM experiment for ISS,[69] the Chinese–French mission SVOM,[70] the Lomonosov/UFFO[71] experiment, an international project led by Korea and Taiwan, the WFM onboard LOFT, a mission currently under assessment study within the ESA/M3 programme.[73] In Fig. 6, we show the confidence level contours in the Ω_M–Ω_Λ plane by using the real data as of 2012 and by adding to them 113 and 363 simulated GRBs (resulting in two samples of 250 and 500 GRBs in total, respectively). The simulated data sets were obtained via Monte Carlo techniques by taking into account the slope, normalization and dispersion of the observed $E_{p,i}$–$E_{\rm iso}$ correlation, the observed redshift distribution of GRBs and the distribution of the uncertainties in the measured values of $E_{p,i}$ and $E_{\rm iso}$. These simulations indicate that with a sample of 250 GRBs (achievable in about five years from now) the accuracy in measuring Ω_M will be comparable to that currently provided by SNe data.

Several authors[62,63,74] are investigating the calibration of the $E_{p,i}$–$E_{\rm iso}$ correlation at $z < 1.4$ by using the luminosity distance versus redshift relation derived from SNe–Ia. The goal is to extend the SNe–Ia Hubble diagram up to redshifts at which the luminosity distance is more sensitive to dark energy properties and evolution. The drawback of this approach is that, with this method, GRBs are no longer an independent cosmological probe.

Other approaches include: cosmographic calibration of the $E_{p,i}$–$E_{\rm iso}$ correlation;[63,74] self-calibration of the correlation with a large enough number of GRBs lying within a narrow ($\Delta z \sim 0.1$–0.2) range of z (promising, but requires a significant sample enlargement); combining the $E_{p,i}$–$E_{\rm iso}$ correlation with other (less tight) GRB correlations;[61,75,76] extending the $E_{p,i}$–$E_{\rm iso}$ correlation by involving other prompt or afterglow properties[77] aimed at reducing the dispersion of the correlation (but with the risk of increasing systematics and lowering the number of GRBs that can be used); testing alternative cosmologies versus the standard ΛCDM model.[78–88] Particularly promising, are the perspectives of the self-calibration method, which will become feasible and accurate as the number of GRBs in the $E_{p,i}$–$E_{\rm iso}$ plane will increase.[60]

Understanding the physical basis for the $E_{p,i}$–$E_{\rm iso}$ correlation is also of fundamental importance for both GRB physics and cosmology, and several groups are working on this issue in the framework of the different scenarios that have been proposed for the origin of GRB prompt emission.[42]

8. Conclusions

Due to their huge radiated energies GRBs can be observed up to $z \sim 10$, therefore they are very powerful cosmological tools, complementary to other probes such as SNe–Ia, clusters or BAO. The correlation between spectral peak photon energy $E_{p,i}$ and intensity ($E_{\rm iso}$, $L_{\rm iso}$, $L_{p,\rm iso}$) is one of the most robust and intriguing properties of GRBs and a promising tool for measuring cosmological parameters. Analyses in the last years provide independent evidence that, if we live in a flat universe, then Ω_M is <1 at $>99.9\%$ c.l. and around ~ 0.3, consistent with current measurements obtained via different methodologies. The simultaneous operation of Swift, Fermi/GBM, Konus-WIND will increase the number of useful samples (z + $E_{p,i}$) at a rate of 15–20 GRB/year, improving the accuracy in the estimate of cosmological parameters. Future GRB experiments (e.g. CALET/GBM, SVOM, Lomonosov/UFFO, LOFT/WFM) will increase dramatically the number of GRB usable in the $E_{p,i}$–$E_{\rm iso}$ plane up to $z \sim 10$ (13.2 Gyrs in terms of look-back time) and therefore it will be possible to use them to follow the dependence on time (if any) of the density of vacuum energy since the early stages of the universe.

In conclusion, GRBs have already provided a direct and independent measurement of Ω_M and simulations show that on a time scale of ~ 5 years they will be able to achieve a comparable accuracy to SNe–Ia. But the surplus value of GRBs is that, in perspective, they can measure w_0 and w_a, namely the evolution of dark energy with time.

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