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C. Arcidiacono, L. Schreiber, G. Bregoli, I. Foppiani, S. Oberti, E. Diolaiti, G. Agapito, A. Puglisi, M. Xompero, F. Cortecchia, M. Patti, M. Lombini, L. Busoni, C. Vérinaud, C. Felini, V. De Caprio, G. Cosentino, P. Ciliegi, M. Bellazzini, P. Feautrier, S. Esposito, R. Ragazzoni, "Numerical simulations of MAORY MCAO module for the ELT," Proc. SPIE 10703, Adaptive Optics Systems VI, 107034I (18 July 2018); doi: 10.1117/12.2311702

SPIE.

Event: SPIE Astronomical Telescopes + Instrumentation, 2018, Austin, Texas, United States

Numerical simulations of MAORY MCAO module for the ELT

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ABSTRACT

MAO (MAORY Adaptive Optics) is the a developed numerical simulation tool for adaptive optics. It was created especially to simulate the performance of the MAORY MCAO module of the Extremely Large Telescope. It is a full end-to-end Monte-Carlo code able to perform different flavors of adaptive optics simulation. We used it to investigate the performance of a the MAORY and some specific issue related to calibration, acquisition and operation strategies. As, MAORY, MAO will implement Multi-conjugate Adaptive Optics combining Laser Guide Stars (LGS) and Natural Guide Stars (NGS) measurements. The implementation of the reference truth WFS completes the scheme. The simulation tool implements the various aspect of the MAORY in an end to end fashion. The code has been developed using IDL and use libraries in C++ and CUDA for efficiency improvements. Here we recall the code architecture, we describe the modeled instrument components and the control strategies implemented in the code.

Keywords: Adaptive Optics, Wave-front Sensing, Numerical Simulation, Multi-conjugate adaptive optics

1. INTRODUCTION

The Adaptive optics (AO) is a technique aiming to enable high-resolution imaging from the ground, beating the limits imposed by optical turbulence. It foresees the real-time compensation of the phase perturbations introduced by the atmospheric turbulence. The simplest configuration of an AO systems is the single-conjugate (SCAO). It includes one deformable mirror (DM) to compensate, and one wavefront sensor (WFS) to measure the phase perturbation residuals in the direction of the reference source. On the ESO - Multi–conjugate Adaptive Optics Demonstrator (MAD)[1–3] the multi deformable mirror version, called Multi-Conjugate Adaptive Optics[4,5] (MCAO), has been demonstrated on sky. In MCAO, it's possible to obtain a correction of the optical turbulence above the telescope valid for a Field of View (FoV) by the means of many reference sources, and not only for the special direction of the reference star as in SCAO case. In the MAD, the MCAO was realized using two deformable mirrors and three Natural Guide Stars (NGS) in the Star Oriented Approach[6] or three to eight NGS

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Adaptive Optics Systems VI, edited by Laird M. Close, Laura Schreiber, Dirk Schmidt, Proc. of SPIE Vol. 10703, 107034I · © 2018 SPIE
CCC code: 0277-786X/18/$18 · doi: 10.1117/12.2311702

in the Layer-Oriented[7–9] version. In both cases a uniform correction for a field of view of 2 arc min has been achieved.[10,11]

We developed the **M**AORY **A**daptive **O**ptics (MAO) numerical simulation tool[12–14] to perform the end to end (E2E) simulations of the Multi–conjugate Adaptive Optics RelaY[15] (MAORY) for the ESO Extremely Large Telescope.[16,17] MAORY is the adaptive optics module of the Extremely Large Telescope (ELT) that will feed the Multi-AO Imaging Camera for Deep Observations Near Infrared spectro-imager[18] (MICADO) through a gravity invariant exit port. MAORY has been foreseen to implement MCAO correction through up to three high order deformable mirrors driven by the reference signals of six Laser Guide Stars (LGSs)[19] feeding as many Shack-Hartmann Wavefront Sensors.[20] A three Natural Guide Stars (NGSs) system provides the low order correction and the truth sensing.
The MAO code has been developed mostly in the IDL language and performs a Monte–Carlo modeling of the MAORY system performance through an extensive usage of the available computing core, both CPUs and GPUs. In this proceeding we recall the code architecture and describe the modeled instrument components and the control strategies we implemented.

2. MAO

MAO was developed using mainly the IDL language: the architecture and the most of the routines are written in IDL. A few high level routines take into account for the system configurations, the simulation skeleton and the main loops. The low level routines provide the repetitive and mathematical jobs. MAO incorporates some of the low level routines of the STARFINDER[21] and of the LOST[22] numerical tools.
The End-to-End simulations of Extremely large telescopes (ELTs) are always starved of resources: efficient simulation tools need optimized mathematical algorithm and optimal computer memory usage to obtain results in a short or, at least, reasonable time. The use of multi–core workstation is mandatory and the requirements on memory are challenging: at least 50Gb memory RAM are needed for a full MCAO ELT simulation. An End-to-End ELT configuration requires in the most important moment, the closed loop phase, to store in the RAM mirror modes, open loop wavefront (WF) arrays of the simulated Adaptive Optics (AO) references and test stars (for Point Spread Function (PSF) and Strehl Ratio (SR) computation), control matrix, slope vectors and a few more service arrays and structures. For the MAORY we typically consider a telescope pupil inscribed on 740×740 square array for a 4.875e-2 meter per pixel, this value sets all the other important ones: considering ≈ 5000 modes for the ground layer Deformable Mirror (DM), ≈ 600 and ≈ 700 respectively for the two post-focal DMs with 2m actuator pitch projected on the primary, a 3×3 constellation of test stars, for 2seconds of run, the simulation needs about 50GB of the available RAM (float numbers).

Many of the IDL original routines are multi-threaded, however, a few ones that are particularly interesting for the code are not. As an example, we give here the case of the Singular Value Decomposition (SVD) in some of the form available in the IDL library (`SVDC`, `LA_SVD`, `IMSL_SVD`). Moreover, some of the IDL-routines that use the thread pool, such as the `FFT` (actually Discrete Fourier Transform `DFT`) are not performing as fast as freely available solutions such as the `fftw` the Fastest Fourier Transform in the West.[23]

High-Performance Computing (HPC) is looking more and more to the use of general purpose graphic processors (GPGPUs). In the recent years, the cost and performance became interesting for a large community of users increasing the availability of mathematical/physical libraries for numerical computation. In particular, we are interested in the use of a GPU to accelerate the numerical computation.
NVIDIA set a new standard with TESLA™Architecture for scientific computing, thanks also to the development of the Compute Unified Device Architecture[24] (CUDA). NVIDIA created a parallel computing platform for the GPUs they produce, which is almost perfectly compatible with the full C standard. Actually, it runs code through a C++ compiler.
From the IDL environment, it's possible to call external (C/C++) library through the dynamically loadable modules (`DLM`). In particular, in this way we have access to parallel optimized C/C++ and CUDA libraries directly from IDL.

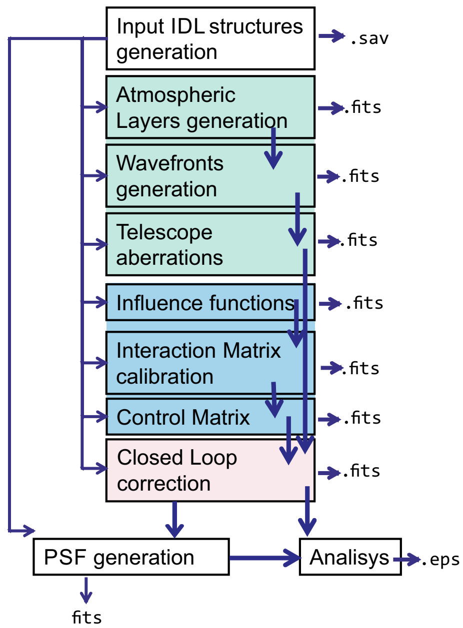

Figure 1. The scheme of the simulation tool.

2.1 MAO Architecture

We designed the simulation tool following a modular approach, focusing on the possibility to re-utilize (recycle) as much as possible of the already simulated cases: the different simulation steps, namely the atmospheric phase screen generation, the open loop wave-front measurements, the inclusion of telescope aberrations, the interaction matrix calibration, the closed loop, the Point Spread Functions (PSFs) generation are performed independently. The results of each of these steps are saved in fits[25] files on the disk and these may be the input for the following modules. See Figure 1. The fits files may used as input for new simulations using, if desired, a somewhat different system configuration.

A full simulation is then divided in standing alone components which play a specific role in the logic the adaptive loop. Some of these logical components of the End-to-End simulation may be independent of the closed loop (as the interaction matrix of the open loop) or even from the adaptive optics instrument simulated (for example the atmospheric phase screens).

2.2 The atmospheric layers generation

The optical turbulence is modeled through a set of phase screens representing turbulent layer at different altitudes. Each phase screen is computed by Discrete Fourier Transform (DFT),[26] using the FFTW library, and applying the Kolmogorov[27,28] or Von Karman[29] models for the phase Power Spectrum. The phase screens are completely static following the frozen flow hypothesis. Indeed each phase screen is randomly generated from a set of variable that has a statistical sense such as the coherence length or the outer-scale.

pixelscale	layer size px	r_0
power spectrum	wavelength	outerscale

Table 1. Input for the phase screens generation.

The phase screens arrays are saved on a `.fits`[25] file, in order to be easily re-used and inspected.

2.3 The open loop wave-front measurements

MAO computes from the phase screens the open loop wavefront (WF) for all the stars. A straightforward geometrical projection of the stars footprint is used for such as computation. Actually, MAO simulates the open loop history as a series of WF for the reference and for the PSF or test star over the field of view of interest. MAO computes the WF of the LGS by considering the geometrical projection of the laser guide reference placed at a finite distance from the phase screen placed at the corresponding layer altitude.

pixelscale	Telescope Diameter	central obstruction ratio
layer altitude	layer wind speed	layer wind direction
NGS star XY direction	LGS star XY direction	simulation step
simulation length	LGS flag	LGS launcher XY ground position
Sodium layer altitude	Sodium layer thickness	LGS beacon direction
Atmosphere file to read	Starting Zenith distance Angle	Zenith distance Speed
Test Star XY position	Pupil Shape definition file	

Table 2. Input for the open loop history generation

MAO considers also the Zenith distance variation during the simulation adjusting both the airmass and LGS/NGS footprint projection on the phase screens.

MAO computes the loop phase history series and it saves it on a `.fits` file for the user defined angular directions and the whole simulated duration of the test.

2.4 Telescope Aberrations

On the open loop phase, the code may add a number of telescope aberrations. The telescope aberrations are optical phase screens that are summed to the existing open loop data. MAO is able to consider a time series of Zernike polynomial coefficients (to take into account telescope vibration for example). MAO uses the possibility to add extra aberrations to mimic the effect of a mis–positioning of the Natural Guide Star WFSs (by adding and extra tip and tilt aberration). Other static (or quasi-static) aberration may be piled up to phase: the code reads user-defined input phase map to be added to the open loop WF history of all, or part of, the simulated direction and reference stars. In this way we may consider for example the scallopping error generated by the E-ELT primary.

pixelscale	Telescope Diameter	open loop history file
Zernike coefficients errors	Derotator errors	Laser Flag
Simulation temporal Step	Simulation length	Load specific aberration keyword
NGS XY position error	NGS XY position rms	Platescale temporal series

Table 3. Input for the aberrations to be summed to the loop history.

It is possible to add error in the positioning of the NGS probes and simulate the effects generate by bearings (NGs's and LGS's) error (speed, wobbling).

On top of the aberrations above is it possible to add the platescale effect modeled as a variable tip-tilt WF proportional to the distance of the star from the optical axis. Actually the input is the temporal history of the platescale

2.5 Influence function and modal base

The deformable mirrors are computed as the linear combination of a modal base.

The actuators base may be defined by the measured or by the expected influence functions. This is the base from which MAO starts to compute a modal base.

pixelscale	Telescope Diameter	central obstruction ratio
Zernike Flag	Karhounen Loeve Flag	open loop history file
Number of DM	Field of View on the DM	Conjugation Altitude
Modal base computation Type	Waffle Mode Optimization flag	actuator Density
Number of iteration for KL atmospheric fitting	Actuator base filename	Zonal Flag
Geometry of the actuators	Pupil Definition	

Table 4. Input generation of the modal base. "Modal base computation type" fixes the kind of modal and the actuator base to generate.

The deformable mirrors are then ready to be used to register the interaction matrix and to close the loops. The tool accepts user-supplied influence functions or it may compute those analytically. The influence function base can be used to fit Zernike modes or to compute the Karhunen-Loeve[30] expansion which best fits the stochastic wave–fronts induced by atmospheric turbulence.

Again, the result may be easily inspected since the computed modes are stored in a (large) `.fits` file.

2.6 The calibration of the interaction matrices

The modal base selected as input is used to register the slopes as they were measured by the WFS modules.

pixelscale	Telescope Diameter	central obstruction ratio
Input Interaction Matrix filename	input modal base filename	Output modal base filename
Number of modes to take	Condition number for SVD truncation	Open Loop Filename
Illumination Threshold for valid Sub-ap	Linear WFS definition	Interaction Matrix Definition
Number of modes to truncate	Zonal flag	Number of DM
Control Matrix Filtering	MMSE flag	r_0
Layers Altitude	Conjugation Altitude	Actuator Base

Table 5. Input generation of the control matrix computation. The "Interaction Matrix Definition" is a placeholder for the input of the interaction matrix computation.

This procedure may consider possible environment conditions (noise) and simulate the calibration measurement as the were obtained on-sky (using newly generated phase screens).

In the calibration simulation process, the data useful for reference slopes computation are saved and then analyzed in this block.

The interaction matrix and the reference slopes vector to be used in closed loop are saved in a `.fits` file.

2.7 Control Loop

The feedback reaction of the system is defined mainly through the computation of the control matrix. It may be easily the (pseudo) inverse of the array built using the DM - WFS interaction matrices and selecting the valid sub-aperture, also known as Least-Square Estimator (LSE), possibly truncated (TLSE). Or it may be regularized considering a Bayesian inference approach using the prior knowledge about noise and turbulence statistics to compute a Minimum Mean Square Error (MMSE) estimator.

Another possibility is to use an iterative algorithm instead of a control matrix in order to compute the DM commands vector. We implemented a Kaczmarz's algorithm,[31] avoiding in this way the computation of the control matrix through the inversion of the DM-to-WFS interaction matrix.

The user may pass a vector of weights to tune the tomographic re-partition of the turbulence on the different DM. Moreover he/she has the chance also to tune the regularization of the control matrix by passing a coefficient.

pixelscale	Telescope Diameter	central obstruction ratio
WFS wavelength(s)	Input modal base filename	Output modal base filename
Number of DM	Pupil Definition	Conjugation Altitude
Metapupil definition	Open Loop Filename	NGS XY position
LGS XY position	SH LGS WFS definition	SH NGS WFS definition
SH Reference WFS definition	Number of modes to calibrate	Linear WFS definition
Pyramid NGS WFS definition	XY ground position of the LGS launcher	Sodium layer altitude
Sodium Layer Thickness	Reference Slope Computation Method	Initial Zenith distance
Zenith Distance speed	XY Slope error for the linear WFS	Piston WFS definition
SH LGS definition	SH LGS Threads info	

Table 6. Input generation of the control matrix computation.

2.8 The closed loop

The core of the simulation is the closed loop phase. The WFS modules compute the vector signals (X- and Y-slopes) from the residual phase (the open loop wavefront subtracted by the DM). Vector slopes multiplied by the control matrix returns the vector coefficients for the modal base linear composition, in the form of the differential coefficients to be added to the one defining the existing mirror shape. It's possible to consider at this level the sodium layer temporal evolution.

For the DM description (DM controller), the user may choose to have a pure integrator or a second order low pass filter.

In the closed loop block it is possible to set:

- slopes filtering (NGS modes from LGS Slopes and vice versa);
- Tip Tilt linear extrapolation (prediction);
- Modal Gain value and optimization;
- Bootstrap method;

Is it possible to use a pseudoopen loop control[32] (POLC) and consider FoV optimization options in order to improve performance.

Eventually, the Closed Loop history of the NGS, LGS and test stars are saved for analysis on `.fits` file.

2.9 The Point Spread Functions (PSFs)

The user select the wavelength at which the PSF is computed and the pixelscale. This module of the code makes an extensive use of the GPU. The user may desire more wavelengths, considering a bandwidth. Actually the user may decide to compute the Strehl Ratio (SR) on the PSF or on the Close Loop WF history using the Marechal approximation.

The PSF generation routines extensively uses the available GPUs.

2.10 Analysis

We developed the first set of tools to allow a quick look at the data and obtain interesting quality merit function from the PSF and from the Closed Loop history file. Typically we look into the SR map over the FoV and to the residual error time evolution on the various directions considered.

3. CONCLUSIONS

We are using the simulation tool to perform trade-off analysis and to take confidence with the MAORY system. We take advantage of the pseudoopen loop control[32] (POLC) and FoV optimization.
Next step would be the implementation of the truth sensor by adding a new loop on the NGS stars. This loop would allow the computation of the reference slopes to be applied to the LGS loop.

Another important update will be considering the rotation of the modal bases (the DMs, or some them) with respect to the WFS and the way the MAORY will take care of it.

ACKNOWLEDGMENTS

The first author wishes to thank all the MAORY consortium for the great work we are doing together. This work has been partly supported by the Italian Ministero dell'Istruzione, dell' Università e della Ricerca (Progetto Premiale E-ELT 2012 ref. Monica Tosi).

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