The Exposure Time Calculator for HIRES@ELT

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Abstract

The Exposure Time Calculator (ETC) is a tool to predict the performances of an instrument. It is extremely useful during all the phases of a project, from the design to the observations. Here we present a detailed description of the ETC for HIRES@ELT, the high resolution optical-infrared spectrograph for the Extreme Large Telescope.

Introduction

HIRES is the high-resolution spectrograph of the ESO Extremely Large Telescope. It consists of three fibre-fed spectrographs providing a wavelength coverage of 0.4-1.8 μm at a spectral resolution of ~100,000 (Marconi et al. 2020, arXiv201112317).

The Exposure Time Calculator (ETC) is a tool to predict the global performances of the spectrometer for different parameters and environmental conditions.
The ETC can compute for a compact source the limiting magnitude achievable at a given wavelength, in a given exposure time and at a given signal to noise ratio and it can compute for an extended source the limiting surface brightness achievable at a given wavelength, in a given exposure time and at a given signal to noise ratio. Moreover, the ETC can compute the signal to noise ratio achievable at a given wavelength, in a given exposure time and at a given magnitude (for a compact object) or at a given surface brightness (for an extended source).

The ETC accounts for a number of instrumental and environmental parameters that can be modified, allowing the user to quantify their effect on the overall performances of the spectrometer.
The ETC works in seeing limited mode.

The program to compute the limiting magnitude and the surface brightness can be found at
http://tirgo.arcetri.astro.it/nicoletta/etc_hires_mag.html
The program to compute the S/N ratio for a compact source can be found at
http://tirgo.arcetri.astro.it/nicoletta/etc_hires_sn_com.html
The program to compute the S/N ratio for an extended source can be found at
http://tirgo.arcetri.astro.it/nicoletta/etc_hires_sn_ext.html

Description

The magnitudes are in AB units, i.e.

$$m(AB) = -2.5\log_{10}\left(\frac{f_\nu}{1\ \mathrm{erg\ cm^{-2}\ s^{-1}\ Hz^{-1}}}\right) - 48.60$$

$$m(AB) = -2.5\log_{10}\left(\frac{f_\lambda}{1\ \mathrm{erg\ cm^{-2}\ s^{-1}}\ \mu\mathrm{m}^{-1}}\right) - 5.0\log_{10}\left(\frac{\lambda}{1\ \mu\mathrm{m}}\right) - 12.40$$

$$m(AB) = -2.5\log_{10}\left(\frac{N_\lambda}{1\ \mathrm{photon\ cm^{-2}\ s^{-1}}\ \mu\mathrm{m}^{-1}}\right) - 2.5\log_{10}\left(\frac{\lambda}{1\mu\mathrm{m}}\right) + 16.85$$

The magnitudes in the AB system can be converted to the Vega system by using the conversion factors of Table 1.

Table 1: AB-Vega mag photometric system conversion factors.

band	λ_{eff} (μm)	m_{AB}-m_{Vega}
U	0.36	0.79
B	0.43	-0.09
V	0.55	0.02
R	0.64	0.21
I	0.80	0.45
J	1.24	0.91
H	1.65	1.39
Ks	2.16	1.85

In case of limiting magnitude and limiting surface brightness computations, the input parameters to the ETC are as follows.

- WL: required wavelength, in μm.
- EXPTIME: required exposure time, in seconds. This is the total exposure time. If the observation consists of several, shorter exposures, the user must update the parameter NDIT. For an IR detector the exposure time for a single observation (NDIT = 1) is typically shorter than ~ 30 minutes.
- NDIT: number of separate read-outs to achieve the requested exposure time. It is suggested that this value be updated if the exposure time is higher than ~ 30 minutes. Default value NDIT = 1.
- SN: required signal to noise ratio. This quantity is computed per resolution element on the extracted spectrum.
- RPOW: resolving power λ/Δλ.

- DAPE: sky-projected angular diameter of the spectrometer aperture. Deafult values for seeing-limited DAPE=0.87 arcsec for $\lambda \leq 0.95$ μm and DAPE=0.71 arcsec for $\lambda > 0.95$ μm. Note: the program assumes a circular aperture. Different geometries can be adjusted by inserting the equivalent diameter of the aperture. The result is independent on the shape of the aperture.

- EFFI: efficiency of the instrument. This parameter also includes the quantum efficiency of the detector. Default value is EFFI = 0.10.

- EFFT: efficiency of the telescope outside the atmosphere. Default values are obtained by interpolating Table 2, delivered by ESO (see "Common ICD between the E-ELT Nasmyth Instruments and the Rest of the E-ELT System", Document Number: ESO-253082).

Table 2: Telescope efficiencies.

WL (μm)	EFFT
0.36	0.13
0.40	0.28
0.45	0.44
0.55	0.58
0.65	0.64
0.80	0.68
1.25	0.80
1.65	0.83
2.60	0.84

- AM: airmass. Default value AM = 1.2.

- SKYBCK: sky background at the selected wavelengths, in AB magnitudes per arcsec^2 at AM = 1. Default values are interpolated from Table 3, which lists the approximate continuum background in between airglow sky-lines under dark conditions.

Table 3: Sky backgrounds.

WL (μm)	SKYBCK (AB mag/arcsec^2)
0.36	22.50
0.44	22.50
0.55	21.80
0.64	21.50
0.80	20.50
1.05	20.50
1.25	20.00
1.65	19.50
2.60	19.50

The effect of the moon is not taken into account. This effect can be calculated by the user adopting Table 4.

Table 4: Sky background variations for different moon phases.

band	Δmag at given days from new moon 0	3	7	10	14
U[a]	0	-0.22	-1.47	-2.50	-4.25
B[a]	0	-0.33	-1.79	-2.87	-4.58
V[a]	0	-0.20	-1.29	-2.25	-3.82
R[a]	0	-0.11	-0.84	-1.63	-3.04
I[a]	0	-0.03	-0.31	-0.74	-1.78
Y[b]	0	-	-	-	-1.50
J[b]	0	-	-	-	-1.00
H[b]	0	-	-	-	-0.50
K[c]	0	-	-	-	-0.00

[a] values used by ESO; [b] Sullivan and Simcoe 2012; [c] never calculated before.

- TBCK: ambient temperature, used to calculate the thermal background, which dominates over the sky background at the longer wavelengths. Default value TBCK = 283 K.

- EBCK: total emissivity of telescope and instrument, used to calculate the thermal background, which dominates over the sky background at the longer wavelengths. It includes also the atmospheric absorption. Default value EBCK = 0.20.

- DTEL: diameter of telescope, default value = 38.5m.

- COBS: fractional diameter of central obscuration of the telescope. Default value = 0.28 (i.e. 9% in area).

- SLE: slit efficiency, i.e. fraction of the light from the astronomical target falling inside the spectrometer slit. Default values are valid for compact source in typical seeing condition and interpolated from Table 5, delivered by ESO (for more details see https://www.eso.org/observing/etc/doc/elt/etc_spec_model.pdf). An extended source is independent of the slit efficiency.

Table 5: Encircled energies.

DAPE (arcsec)	U (0.36 μm)	B (0.44 μm)	V (0.55 μm)	R (0.64 μm)	I (0.80 μm)	J (1.25 μm)	H (1.65 μm)	Ks (2.16 μm)
0.0044	0.00229	0.00263	0.00282	0.00325	0.00380	0.00480	0.00474	0.00646
0.0200	0.04780	0.05230	0.05840	0.06850	0.07920	0.09680	0.11600	0.15300
0.1000	1.19000	1.30000	1.46000	1.67000	1.94000	2.42000	2.93000	3.76000
0.2000	4.66000	5.08000	5.68000	6.47000	7.46000	9.13000	11.00000	13.70000
0.4000	17.10000	18.40000	20.37000	22.70000	25.60000	30.10000	34.70000	40.10000
0.6000	33.50000	35.60000	38.50000	42.00000	46.10000	51.80000	56.80000	61.90000
0.8000	50.00000	52.40000	55.50000	59.10000	63.00000	68.00000	71.90000	75.60000
1.0000	64.00000	66.10000	68.80000	71.80000	74.80000	78.50000	81.20000	83.60000
1.6000	88.00000	88.30000	88.90000	89.80000	90.70000	91.80000	92.60000	93.30000

- DPIX: physical size of a detector pixel. Default value DPIX =10 μm for λ≤0.95 μm and DPIX = 15 μm for λ>0.95 μm.

- FCAM: focal aperture of the camera. Default value FCAM = 1.5.

- PIXBINNED: on-chip binning factor (only for CCDs at λ≤0.95 μm). Default value PIXBINNED = 1.

- RON: read-out noise of the detector. Default value RON = 1 e^-/pix for $\lambda \le 0.95$ µm and RON = 3 e^-/pix for $\lambda > 0.95$ µm.

- DARKCUR: dark current of the detector. Default DARKCUR = 1 e^-/pix/hr for $\lambda \le 0.95$ µm and DARKCUR = 4 e^-/pix/hr for $\lambda > 0.95$ µm.

In case of signal to noise ratio computation, the input parameters to the ETC are the parameters reported above with the exception of the SN parameter, that becomes the output parameter, while the required input parameter is:

- MAG: magnitude, in AB units for compact source;
  or
- SB: surface brightness, in AB mag/arcsec^2 for extended source (in this case there is not the SLE parameter).

The derived quantities are computed as follows.

- Telescope area
  $$ATEL = \pi/4 \cdot 10^4 \cdot DTEL^2 \cdot (1 - COBS^2) \quad \mathrm{cm}^2$$

- Pixel sky projected angular size
  $$ANPIX = 0.036 \left(\frac{FCAM}{1.5}\right)^{-1} \left(\frac{DPIX}{10\mu\mathrm{m}}\right) \left(\frac{DTEL}{38.5\mathrm{m}}\right)^{-1} \quad \mathrm{arcsec}$$

- Number of pixels corresponding to spectrometer aperture
  $$PIXAPE = \frac{\pi/4 \cdot DAPE^2}{ANPIX^2} \quad \mathrm{pixels}$$

- Total number of read-out pixels per resolution element
  $$TOTPIXRE = INT(PIXAPE/PIXBINNED)$$

- Detector noise over detector area corresponding to spectrometer aperture
  $$NOISEDET = \sqrt{TOTPIXRE \cdot (NDIT \cdot RON^2/PIXBINNED + DARKCUR \cdot EXPTIME)} \quad e^-$$

- The variation with the airmass is considered as follow:
  $$SKYBCKAM = SKYBCK - 0.4 \cdot (AM - 1)$$

- Background flux in spectrometer aperture

$$\sigma_{sky} = \frac{10^{(16.85 - SKYBCKAM)/2.5}}{RPOW} \quad \text{photons cm}^{-2}\ \text{s}^{-1}\ \text{arcsec}^{-2}$$

$$\sigma_{th} = \frac{1.4 \cdot 10^{12} \cdot EBCK \cdot \exp\left[-14388/(WL \cdot TBCK)\right]}{WL^3 \cdot RPOW} \quad \text{photons cm}^{-2}\ \text{s}^{-1}\ \text{arcsec}^{-2}$$

$$NBCK = EFF \cdot ATEL \cdot \pi/4 \cdot DAPE^2 \cdot (\sigma_{sky} + \sigma_{th}) \qquad \text{e}^-\ \text{s}^{-1}$$

where

$$EFF = EFFI \cdot EFFT \cdot AT$$

is the total efficiency and AT is the atmospheric transmission.
The default values are interpolated from Table 6 and calculated at the appropriate AM, according to the following definitions (see also the related ESO document *https://www.eso.org/observing/etc/doc/elt/etc_spec_model.pdf*).

$$e^{-\tau_\lambda \cdot AM} \qquad \alpha = e^{-\tau_\lambda \cdot AM} \qquad AT = \alpha^{AM}$$

Table 6: Atmospheric transmissions.

WL (μm)	α (AM = 1)
0.36	0.67
0.44	0.83
0.55	0.90
0.70	0.98
0.90	0.99
1.00	1.00
1.25	1.00
1.65	1.00
2.16	1.00
2.60	1.00

- Background noise in spectrometer aperture and resolution element

$$NOISEBCK = \sqrt{NBCK \cdot EXPTIME}$$

- Total noise per resolution element

$$NOISETOT = \sqrt{NOISEBCK^2 + NOISEDET^2 + NOBJ}$$

where NOBJ is the total number of photo-electrons per resolution element produced by the object in the given exposure time.

- Signal to noise per resolution element

$$SN = \frac{NOBJ}{NOISETOT}$$

which can be written as

$$SN^2 = \frac{NOBJ^2}{NOISEBCK^2 + NOISEDET^2 + NOBJ} \qquad \textit{(ETC\_1)}$$

Solving for NOBJ yields

$$NOBJ = \frac{SN^2}{2} \cdot \left(1 + \sqrt{1 + 4\,\frac{(NOISEBCK^2 + NOISEDET^2)}{SN^2}}\right) \qquad \textit{(ETC\_2)}$$

- Object signal per resolution element on the detector

$$NOBJ = \frac{SLE \cdot EFF \cdot ATEL \cdot EXPTIME}{RPOW} \cdot 10^{(16.85-MAG)/2.5} \qquad \textit{(ETC\_3)}$$

Combining equations (ETC_2) and (ETC_3) and adopting MAG = MAGLIM yields the following explicit expression for MAGLIM, the limiting magnitude in AB units achievable for a compact source (i.e. a possible output of the program).

$$\begin{aligned} MAGLIM = \; & 16.85 \; - 2.5 \cdot \log_{10}\left(\frac{RPOW \cdot SN^2}{2 \cdot EXPTIME \cdot SLE \cdot EFF \cdot ATEL}\right) - \\ & + \; 2.5 \cdot \log_{10}\left(1 + \sqrt{1 + 4\,\frac{NOISEBCK^2 + NOISEDET^2}{SN^2}}\right) \end{aligned}$$

Combining equations (ETC_1) and (ETC_2) yields the following explicit expression for SN in case of a compact source (i.e. a possible output of the program).

$$SN = \sqrt{\frac{\left(\frac{SLE \cdot EFF \cdot ATEL \cdot EXPTIME}{RPOW} \cdot 10^{(16.85-MAG)/2.5}\right)^2}{NOISEBCK^2 + NOISEDET^2 + \frac{SLE \cdot EFF \cdot ATEL \cdot EXPTIME}{RPOW} \cdot 10^{(16.85-MAG)/2.5}}}$$

The same procedure applied in case of extended source, yields the following explicit expression for SBLIM, the limiting surface brightness in AB magnitude/arcsec^2 (i.e. a possible output of the program).

$$\begin{aligned} SBLIM = \; & 16.85 \; - 2.5 \cdot \log_{10}\left(\frac{RPOW \cdot SN^2}{2 \cdot EXPTIME \cdot EFF \cdot ATEL}\right) - \\ & + \; 2.5 \cdot \log_{10}\left(1 + \sqrt{1 + 4\,\frac{NOISEBCK^2 + NOISEDET^2}{SN^2}}\right) + \\ & + \; 2.5 \cdot \log_{10}\left(\pi/4 \cdot DAPE^2\right) \end{aligned}$$

and the following explicit expression for SN (i.e. a possible output of the program), adopting SBLIM = SB.

$$SN = \sqrt{\frac{\left(\frac{EFF \cdot ATEL \cdot EXPTIME}{RPOW} \cdot \frac{\pi}{4} \cdot DAPE^2 \cdot 10^{(16.85-SB)/2.5}\right)^2}{NOISEBCK^2 + NOISEDET^2 + \frac{EFF \cdot ATEL \cdot EXPTIME}{RPOW} \cdot \frac{\pi}{4} \cdot DAPE^2 \cdot 10^{(16.85-SB)/2.5}}}$$