(Sect. 4.1.3). We adopt the parametrization of Mattsson et al. (2012), where the grain growth timescale, $\tau_{\rm grow}$, is parametrized by a tuning parameter, $\epsilon_{\rm grow}$:

$$\frac{1}{\tau_{\rm grow}(t)} = \epsilon_{\rm grow}\frac{\psi(t)}{M_{\rm gas}(t)}\left(Z(t) - Z_{\rm dust}(t)\right), \tag{13}$$

where $\psi(t)$ is the SFR as a function of time t. This tuning parameter encompasses our uncertainty about grain sizes, sticking coefficients, and the fraction of cold clouds where dust growth occurs[23].

Dust destruction by SN II blast waves. SN II shock waves destroy dust by kinetic sputtering and grain-grain collisional vaporization (e.g., Jones et al. 1994). We adopt the dust destruction timescale, $\tau_{\rm dest}$, of Dwek & Scalo (1980):

$$\frac{1}{\tau_{\rm dest}(t)} = \frac{m_{\rm gas}^{\rm dest} R_{\rm SN}(t)}{M_{\rm gas}(t)}, \tag{14}$$

where $m_{\rm gas}^{\rm dest}$ is a tuning parameter quantifying the effective mass of ISM swept by a single SN II blast wave within which all the grains are destroyed.

5.2. Modeling the evolution of individual galaxies

We use the dust evolution model of Sect. 5.1 to fit, for each galaxy: $M_{\rm dust}$, $M_{\rm gas}$, $M_\star$, SFR, and Z. Dust destruction in X-ray emitting gas is not taken into account in this model. We therefore exclude the ETGs ($T \leq 0$) which would not be consistently fitted (cf. Sect. 4.1.2). The sample we model here contains 556 sources.

5.2.1. The model grid

We have generated a large grid of models that we interpolate to find the best evolution tracks for each of our sources. We assume a delayed SFH (Lee et al. 2010):

$$\psi(t) = \psi_0 \frac{t}{\tau_{\rm SFH}} \exp\left(-\frac{t}{\tau_{\rm SFH}}\right), \tag{15}$$

parametrized by the SFH timescale, $\tau_{\rm SFH}$, and a scaling factor, ψ_0. We assume a Salpeter (1955) IMF in order to be consistent with the observed SFR and stellar mass estimates (Sect. 2.2). Exchanges of matter between galaxies and their environment can have a significant role in regulating the global dustiness at all redshifts (e.g., Jones et al. 2018; Ohyama et al. 2019; Sanders et al. 2020; Burgarella et al. 2020). We account for inflow and outflow, assuming their rates are proportional to SFR: $R_{\rm in/out}(t) = \delta_{\rm in/out} \times \psi(t)$. Table 6 gives the parameters of our dust evolution model grid. We perform log-log interpolation of this grid in order to estimate the tracks corresponding to any arbitrary combination of parameters.

The SFH we have adopted here, to compute the chemical evolution, is different from the SFH used by Nersesian et al. (2019) to model the SEDs and estimate the observed SFR and $M_\star$ (cf. Sect. 2.2.1). Ideally, adopting the same SFH would be more consistent. However, adding a second functional form to Eq. (15), accounting for a potential recent burst, would push this model beyond our current computational capabilities, by increasing the number of free parameters. At the same time, the elemental and dust enrichment by this recent burst would probably be negligible. Alternatively, we could have also modeled the SED, using solely Eq. (15). The problem, in this case, is that the use of a single stellar population biases the estimate of $M_\star$ (cf. Appendix B.2).

[23] We did not implement the parameter f_c, the fraction of cold clouds, introduced by De Vis et al. (2017a), as its effect can be encompassed in $\epsilon_{\rm grow}$ and $m_{\rm gas}^{\rm dest}$ (Eq. (14)).

Table 6. Dust evolution parameter grid.

Parameter	Notation	Range	Step
Individual galaxy parameters			
Age [Gyr]	t	$[0.001, 15]$	0.15
SF timescale [Gyr]	$\tau_{\rm SFH}$	$[0.1, 30]$	0.57 (ln)
SFR scale [Gyr^{-1}]	ψ_0/M_0	$[0.0125, 12.5]$	0.26 (ln)
Inflow rate/SFR	$\delta_{\rm in}$	$[0.05, 4]$	0.37 (ln)
Outflow rate/SFR	$\delta_{\rm out}$	$[0.05, 4]$	0.13 (ln)
Common dust evolution parameters			
SN condensation	$\delta_{\rm SN}$	$[0.001, 1]$	0.37 (ln)
Grain growth	$\epsilon_{\rm grow}$	$[100, 10\,000]$	0.17 (ln)
SN dest. [$M_\odot$]	$m_{\rm gas}^{\rm dest}$	$[50, 1500]$	0.34 (ln)

Notes. Individual galaxy parameters control the particular SFH of each galaxy. Common dust evolution parameters are fitted to the whole sample, assuming their values are the same for each galaxy. The grid step (4th column) is logarithmic for all parameters except the time grid. The SFR scale is normalized to the total initial mass of the galaxy, M_0. All extensive quantities computed by the model are proportional to M_0, while intensive quantities are independent of M_0. In practice, we generate a grid for an arbitrary $M_0 = 4 \times 10^{10}\,M_\odot$. This parameter cancels out in the process, as we are fitting ratios (Sect. 5.2.2).

5.2.2. Hierarchical Bayesian dust evolution inference

To constrain our dust evolution model, we consider the posterior inference of our reference run (Sect. 3.2) as a set of observational constraints. We fit the four independent, intensive quantities: s$M_{\rm dust}$, s$M_{\rm gas}$, sSFR, and Z. The complex PDF, including the intricate parameter correlations, is preserved in this process. We make the restrictive assumption that the three tuning parameters, $\delta_{\rm SN}$, $\epsilon_{\rm grow}$, and $m_{\rm gas}^{\rm dest}$ are universal and are therefore identical in every galaxy. Consequently, we assume that the difference between galaxies is solely the result of their particular individual SFH. We vary the age of the galaxy, t, the two SFH parameters, $\tau_{\rm SFH}$ and ψ_0, and the inflow and outflow rates, $\delta_{\rm in}$ and $\delta_{\rm out}$.

On the universality of the tuning parameters. As we have seen, the three tuning parameters hide effects of the dust constitution, size distribution, fraction of cold clouds, etc. These quantities could vary across different environments. However, as we show in Sect. 5.3.1, the SN II dust condensation dominates in the low-metallicity regime, while grain growth and SN destruction are important above the critical metallicity. Thus, the parameters that we constrain are representative of the regime where they dominate the dust budget. Exploring their variation with the environment is probably premature. We are, however, able to infer the variation of the timescales across galaxies from Eqs. (11)–(14). In addition, if we were to infer one set of tuning parameters per galaxy, it would imply that they vary as a function of the galaxy's global parameters. In order to be consistent, we would then need to implement these variations of the tuning parameters in the dust evolution model, and change their value at each time step, accordingly.

The model prior. We have built a HB model to infer these parameters. The prior of the five SFH-related parameters is a