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## Shall Numerical Astrophysics Step Into the Era of Exascale Computing?

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**Abstract.** High performance computing numerical simulations are today one of the more effective instruments to implement and study new theoretical models, and they are mandatory during the preparatory phase and operational phase of any scientific experiment. New challenges in Cosmology and Astrophysics will require a large number of new extremely computationally intensive simulations to investigate physical processes at different scales. Moreover, the size and complexity of the new generation of observational facilities also implies a new generation of high performance data reduction and analysis tools pushing toward the use of Exascale computing capabilities. Exascale supercomputers cannot be produced today. We discuss the major technological challenges in the design, development and use of such computing capabilities and we will report on the progresses that has been made in the last years in Europe, in particular in the framework of the ExaNeSt European funded project. We also discuss the impact of these new computing resources on the numerical codes in Astronomy and Astrophysics.

### 1. Introduction

The last decade has seen the advent of numerous digital sky surveys across a range of wavelengths (e.g the Cosmic Microwave Background (CMB) experiments (Ade et al. 2014; Spergel et al. 2003), or the Sloan Digital Sky Survey (Eisenstein et al. 2011)), with terabytes of data and often with tens of measured parameters associated to each observed object. Moreover, during the next decade, new highly complex and massively large data sets are expected from novel and more complex scientific instruments that might provide new insights into the knowledge of fundamental laws of physics at cosmological scales and on the formation and evolution of cosmic structures (e.g. the Square Kilometer Array (SKA) (Dewdney et al. 2009), the Cherenkov Telescope Array (CTA) (Acharya et al. 2013), the Extremely Large Telescope E-ELT (de Zeeuw et al. 2014), the James Webb Space telescope (Gardner et al. 2006), etc).

At the same time, a new generation of large surveys, such as the ground based Large Synoptic Survey Telescope (LSST Science Collaboration 2009) (LSST), or the Euclid satellite missions (Laureijs et al. 2012) might shed light on the nature of dark energy and dark matter and possibly revolutionize modern physics.

Handling and exploring these new data volumes, and actually making real scientific discoveries, poses considerable technical challenges, in particular regarding access to a new generation of computing facilities and computational HPC algorithms.

In Astronomy and Astrophysics (A&A), High Performance Computing (HPC) numerical simulations are today one of the more effective instruments to compare observations with theoretical models. They enable Astronomers to understand nuanced predictions, as well as to shape experiments more efficiently. They are mandatory during the preparatory and operational phases of new scientific experiments. They also help capture and analyze the torrent of experimental data being produced by the new generation of scientific instruments. Computational modeling can illuminate the subtleties of complex theoretical models, making HPC infrastructures into theoretical laboratories to test physical processes.

The more accurate these theoretical experiments are, the more efficient the future large scale surveys will be in solving the mysteries of our Universe, so the size and complexity of the new experiments require extremely large computational resources, pushing toward the use of Exascale computing capabilities.

The development of Exascale computing facilities with machines capable of executing  $O(10^{18})$  floating point operations per second (FLOPS) will be characterized by significant and dramatic changes in computing hardware architecture from current petascale capable super-computers. To build an Exascale resource we need to address some major technology challenges related to Energy consumption, Network topology, Memory and Storage, Resilience and of course Programming model and Systems software.

From a computational science point of view, the architectural design of existing peta-scale supercomputers, where computing power is mainly delivered by accelerators (GPU, FPGA, Cell processors etc.), already impacts on scientific applications. This will become more evident on the future Exascale resources that will involve millions of processing units causing parallel application scalability issues due to sequential application parts, synchronizing communication and other bottlenecks. Future applications must be designed to make systems with this number of computing units efficiently exploitable.

An approach based on hardware/software (HW/SW) co-design is crucial to enable Exascale computing by solving the application-architecture performance gap (the gap between the capabilities of the HW and the performance released by HPC SW) and contributing to the design of supercomputing resources that can be effectively exploited by real scientific applications.

This paper will summarize the major challenges facing Exascale computing and how much progress has been made in the last years in Europe. We will present the effort done by the ExaNeSt EU funded project to build a prototype of an Exascale facility based on ARM CPUs and accelerators, designed using a HW/SW co-design approach, where Astrophysical codes are playing a central role in defining network topology and storage systems. Finally we will discuss how the co-design will impact on Numerical Codes that must be re-engineered to profit from the Exascale supercomputers.

## 2. Technical challenges in Exascale computing

A new generation of supercomputer able to perform  $10^{18}$  FLOPs, is expected to be available for the end of 2020. They will be between ten and one hundred times faster

than actual Tier-0 HPC facilities (as listed in the TOP500 (<http://www.top5000.org>) rank).

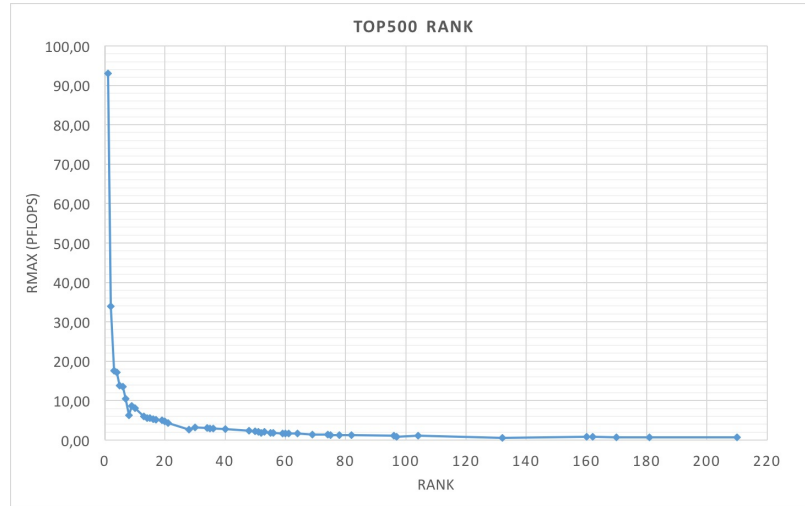


Figure 1. Performance measurement in PFLOPs for the first 200 most powerful supercomputers in the World. Performances are measured with the HPL (matrix operations) package (Petitet et al. 2016). The first supercomputer in the Top 500 rank is the Chinese Sunway (MPP) that performs 93 PFLOPs with 10 million cores.

The realization of an Exascale supercomputer requires significant advances in a variety of technologies both HW and SW. A series of studies in the last years in Europe (<http://www.etp4hpc.eu>), USA (Lucas et al. 2014) and Japan (Kobayashi 2015) categorize the technological challenges faced in a few core research topics: (a) High performance interconnect technology; (b) Memory technology (both DRAM and non volatile low-latency high density data storage); (c) System software and runtime system that ensures that applications are able to maximize the capabilities of the underlying HW; (d) Programming systems; (e) Data management for simulated and observed data; (f) Resilience.

Algorithms and scientific SW improvements contribute to the increasing of the computational capabilities as much as the HW innovation. Algorithms and code re-engineering, supported by the proper programming model and system SW is mandatory to exploit the Exascale platforms.

Beside the technological challenges listed above, there are other two crucial aspects that are also deeply related to all of them: the energy efficiency and a new approach towards the evaluation of the computing performance (Sustained performance) of super computers

With current semiconductor technologies, all proposed Exascale designs would consume hundreds of megawatts of power. If we just scale up the actual most powerful supercomputer to reach Exascale capacity, it will require about one GW to be operated. New designs and technologies are needed to reduce this energy requirement to a more manageable and economically feasible level. And those technologies involve not only the design of the CPUs (that should implement a simpler but energy efficient design (Lucas et al. 2014)), but also all the other aspects including algorithms and SW. The goal is to operate and Exascale facility with less than 50 MW.

## 2.1. Sustained Performance versus Peak Performance

In the last decade, the peak performance of high-end computing systems has been incredibly boosted by aggregating a huge number of nodes, each of which consists of multiple fine-grain cores, because the LINPACK benchmark (Petitet et al. 2016) used to rank facilities in the top 500, is only computation-intensive.

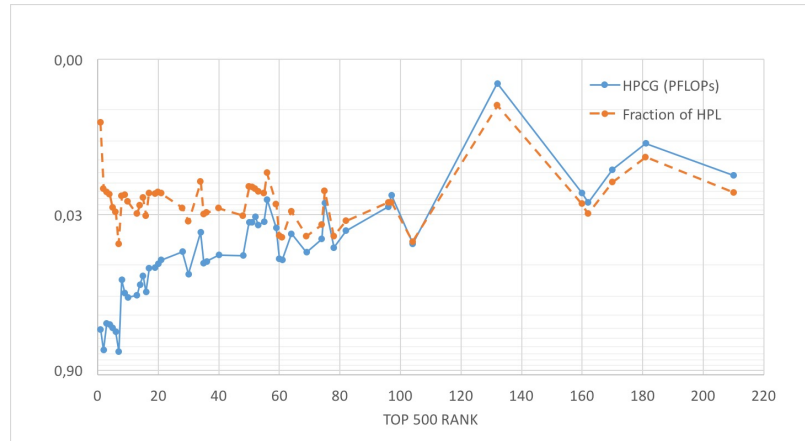


Figure 2. HPCG PFLOPs for the first 200 most powerful supercomputers in the World. We present the sustained performance of the first 200 most powerful supercomputers in the World as measured with HPCG. As reference we plot also the ratio between the HPCG and HPL performance. Sustained performance of the Chinese Sunway is  $10^{-3}$  less than the HPL measured performance.

However, many scientific applications, including N-Body Hydrodynamic Cosmological simulations, are memory-intensive: they are not only consuming floating point operations but they also need to access data in memory.

We measure the application behavior in terms of bytes per flop (B/F), that can be defined as a ratio of the memory throughput in bytes/s to the computing performance in flop/s of an HPC system. A recent paper from (Kobayashi 2015) shows that scientific applications commonly need 0.5B/F or more in the case of Astrophysical applications ( $> 1B/F$ ).

Dongarra et al. (2015) develop a new benchmark tool to account for sustained performance: the HPCG.

As shown in fig. 2, the sustained performance of actual supercomputers is not even in the PFLOPS, and scaling up to Exascale in sustained performance means a jump of more than 4 orders of magnitude.

Moreover, applications that implement a high ( $\approx 1$ ) B/F, impact also on the power consumption. For example, today extremely low power, high frequency GDDR5 RAM consumes about 4.3 W / 64 Gbs. To feed a modest 0.2 B/F for a sustained  $10^{18}$  FLOPS requires about 12MW of power just for memory access.

## 3. The ExaNeSt approach to Exascale

With architecture evolution, the HPC market has undergone a fundamental paradigm shift. The adoption of low-cost, Linux-based clusters extended HPC's reach from its

roots in modeling and simulation of complex physical systems to a broader range of industries, ranging from cloud computing and deep learning, to automotive and energy, many of which were originally served by datacenters. ExaNeSt (Katevenis et al. 2016) belongs to a group of ongoing EU projects supporting the next step forward in this direction. Today, low-power microprocessors dominate the embedded, smartphone and tablet markets, outnumbering x86 devices both in volume and in growth rate. If these trends continue, we can expect to see such energy-efficient servers benefiting from the same economies of scale that in the past favored general-purpose personal computers (over mainframes) and more recently commodity clusters (over custom supercomputers).

ARM is the industry leader in power-efficient processor design. ARM processors consume about 2 to 3 times less electrical energy for a given amount of computation relative to Intel-based processors, and are widely used in embedded consumer electronics, including mobile phones and tablets. As a result, many research and industry programs see ARM-based energy-efficient servers as a potential successor to x86 and POWER-based servers in hyperscale datacenters supercomputers.

ExaNeSt partners with a number of concurrent FET-HPC projects that aim to collectively answer the HPC challenges described in the strategic vision statement of ETP4HPC. Common across all projects is the technological approach for scalable, low-power and economically viable solution for compute, as is being refined and realized in the EuroServer (Marazakis et al. 2016). EuroServer has common participants across ExaNeSt and various other consortia. Here is a summary of the projects that are aligned with this approach:

- ExaNeSt (Katevenis et al. 2016): is responsible for the physical deployment characteristics to support the required compute density, along with the storage and interconnect services.
- ExaNoDe (<http://www.exanode.eu>): focuses on the delivery of low-power compute elements for HPC.
- ECOSCALE (Lawereins 2016): focuses on integrating and exposing the acceleration capabilities of FPGAs in HPC.
- Eurolab-4-HPC (<http://www.eurolab4hpc.eu>): is a supporting action CSA that is to support the actions and initiatives around delivery strategy for ETP4HPC vision.

Together, these and other initiatives, at local and international level, have the goal to deliver a European solution for high performance computing and to ensure Europe's leadership in HPC.

### **3.1. The co-design approach for software and hardware**

An intensive co-design effort is essential to build an Exascale system. The design of the ExaNeSt platform is tailored to real scientific and industrial applications used to define the requirements for the architecture and, at a later stage, to evaluate the final solution. The behavior of scientific applications is analyzed to optimize the interconnect and the data management of the platform both in terms of HW and SW. Applications have been instrumented to provide their network and I/O traces and the traces have been analyzed to understand the type and kind of message exchanged by application tasks (Fig 3).

On the other side applications will be re-engineered to exploit the ExaNeSt platform, parallel algorithms will be adapted to it, and new algorithms and implementations will be developed to approach the computational capabilities of the new co-designed HW.

### 3.2. ExaNeSt Technology: challenges and solutions

Presently, ExaNeSt is designing two prototype systems based on packaging technology by Iceotope (<http://www.datacenterknowledge.com/archives/2013/03/04/iceotope-liquid-cooling-in-action/>). The overall objective is to stress our complete solution (interconnect, storage, systems software) using real-world HPC applications, which will be ported to the prototypes. The systems that are developed will be based on Xilinx Ultrascale+ FPGAs, with quad ARM Cortex-A53 64-bit cores. Each compute daughter-board will consist of four FPGAs (i.e. with a total of 16 cores) and an NVMe in-node SSD storage device.

The first prototype (Track-1) will exploit existing liquid immersion technology from Iceotope, able to cool the thermal drive of 800W per blade. Each blade in Track-1 will host 4-8 compute daughter-boards (16-32 FPGAs). The total size of Track-1 prototype will depend on the cost of Xilinx FPGAs: with our current price estimates, it is expected to range between 6 and 16 blades. Track-2 prototype will use novel liquid cooling, developed by Iceotope during course of the ExaNeSt project. The new cooling technology will enable even denser designs, with 16 compute daughter-boards per blade.

#### 3.2.1. Rack-level shared memory

The ExaNeSt project develops architectures for systems with densely-packed compute, memory and storage devices, interconnected using high-performance interconnects. For many big data and HPC applications, the access to fast DRAM memory is a key to performance.

In our design, the memory attached to each compute node has modest size – i.e., tens of GB per compute node; in order to make many hundreds of DRAM available to each compute node, we will plan to enable remote memory sharing.

Our memory architecture will be based on *Unimem*, first developed within EuroServer (Marazakis et al. 2016). With Unimem, a node can access parts of memories located in remote nodes. To eliminate the complexity and the costs of system-level coherence protocols (Laudon & Lenoski 1997), the Unimem architecture defines that each physical memory page can be cached at only one location. In principle, the node that caches a page can be the page owner (the node with direct access to the memory device) or any other remote node; however, in practice, it's preferred that remote nodes do not cache pages.

In ExaNeSt, we are extending Unimem to operate efficiently on a large installation with real applications. One important enhancement is to enable a *virtual* global address space, rather than a physical one, as was the case in Euroserver. This improves security, allows page migration, and can also simplify multi-programming, just as virtual memory did in the past for single node systems.

### 3.2.2. In-node Data Storage

Modern HPC technology promises “true-fidelity” scientific simulation, enabled by the integration of huge sets of data coming from a variety of sources. As a result, the problem of “big data” in HPC systems is rapidly growing, fueling a shift towards *data-centric HPC architectures*. Low-power, low-cost, fast non-volatile memories (NVM) (e.g. flash-based) is a first key enabling technology for data-centric HPC. The decreasing cost of low-power fast non-volatile memories (NVM) (e.g. flash-based) is changing the computing landscape dramatically. These devices promise to narrow the storage-processor performance gap with low latency (tens of microseconds vs. 10+ milliseconds) and high I/O operations per second (IOPS) performance at capacities of several hundreds GBytes. ExaNeSt will place these storage devices with the compute nodes rather than in a centralized location. We propose extensions to a parallel file system to take advantage of such devices as a cache layer. Moreover, we design cache maintenance protocols based on the concepts of the Unimem memory consistency model. Our high-level goal is that as long as the processors stay within the (extended) coverage offered by their local (in-node) storage devices, they achieve low-latency and low-power access to data, avoiding the movement of data across long distances.

### 3.2.3. Unified Interconnect

The ExaNeSt project focuses on a tight integration of fast NVM devices at the node level using Unimem to improve on data locality. The presence of distributed low-latency devices introduces new challenges for the underlying interconnect. In this project, we advocate the need for a unified system interconnect that will merge inter-processor traffic with a major part (if not all) of storage traffic. This consolidation of networks is expected to bring significant cost and power benefits, as the interconnect is responsible for 35% of the power budget in supercomputers and consumes a lot of power even when it idles (Hoefler 2010).

The most advanced inter-processor interconnects, although customized to provide ultra-low latencies, typically assume benign, synchronized (application-clocked) processor traffic. Pulling the bursty storage flows inside a unified interconnect may negatively affect performance, thus calling for advanced Quality-of-Service (QoS) support. We will address the design of a suitable unified interconnect for exascale systems by splitting the work into two major parts. In the first part, we address the interconnect within a rack, examining suitable low-power electrical and optical technologies and appropriate topologies. We address system packaging and topology selection in tandem, aiming at multi-tier interconnects (Kodi et al. 2014), to address the disparate needs and requirements at separate building blocks inside the rack (e.g. mezzanine board, chassis, etc.). A set of alternative state-of-the-art photonic and optical link technologies will be explored.

## 4. ExaNeSt Applications and Co-design: Astrophysical Numerical simulations

A set of relevant and ambitious code from different scientific areas has been identified for co-design, including HPC for astrophysics, nuclear physics, neural networks and big data.

In A&A, often the scientific problems under investigation involve either complex physics, a very large dynamical range, or both. This kind of computation requires high



resolution, that translates in a very large number of computational elements - usually particles.

In the ExaNeSt project we use the TreePM+SPH code GADGET-3, evolution of the public code GADGET-2 (Springel 2005) and PINOCCHIO semi-analytical code (Monaco et al. 2002). In simulations used in ExaNeSt, besides gravity and hydrodynamics, the following processes are modeled: radiative cooling of the gas, star formation, chemical evolution, energy feedback from exploding stars, a uniform time-dependent UV background, evolution of black holes and energy outputs from them.

These numerical computations are dynamical in space and time: the computation evolves from an initially homogeneous, easy to balance configuration, to an extremely dis-homogeneous one. Furthermore, galaxies move with respect to each other, possibly colliding and clustering together. This behavior makes the load-balancing a severe issue and those codes an excellent and challenging candidate to design and optimize the interconnect of a supercomputer.

In ExaNeSt project, we used two different cosmological simulations. The first one is a portion of the Universe, a cube having a side of 25 Mpc, with relatively low resolution (details in Barai et al. (2015)). This simulation does not resolve the internal structure of galaxies and is thus relatively easy to balance in all its phases. The second one follows the birth and the evolution of a single galaxy. Here, the resolution is higher and the dis-homogeneity towards the end of the simulation is larger (details in Murante et al. (2015)). In neither case we run the full simulation, which would be very time-consuming. So we instrument the code and run the simulation to collect network traces for small temporal intervals at the beginning of the simulation, at intermediate times and towards the end, so as to sample the different dynamical situations, that can reflect in different communication patterns and workload balances. An example of traces collected is shown in Fig. 3.

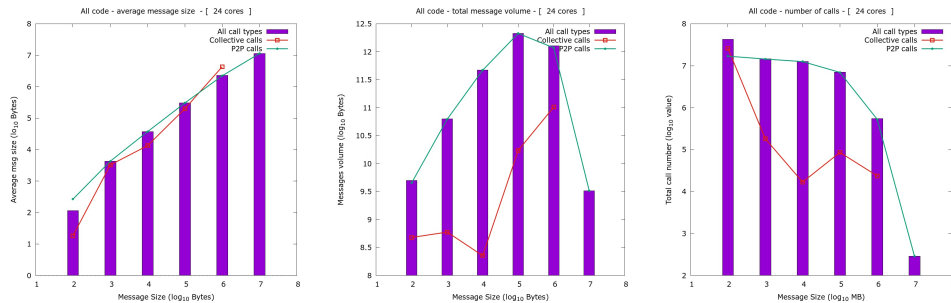


Figure 3. The plots show the average message size [left panel] the total amount of data [central panel], both in  $\log_{10}$  Bytes, and the total number of calls to MPI\_ functions [right panel]. All data have been binned per message size to which they refer to (x axis,  $\log_{10}$  Bytes).

## 5. Implementing an exascale Astrophysical HPC application

GADGET is a full-fledged high-performance code for cosmological simulations. However, some architectural concerns arise on the verge of the exascale era.

Parallel machines are increasingly built with (cc)NUMA hierarchical architectures in which multiple cores have access to a hierarchy of memories. Multiple cache memories are placed at the lowest (fastest) levels of the hierarchy, and every core may have individual caches, while groups of cores may share higher level caches or single memory controllers. On such systems it is crucial that codes take advantage of spatial and temporal locality of data, which also requires them to be NUMA-aware. This concept translates into the "affinity" that a given memory segment has with a particular computing core. The GADGET memory model is not NUMA-aware and tries to exploit memory locality only with some basic stratagem. Hence the re-design of the code must start from the re-design of the memory model so as to consider the different affinity of different memory regions. Due to the extreme diversity of physical processes being modeled and algorithms implemented, this a difficult task that does not have a unique solution and may require memory layout transformations in some points.

Secondly, the code has been conceived as the parallel generalization of a serial code, in a pre-multithread era. Hence the workflow is rigidly procedural, meaning by this that all the tasks perform the same operations - possibly individually using more threads in local loops - with frequent synchronization via MPI messages. Fig. 3 reveals at glance that the number of communications that exchange very small amount of data (few to hundreds of bytes) largely outnumber those that exchange large data chunks (0.1-10 or more MB), although the latter account for most of the total data traffic on the machine network. As a consequence, the network latency can represent a significant fraction of both the communication and running times. In view of these two facts, and the strongly hierarchical architecture foreseen for the exascale machines, it is compulsory to adopt a different code design i.e. to decompose the workflow in as-small-as-possible single tasks with clear dependencies on, and conflicts between, each other from both the point of view of operations to be performed and data to be processed. In such a way, the workflow would be translated in a queue system where idling threads perform the first available tasks on not-under-use data. Synchronization of operations should pass as much as possible through RDMA operations and the queue system itself. Moreover, 'encouraging' threads that reside on the same group of cores to undergo similar tasks, or tasks operating on the same data, should lead to a more efficient exploitation of memory affinity and locality (even if redundancy of some data may be required).

We will start from a stripped-down version of the publicly available code, that incorporates only the gravity and hydrodynamical solvers, and the star formation plus the stellar evolution modules. We plan to separately develop mini-apps for each of the core algorithms, designed ab-initio as "atomic" inter-dependent tasks. In this way we will be able to develop different algorithms and strategies for each of the "pillars" detailed above in a much easier way than in a unique monolithic code.

## 6. Conclusions

Today supercomputers must be considered as theoretical laboratories necessary to Astronomers as much as any observational facilities. New generations of super computers will be able to perform  $10^{18}$  FLOPS however it is unlikely that Exascale is achievable without disruptive changes in the way the super computers will be built and in the way we will use them. Astronomers will be obliged to re-engineer their applications in terms of a new paradigm based on task based programming, innovative resilience and

heterogeneous computing (CPUs/GPUs/HW accelerators). Moreover, it will be necessary to bring computation close to the data. As a consequence, only applications that implement a low B/F will exploit Exascale computations. This will be in practice extremely complex when dealing with Big Data, opening new challenges for the A&A community in prevision of the new experiments.

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