Date	**01/12/2009**
Issue	**1**
Revision	**0**
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MARSIS RADAR: IONOSPHERE PHASE DISPERSION COMPENSATION

Issue 2, Rev 0

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1. INTRODUCTION

One of the main challenges that the Marsis radar (on-board the Mars Express Mission) needs to deal with are the effects introduced in an electromagnetic wave by the Mars ionosphere. According to the recent literature [1], the profile of the electron density and thus of the plasma frequency for increasing altitude raises sharply towards a single well-defined maximum located at an altitude of about 100-150 Km and then drops off smoothly as the altitude increases.

The value of the maximum plasma frequency $f_{p,max}$ is a function of the solar flux density, as well as of the Solar Zenith Angle (SZA), which is dictated by the illumination condition (day/night). According the behavior of fig.1.

on day time the $f_{p,max}$ can be as high as 3-4 MHz

- on night time the maximum value of $f_{p,max}$ should be about 0.8-1 MHz.

We can notice that the phase distortion arising from the Ionosphere produces a delay, an increase of sidelobes level (after matched filter), a distortion of the waveform shape and a loss of signal to noise ratio. Propagation effects on the MARSIS Radar Signals [2], taking into account the very large fractional bandwidth, will be investigated and a compensation technique will be proposed.

2. IONOSPHERE MODELS

In order to characterize the distribution of the plasma frequency vs. height and to quantify the amount of distortion due to the ionosphere propagation the “gamma” model can be used.

By comparison of Night [3] time and Day time ionosphere profile [4] with the

2.1 Gamma model

(1) $$f_P(z) = f_{p-max} \frac{z-h_0}{b} e^{1-\frac{z-h_0}{b}} \delta_{-1}(z-h_0)$$

where

f_{p-max} : maximum plasma frequency

b : shape factor

h_0 : beginning of the ionosphere layer

we have the behaviors of Fig.2 and the corresponding phase errors (s. Fig.3) are very small, with reference in particular to the requirements of the compressed return echoes; requirements given in terms of pulse widening (<20%) and side lobe level to allow the subsurface interface detection (the level of the return echoes, with reference to the surface echo, can decrease linearly with depth until 60 dB for depth bigger of 3 Km), as will appear more clear in the following.

A suitable range for the parameter b of eq.(1) appears (s.Fig.4):

(2) $$20 < b < 50 \text{ Km}$$

We can remember that the phase distortion induced by plasma layer of thickness L as a function of the frequency f depends on the profile of the plasma frequency and can be expressed as:

(3) $$\Delta\phi_{GAMMA}\left(f,b,f_{P,MAX}\right)=\frac{4\pi}{c}f\int_{0}^{L}\left[\sqrt{1-\left(\frac{f_P(z)}{f}\right)^2}-1\right]dz$$

so that the compensation procedure is related to the estimation of b and $f_{p,max}$
In order to reduce the estimation complexity, we can approximated the eq.(3) with the following:

(4) $$fit(f)=a_o+a_1(f-f_o)+a_2(f-f_o)^2+\ldots a_\nu(f-f_o)^\nu$$

where f_0 is the carrier frequency. We can notice that a_o not introduce distortion, a_1 introduce only time displacement and in any case a coarse estimation of a_1 can be obtained from the extra time delay respect to the free-space delay (taking into account the accuracy in the knowledge orbital parameter), moreover the estimation of $a_2 \ldots a_\nu$ can be obtained by signal optimization procedure. We wish consider the possibility of estimate a_2 by contrast method applied on the received signals and to estimate $a_3 \ldots a_\nu$ by theoretical extrapolation technique: using closed loop approach, as discussed in the following: in any case we must minimize the ν.
In order to find the coefficients of the polynomial fit(f) of degree ν, that fits the integral of Gamma model in a least squares sense, we have used of the Levenberg's and Macquardt algorithm, obtaining the values of Tab.1 for ν=3 and ν=4, by considering the bandwidth of 1 MHz and night and day side operation with extreme b values.

							ν=3		ν=4		
b (km)	**h** (km)	**h_o** (Km)	**f_o** (MHz)	**f_{p-max}** (MHz)	**a_{o-bf}**	**a_{1-bf}** (MHz^{-1})	**a_{2-bf}** (MHz^{-2})	**a_{3-bf}** (MHz^{-3})	**a_{2-bf}** (MHz^{-2})	**a_{3-bf}** (MHz^{-3})	**a_{4-bf}** (MHz^{-4})
50		120	1.8	0.65	-186	108	-70	45	-64	45	-29
50	800	120	1.8	0.8	-285	170	-118	80	-106	80	-57
50	800	120	1.8	1	-456	285	-224	174	-191	174	-147
20	800	120	1.8	0.65	-464	270	-177	112	-161	112	-73
20	800	120	1.8	0.8	-713	426	-296	201	-264	201	-143
20	800	120	1.8	1	-1139	714	-559	436	-478	436	-368
50	800	120	5	2	-637	135	-30	7	-30	7	-2
50	800	120	5	3	-1495	348	-90	25	-88	25	-8
50	800	120	5	4	-2864	803	-301	139	-283	139	-79
20	800	120	5	2	-1593	338	-75	17	-74	17	-4
20	800	120	5	3	-3739	870	-225	63	-221	63	-19
20	800	120	5	4	-7160	2010	-752	349	-709	349	-197

Tab.1

By considering the range compressed signals, after the compensation (with the best fitting parameters given in Tab.2.1; the Hanning weighting function is used, to allow the required low level of sidelobes) of the ionosphere distortion (gamma model), as shown in Fig.5 and Fig.6, we can conclude that the compensation of the ν=4 term is wanted, also if estimation and software constraints must be taken into account.

2.2 Uniform model

In order to reduce the estimation complexity, an alternative simplified ionosphere model characterized by a constant plasma frequency $f_{p,eq}$ and thickness L_{EQ} can be introduced, so that the eq. (3) becomes:

(5) $$\Delta\phi_{EQ}(f) = \frac{4\pi L_{EQ}}{c} f \left(\sqrt{1 - \left(\frac{f_{p,eq}}{f}\right)^2} - 1 \right) = 2\pi\tau_0 \left(\sqrt{f^2 - f_{p,eq}^2} - f \right)$$

Moreover a *one-dimensional* version of the uniform equivalent model can be introduced, if the ionosphere equivalent thickness is fixed to an average value $L_{EQ}=L_m$ (80 km) (τ_o=533 μsec).
In this case the phase errors increase according the behavior of Fig.7, but these values can be accepted, as is shown in Fig.8, where the phase distortion effects on the impulsive response are shown. This estimation procedure entails to obtain $f_{p,eq}$, ; Fig.9 shows the behavior of $f_{p,eq}$ vs. the gamma parameters; we can notice that $f_{p,eq}/f_o$ doesn't depend on b, it depends on $f_{p,max}/f_o$ only.
Moreover we can write:

(6) $$\Delta\phi_{EQ}(f) = 2\pi\tau_0 \left(\sqrt{f^2 - f_{p.eq}^2} - f \right) \cong a_0 + a_1(f - f_0) + a_2(f - f_0)^2 + a_3(f - f_0)^3 + \dots$$

where

$$a_0 = 2\pi\tau_0 \left(\sqrt{f_0^2 - f_{p.eq}^2} - f_0 \right) \qquad [rad]$$

$$a_1 = 2\pi\tau_0 \left(\frac{f_0}{\sqrt{f_0^2 - f_{p.eq}^2}} - 1 \right) \qquad [rad/Hz]$$

$$a_2 = -2\pi\tau_0 \left(\frac{f_{p.eq}^2}{2(f_0^2 - f_{p.eq}^2)^{\frac{3}{2}}} \right) \qquad [rad/Hz^2]$$

$$a_3 = 2\pi\tau_0 \left(\frac{f_0 f_{p.eq}^2}{2(f_0^2 - f_{p.eq}^2)^{\frac{5}{2}}} \right) \qquad [rad/Hz^3]$$

$$a_4 = -2\pi\tau_o \left(\frac{4 f_o^2 f_{p.eq}^2 + f_{p.eq}^4}{8\left(f_o^2 - f_{p.eq}^2\right)^{\frac{7}{2}}} \right) \qquad \left[rad/Hz^4\right]$$

To estimate a_2 (that is the most important term of the phase dispersion after the linear one) we use the contrast method, which allows also the estimation of f_p.

The 3rd (and eventually 4th) order phase term shall be obtained by a_2 according to the previous equations, as matter of fact we can select among 2 different methods to obtain the cubic term:

- according to the single parameter equivalent model from the term a_2 we can obtain the equivalent plasma frequency and then the term a_3
- for low value of the ratio fp.eq/fo, $a_3 \approx -a_2$ /fo, this last solution is easier to implement than the preceding one but gives bigger error as more the plasma frequency is close to the transmitted frequency (especially during the daytime when we operate with the higher frequencies)

3. CONTRAST METHOD: Design Approach

Considering now only the effect of the quadratic phase term on the chirp signal compression, we can relate easily the quadratic phase distortion in the spectrum with the mismatching in the chirp slope μ of matched filter in the receiver.

Taking into account the behavior of Fig.10 [5], we can define the requirement in the mismatching allowed

(7) $$\gamma TB \le 2$$

where (according to the MARSIS design):

T is the chirp duration (250 μsec)

B is the chirp bandwidth (1 MHz)

$\gamma = \frac{\Delta\mu}{\mu}$ is the mismatching factor

$$\mu = \frac{2\pi B}{T} = 2.51*10^{10}\sec^{-2} \rightarrow \gamma \le 8*10^{-3}$$

therefore the required *accuracy* in the compensation of the quadratic term coefficient (a_2) is:

(8) $$\Delta a_2 = \frac{\pi\gamma T}{B} = 6*10^{-12} \text{ [rad/Hz}^2\text{]}$$

Moreover the *research area* (in terms of a_2) during the tracking phase is function of the ionosphere variation in the space covered by the orbiter during each synthetic aperture (integration) time (since the compensation is updated adaptively every frame).

In this case we have supposed that the maximum variation of $|\Delta f_{p.eq}| \approx |\Delta f_{p.max}| \approx \pm 50\text{KHz}$, so that we can assume:

(9) $$a_2 = \pm 30*10^{-12} \text{ [rad/Hz}^2\text{]}$$

Moreover we can notice that the error, on the cubic term, accepted can be evaluated by considering the distortion introduced from an approximated sinewave term (obtained through the subtraction of the cubic term from the best fitting linear term), the corresponding amplitude must be less then 1 rad:

(10) $$\Delta a_3 \left(\frac{B}{2}\right)^2 (f-f_0) - \Delta a_3 (f-f_0)^3 \Big|_{f-f_0=\frac{B}{4}} \approx \Delta a_3 B^3 \frac{1}{21} \approx 5*10^{16}\Delta a_3 < 1 \quad \text{rad}$$

$$\rightarrow \quad \Delta a_3 < 20*10^{-18} \quad rad/Hz^3$$

In order to evaluate the error introduced by

(11) $$a_3 = -a_2/fo$$

we can write (s.eq.(5))

(12) $$\frac{a_3}{a_2} = -\left(\frac{f_o}{(f_0^2 - f_{p.eq}^2)}\right) \approx -\frac{1}{f_o} - \frac{f_{p.eq}^2}{f_o f_o^2} \approx -\frac{1}{f_o}\left(1 - \frac{a_2 f_0}{\pi\tau_0}\right)$$

therefore from eq (10) we can have:

$$\Delta a_3 \approx (a_2)^2 \frac{1}{\pi\tau_o} < 20*10^{-18} \quad rad/Hz^3 \rightarrow (a_2)^2 < 20*10^{-18}\pi\tau_o \approx 3.4*10^{-20} \rightarrow a_2 < 200*10^{-12}$$

The a_2 can be estimated $(\hat{a}_2)$ through the Amplitude Contrast Maximization technique [6] (s.Annex 1), that is the estimation of the phase distortion of the received signal based on the principle that the output of the matched filter is maximally sharp, when the reference function matches perfectly the phase distortion spectrum of the received signal. Moreover a_3 can be obtained from eq. (12): τ_o can be adapted in order to improve the results.
According to the previous data, some results are shown in Fig.11, to be compared to the data of Fig.5.
The estimation can improved by inserting also the

(13) $$a_4 = -a_3/fo$$

and the results are shown in Fig.12, and must be compared with the behavior of Fig.6.
Tab. 2 shows the error of estimation of a_2 and a_3 in the previous cases (s Tab.1)

			ν=3		ν=4		
b (km)	**f_o** (MHz)	**$f_{p\text{-}max}$** (MHz)	**Δa_2** (MHz^{-2})	**Δa_3** (MHz^{-3})	**Δa_2** (MHz^{-2})	**Δa_3** (MHz^{-3})	**Δa_4** (MHz^{-4})
50	1.8	0.65	-2	-2	-4	-4	7
50	1.8	0.8	-4	-5	-2	-15	20
50	1.8	1	-18	-11	-14	-39	73
20	1.8	0.65	-5	5	-2	-9	16
20	1.8	0.8	-16	21	-10	-11	37
20	1.8	1	-39	72	-47	-9	133
50	5	2	-3	0	2	-1	0
50	5	3	-2	-3	1	-4	3
50	5	4	-9	-30	-22	-32	57
20	5	2	-1	1	-3	0	1
20	5	3	-3	6	-4	7	6
20	5	4	-19	97	-30	66	113

Tab.2

Eq.s (12), (13) $\Delta a_2 = \hat{a}_2 - a_{2.bf} \Leftrightarrow \Delta a_3 = \hat{a}_3 - a_{3.bf} \Leftrightarrow \Delta a_4 = \hat{a}_4 - a_{4.bf}$

In order to improve the estimation technique, taking into account the approximation due to limited number of the parameters a_i and very large fractional bandwidth, we have analyzed the best fitting data and the optimum relations between a_2, a_3 and a_4 can be estimated (s.Annex 2), so that the eq.s (12),(13) become:

f_o=1.8 MHz:

(14)
$$a_3 \approx -\frac{a_2}{f_{o1}}\left(1-\frac{a_2 f_{01}}{\pi\tau_{01}}\right) \qquad f_{01} = 1.4MHz$$
$$a_4 \approx -\frac{a_2}{1.1\cdot f_{01}^2}\left(1-\frac{a_2(1.1\cdot f_{01})}{0.5\cdot\pi\tau_0}\right) \qquad \tau_{01} = 700\mu\sec$$

f_o=5 MHz:

(15)
$$a_3 \approx -\frac{a_2}{f_{o1}}\left(1-\frac{a_2 f_{01}}{\pi\tau_{01}}\right) \qquad f_{01} = 2.8MHz$$
$$a_4 \approx -\frac{a_2}{0.95\cdot f_{01}^2}\left(1-\frac{a_2(0.95\cdot f_{01})}{0.5\cdot\pi(0.7\cdot\tau_0)}\right) \qquad \tau_{01} = 1600\mu\sec$$

so that the data of Tab.3 and the behaviors of Fig.13 and Fig.14 arrive.

			ν=4		
b (km)	**f_o** (MHz)	**$f_{p\text{-}max}$** (MHz)	**Δa2** (MHz^{-2})	**Δa3** (MHz^{-3})	**Δa4** (MHz^{-4})
50	1.8	0.65	2	2	-9
50	1.8	0.8	4	-3	-7
50	1.8	1	-2	-19	10
20	1.8	0.65	11	5	-28
20	1.8	0.8	14	6	-47
20	1.8	1	-1	10	-3
50	5	2	2	3	-2
50	5	3	1	7	-6
50	5	4	-4	-20	23
20	5	2	3	9	-7

20	5	3	7	22	-19
20	5	4	-3	6	-1

Tab.3

We can notice that in the last figures (15, 16, 17), the increasing of side lobe due to Fresnel ripple to the level [7]:

(16) $$\text{S.L.F.} = 20\log(\tau\, B)+3 = = 20\log(250*1)+3 = 51\ \text{dB}$$

was software compensate, by ripple extraction (in the 1 MHz bandwidth) by FFT of reference function (Ref-fun-def(f)) and performing Ref-prod (s.Annex 1).

4. CONTRAST: Evaluation of Noise Effects

By analytical point of view, we can assume for simplicity a gaussian pulse $X_r(t)$ after detection

(17) $$X_r(t) = A\, e^{\left(-\frac{(t-t_0)^2}{2\sigma_r^{\,2}}\right)} + n(t)$$

with

(18) $$\sigma_r^{\,2} = \left(\frac{\tau_0 w}{2}\right)^2$$

where

τ_0 is the minimum pulse duration (after ideal matched filter).

t_0 is the delay time due to ionosphere phase dispersion.

w is the pulse duration widening due to mismatching referred to ionosphere quadratic error.

The *Contrast* of the signal X_r is defined by the ratio of the standard deviation of the signal to its average value and we obtain

(19) $$C = \sqrt{\frac{2\gamma}{w\sqrt{\pi}} \cdot \frac{\mathrm{Erf}\left(\frac{2}{w}\cdot\left(\frac{\gamma}{2}-\alpha\right)\right) + \mathrm{Erf}\left(\frac{2}{w}\cdot\left(\frac{\gamma}{2}+\alpha\right)\right)}{\left\{\mathrm{Erf}\left(\frac{\sqrt{2}}{w}\cdot\left(\frac{\gamma}{2}-\alpha\right)\right) + \mathrm{Erf}\left(\frac{\sqrt{2}}{w}\cdot\left(\frac{\gamma}{2}+\alpha\right)\right)\right\}^2} - 1}$$

where $\gamma = T/\tau_0$, $\alpha = t_0/\tau_0$ being T the integration time

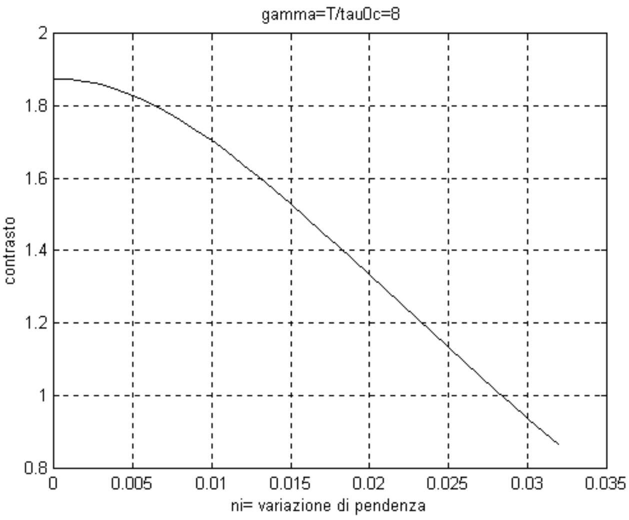
For $T \gg \tau_0$ and $\gamma \gg \alpha$ the previous equation becomes

(20)
$$C_W = \sqrt{\frac{\gamma}{w\sqrt{\pi}} - 1}$$

where w is the -3dB widening factor of the pulse due to the quadratic slope mismatching and it can be related to the quadratic phase distortion by the following relation

(21)
$$w \cong 1 + 0.025(vTB)^2 = 1 + \delta x$$

where $\nu = \frac{\Delta\mu}{\mu}$ is the mismatching factor and μ is the chirp slope defined by $\mu = \frac{2\pi B}{T}$



From figure we notice that in absence of slope mismatching the contrast assume is maximum value.

Moreover being $a_2 = \frac{2\pi^2}{\mu}$ we can write

$w \cong 1 + 0.025\left(\frac{\delta a_2 B^2}{\pi}\right)^2$ and the contrast given becomes an explicit function of δa_2

(22)
$$C_X = \sqrt{\frac{\gamma}{w\sqrt{\pi}} - 1} = \sqrt{\frac{\gamma}{(1+\delta x)\sqrt{\pi}} - 1} = \sqrt{\frac{\gamma}{\sqrt{\pi}}(1-\delta x) - 1} = \sqrt{\frac{\gamma}{\sqrt{\pi}} - 1 - \frac{\gamma}{\sqrt{\pi}}\delta x} \approx \sqrt{\frac{\gamma}{\sqrt{\pi}} - 1}\left(1 - \frac{\frac{\gamma}{2\sqrt{\pi}}\delta x}{\frac{\gamma}{\sqrt{\pi}} - 1}\right)$$

(23)
$$C_X \approx C - \frac{(C^2+1)}{2C}\delta x$$

(24)
$$C_n \approx \frac{\sqrt{\sigma_{X_r}^2 + \gamma\sigma_n^2}}{\overline{X}_r + \gamma\overline{x}_n \pm \sqrt{(2-\frac{\pi}{2})\gamma\sigma_n^2}} \approx \frac{\sigma_{X_r}\left(1+\gamma\frac{\sigma_n^2}{2\sigma_{X_r}^2}\right)\left(1-\gamma\frac{\overline{x}_n}{\overline{X}_r} \pm \sqrt{(2-\frac{\pi}{2})\gamma}\frac{\sigma_n}{\overline{X}_r}\right)}{\overline{X}_r} \approx$$
$$\approx C + \gamma C\frac{\sigma_n^2}{2\sigma_{X_r}^2} - \gamma C\frac{\overline{x}_n}{\overline{X}_r} \pm C\sqrt{(2-\frac{\pi}{2})\gamma}\frac{\sigma_n}{\overline{X}_r}$$

(25)
$$C\sqrt{(2-\frac{\pi}{2})\gamma}\frac{\sigma_n}{\overline{X}_r} = \frac{\left(C^2+1\right)}{2C}\delta x$$

(26)
$$\frac{\overline{X}_r}{\sigma_n} \approx \frac{2C^2}{(C^2+1)\delta x}\sqrt{(2-\frac{\pi}{2})\gamma} = \frac{\gamma-\sqrt{\pi}}{\gamma\delta x}\sqrt{2(4-\pi)\gamma}$$

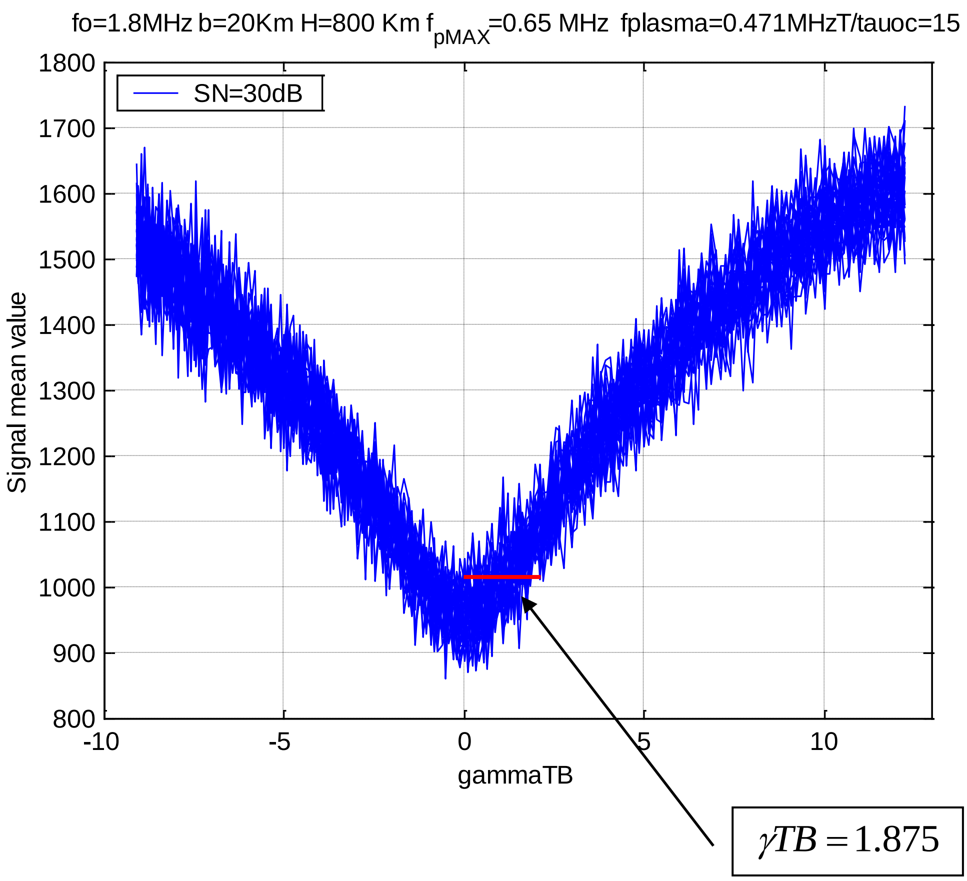
$$\overline{X}_r = A \qquad \frac{S}{N} = \frac{A}{\sqrt{2}\sigma} = \frac{\gamma-\sqrt{\pi}}{\gamma\delta x}\sqrt{\gamma(4-\pi)}$$

The maximum value of the contrast can be also obtained by selection of the minimum value of the mean value.

(27)
$$E = A^2\tau = A_0^2(1-\delta A)^2\tau(1+\delta x) = \cos t \rightarrow 2\delta A \approx \delta x \rightarrow A\tau = A_0\tau(1+\frac{\delta x}{2})$$

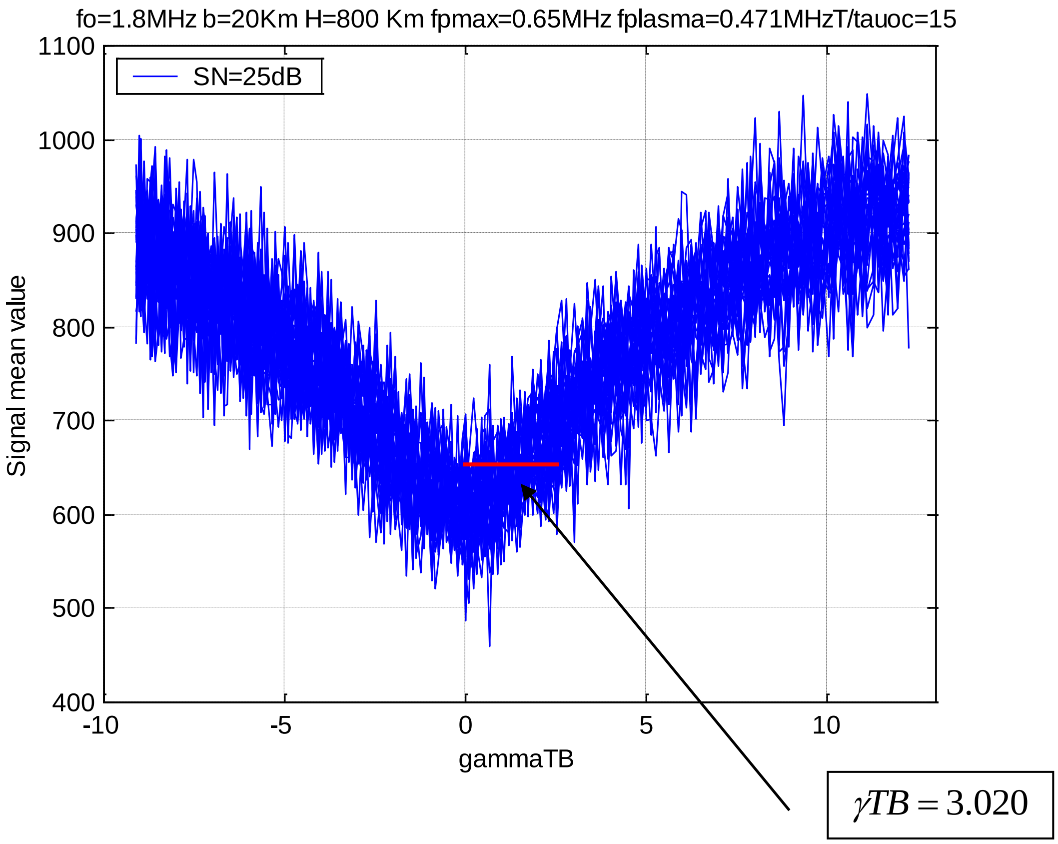
(28)
$$C_n\big|_{max} \rightarrow \overline{X}\big|_{min} \approx \left(\frac{A\tau}{T} + \overline{x}_n \pm \sqrt{(2-\frac{\pi}{2})\frac{\sigma_n^2}{\gamma}}\right)\Bigg|_{min} = \left|\left(\frac{A_0}{\gamma}(1+\frac{\delta x}{2}) + \overline{x}_n \pm \sqrt{\frac{4-\pi}{2\gamma}}\sigma_n\right)\right|_{min}$$
$$\rightarrow \frac{A_0}{\gamma}\frac{\delta x}{2} = \sqrt{\frac{4-\pi}{2\gamma}}\sigma_n \rightarrow \frac{A_0}{\sqrt{2}\sigma_n} = \frac{\sqrt{(4-\pi)\gamma}}{\delta x}$$

EVALUATION OF NOISE EFFECTS



$$\gamma TB = 1.875 \longrightarrow \delta_x = 0.09 \longrightarrow S/N \cong 32 dB$$

EVALUATION OF NOISE EFFECTS



$$\gamma TB = 3.020 \longrightarrow \delta_X = 0.21 \longrightarrow S/N \cong 24dB$$

5. ACQUISITION TRACKING PHASE

After the acquisition phase and before starting the pulse tracking phase, taking into account that we haven't precise preliminary knowledge of a_2, the *research area* increases. Hence the estimation of a_2 term shall be initialized, obviously in order to reduce as more as possible the time necessary to get the optimum value. The initialization can be obtained by the measurement of the extra delay time (or a_1), due to ionosphere, as available in acquisition phase; since for high value of the ratio $f_o/f_{p.eq}$ the coefficient $a_2 \approx -a_1/f_o$ (see section 2) it seems reasonable to choose this value to initialize the coefficient $a_2 \rightarrow \hat{a}_2$

The extra delay time (τ) corresponding to the frequency f_0 is given (s.eq.(5)) by:

(29)
$$\tau = \frac{a_1}{2\pi} = \frac{\tau_0}{\sqrt{1-\frac{f_{p.eq}^{\ 2}}{f_0^{\ 2}}}} - \tau_0 = \tau_0 \varphi\left(\frac{f_{p.eq}}{f_0}\right)$$

The τ should be less than 110 μsec (night time $\rightarrow$ $f_{p.eq}$<1 MHz, f_0 = 1.8 MHz) and 350 μsec (day time $\rightarrow$ $f_{p.eq}$<4 MHz, f_0 = 5 MHz).

We can notice that the τ measurement accuracy is function of:

- orbit knowledge accuracy: uncertainty of orbiter position of 3 sec, radial speed less 1 km/sec entail $\Delta\tau_1$<20 μsec
- surface behavior and altimeter measurement accuracy $\Delta\tau_2$<10 μsec

so that we can obtain:

(30)
$$\hat{a}_1 = a_1 \pm 10^{-4} \rightarrow \hat{a}_2 = a_2 \pm \frac{10^{-4}}{f_0} \qquad \text{[rad/Hz}^2\text{]}$$

in the night time (for high value of the ratio $f_o/f_{p.eq}$

(31)
$$\hat{a}_2 = a_2 \pm 50 * 10^{-12} \qquad \text{[rad/Hz}^2\text{]}$$

In order to improve the estimation of the term a_1, by mean without the effects of orbit and surface un knowledge, in the *two frequency operative modes* we can use the measurement of time delay difference ($\Delta\tau_M$) between the time delay of two signal with different central frequencies f_0 , f_{02}= f_0 +Δf .

(32)
$$\Delta\tau_M = \tau - \tau_2 = \tau_0\left[\varphi\left(\frac{f_{p.eq}}{f_o}\right) - \varphi\left(\frac{f_{p.eq}}{f_{o2}}\right)\right] = \tau_0 \Delta\varphi$$

with
$$\frac{\tau}{\tau_0} = \varphi\left(\frac{f_{p.eq}}{f_0}\right) \qquad \text{and} \qquad \frac{\tau_2}{\tau_0} = \varphi\left(\frac{f_{p.eq}}{f_{02}}\right)$$

Assuming an exact estimation of $\Delta\tau_M$ we can write:

$$(33)\qquad \tau = \Delta\tau_M \frac{\varphi\left(\frac{f_{p.eq}}{f_o}\right)}{\Delta\varphi} = \Delta\tau_M \frac{\frac{1}{\sqrt{1-\left(\frac{f_{p.eq}}{f_0}\right)^2}} - 1}{\frac{1}{\sqrt{1-\left(\frac{f_{p.eq}}{f_0}\right)^2}} - \frac{1}{\sqrt{1-\left(\frac{f_{p.eq}}{f_{02}}\right)^2}}} \cong \Delta\tau_M \frac{\frac{1}{f_0^2}}{\left(\frac{1}{f_0^2} - \frac{1}{f_{02}^2}\right)}$$

Concerning the hypothesis of high value of the ratio f_o/f_p we can write:

$$(34)\qquad a_2 = -2\pi\tau_0\left(\frac{f_{p.eq}^2}{2\sqrt{(f_0^2 - f_{p.eq}^2)}\ (f_0^2 - f_{p.eq}^2)}\right) = -\frac{2\pi\tau_0}{f_0}\left(\frac{f_{p.eq}^2}{2f_0^2}\right)\left(\frac{\tau}{\tau_0}+1\right)^3 = -\frac{2\pi\tau_0}{2f_0}\left(\left(1+\frac{\tau}{\tau_0}\right)^2 - 1\right)\left(1+\frac{\tau}{\tau_0}\right) =$$

$$= -\frac{2\pi\tau_0}{2f_0}\left(\frac{2\tau}{\tau_0} + \left(\frac{\tau}{\tau_0}\right)^2\right)\left(1+\frac{\tau}{\tau_0}\right) = -\frac{\pi\tau}{f_0}\left(2+\frac{\tau}{\tau_0}\right)\left(1+\frac{\tau}{\tau_0}\right) = -\frac{2\pi\tau}{f_0}\left(1+\frac{\tau}{2\tau_0}\right)\left(1+\frac{\tau}{\tau_0}\right) \approx -\frac{2\pi\tau}{f_0}\left(1+\frac{3\tau}{2\tau_0}\right)$$

so that:

$$(35)\qquad a_2 = -\frac{a_1}{f_0}\left(1+\frac{3\tau}{2\tau_0}\right) \rightarrow \hat{a}_2^{'} = \hat{a}_2(1+y) \rightarrow \begin{matrix} y < 0.3 \Leftrightarrow night-time \\ y < 0.98 \Leftrightarrow day-time \end{matrix}$$

In conclusion, taking into account the maximum value of $a_2 < 1000$ rad/MHz^2 , we can conclude that it seems reasonable to choose the coefficient $-a_1/f_o$ to initialize the coefficient a_2 , but some compensation terms should be applied, following the approach of the previous section, taking into account the procedure selected for the start of the tracking phase.
In any case the uncertainty on τ entails an error on the estimation on a_2 :

$$(36)\qquad \Delta a_2 \approx \frac{2\pi\Delta\tau}{f_0}\left(1+\frac{3\tau}{2\tau_0}\right) + \frac{2\pi\tau}{f_0}\frac{3\Delta\tau}{2\tau_0} = \frac{2\pi\Delta\tau}{f_0}\left(1+\frac{3\tau}{\tau_0}\right) = \frac{\Delta\tau}{\tau}\hat{a}_2(1+2y) = \frac{\Delta\tau}{\tau}\hat{a}_2^{'}\frac{1+2y}{1+y} \rightarrow \begin{matrix} 0.33 * \hat{a}_2^{'} \Leftrightarrow night-time \\ 0.13 * \hat{a}_2^{'} \Leftrightarrow day-time \end{matrix}$$

by considering $\Delta\tau = \Delta\tau_1 + \Delta\tau_2 \leq 30$ μsec. The values of $\hat{a}_2^{'}$ are given in fig.3.1.2 and fig.3.2.2 of Annex 3.

6. CONCLUSIONS

In this paper the compensation procedure for phase distortion on the pulse chirp compression are envisaged. In particular the estimation procedure can be reduced to the estimation of chirp slope and of cubic term of frequency series development of ionosphere high frequency uniform equivalent model.
The research range necessary in the estimation of the parameters has been taken into account in particular in the transition phase acquisition tracking; in fact we have pointed out that in the transition between acquisition and tracking (due to the few preliminary information we have on

Mars Ionosphere), the “research area” allowed in a single cycle of the contrast-block is not sufficient.

In addition in annex 3 the quantization step of the frequency variable has been investigated.

ANNEX 1

FLOW CHART CONTRAST METHOD

FLOW CHART CONTRAST

↓

Signal s(t) , 490 samples:

$$s(t) = A\exp\left(j\pi\mu(t-t_o)^2\right) rect_{250\mu\sec}(t-t_0)$$

A=input

f_d=input → Banda della Ref_Fun(f) centrata in f_0

B_c=1 MHz

T=250µsec

µ= B_c /T

t_o=input

a'_{2-in} = input

$\Delta a_2=6.28*10^{-12}$ [rad/Hz^2]

endloop= input → numero prove del ciclo del contrasto

t → 350µsec (490 samples):

$t_i = -125*10^{-6}+1/fs,..-125*10^{-6}+i*1/fs,..225*10^{-6}$

fs=1.4 MHz

↓

S=s(t_i) i = 1,…. , 490
S= 0 i = 491,.. 512

↓

S(f)=FFT{s(t_i)}, 512 samples
f=-fs/2+fs/512+ f_0,…….,fs/2+ f_0
step=fs/512

↓

Ionosferic Distortion (Gamma Model)

IF GAMMA MODEL

$$f_p(z) = f_{p-\max}\frac{z-h_o}{b}\exp\left(1-\frac{z-h_o}{b}\right)\delta_{-1}(z-h_o)$$

b=input (km)
ho=120 Km
H=input (Km)
fo= carrier= input
fp_{max}= input

$$\Delta\phi(f) = \frac{4\pi f}{c}\int_{h_o}^{H}\left[\sqrt{1-\left(\frac{f_p(z)}{f}\right)^2}-1\right]dz$$

f= array_512 elements
f=-fs/2+fs/512+f_0 ,…….,fs/2+f_0
step=fs/512

Ionosferic Distortion (Uniform Model)

IF UNIFORM MODEL

fo= carrier= input
fp= input
h_i= input
L_{eq}= input

$$\Delta\phi(f) = \frac{4\pi f}{c}\int_{h_i}^{h_i+L_{eq}}\left[\sqrt{1-\left(\frac{f_p}{f}\right)^2}-1\right]dz$$

f= array_512 elements
f=-fs/2+fs/512+f_0,…….,fs/2+f_0
step=fs/512

Ionosferic Distortion (Reduced Model)

IF REDUCED MODEL

τ_o= 650 µsec
fo= carrier= input
fp= input

$$a_2 = -2\pi\tau_o\left(\frac{f_p^2}{2\left(f_o^2 - f_p^2\right)^{\frac{3}{2}}}\right)\left[rad/MHz^2\right]$$

$$a_3 = 2\pi\tau_o\left(\frac{f_o f_p^2}{2\left(f_o^2 - f_p^2\right)^{\frac{5}{2}}}\right)\left[rad/MHz^3\right]$$

$$a_4 = -2\pi\tau_o\left(\frac{4 f_o^2 f_p^2 + f_p^4}{8\left(f_o^2 - f_p^2\right)^{\frac{7}{2}}}\right)\left[rad/MHz^4\right]$$

$$\Delta\phi(f) = a_2(f - f_0)^2 + a_3(f - f_0)^3 + a_4(f - f_0)^4$$

f= array_512 elements
f=-fs/2+fs/512+f_0,…….,fs/2+f_0
step=fs/512

Ionosferic Distortion (TEC Model)

IF TEC MODEL

$$f_o = input$$

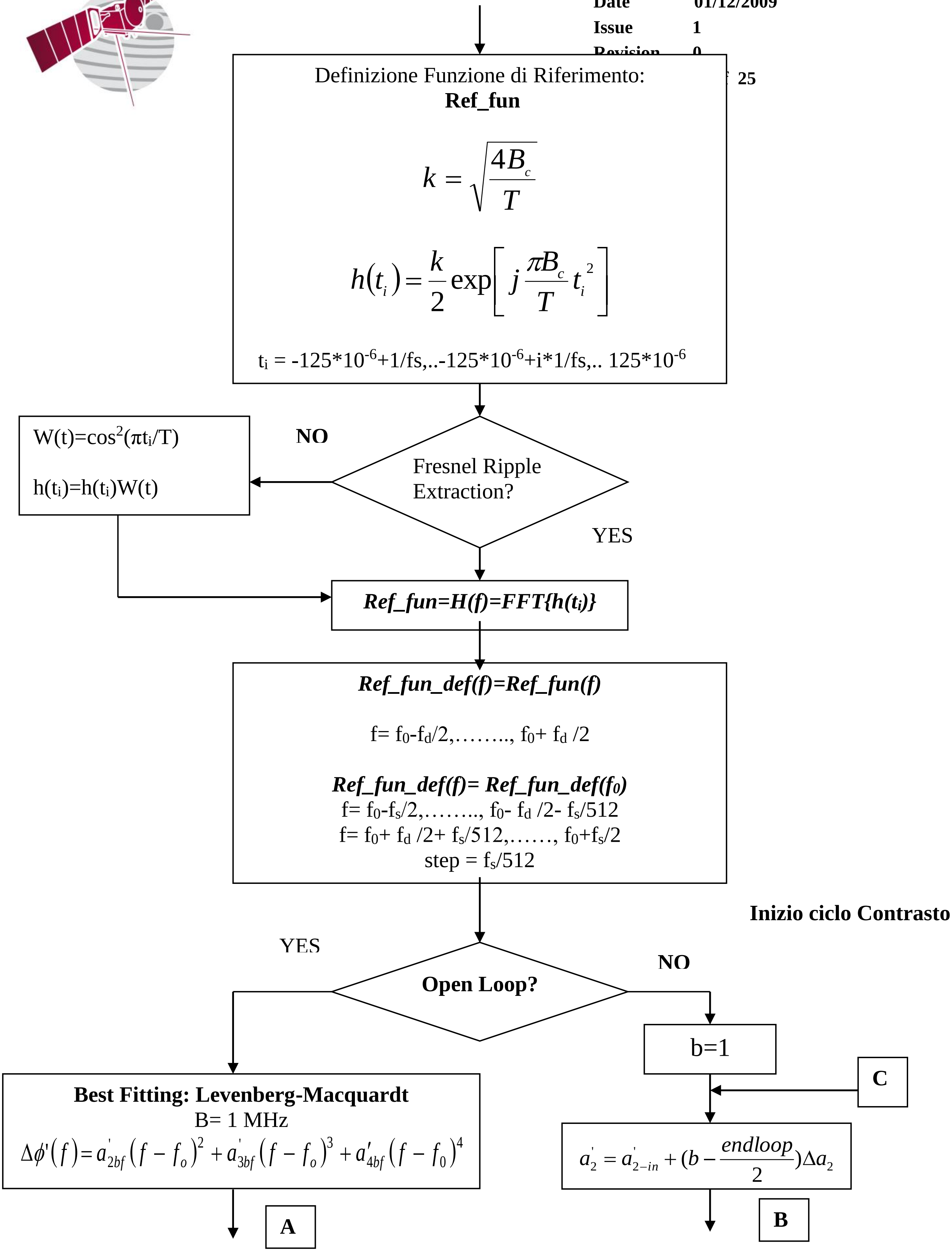
$$a_2 = input \quad \left[rad/MHz^2\right]$$

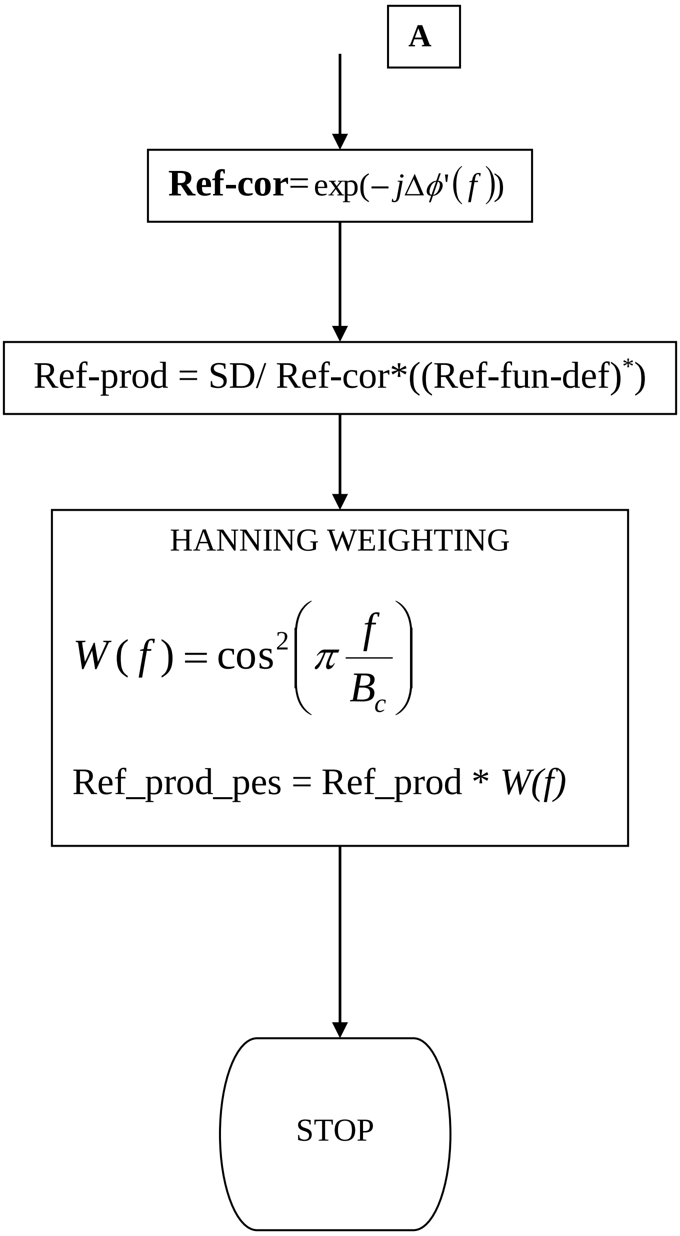
$$a_3 = -\frac{a_2}{f_o} \quad \left[rad/MHz^3\right]$$

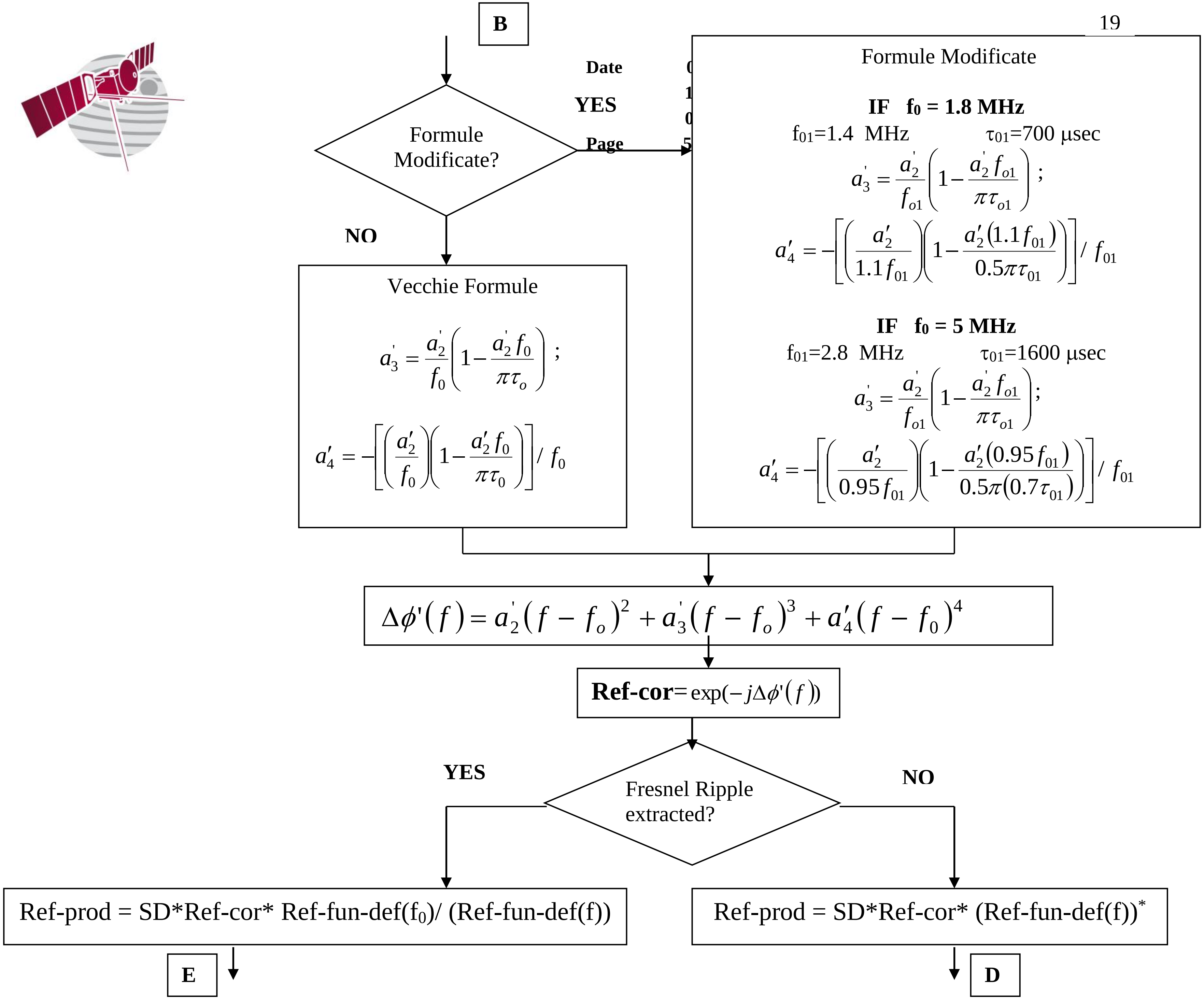
$$a_4 = -\frac{a_3}{f_o} \quad \left[rad/MHz^4\right]$$

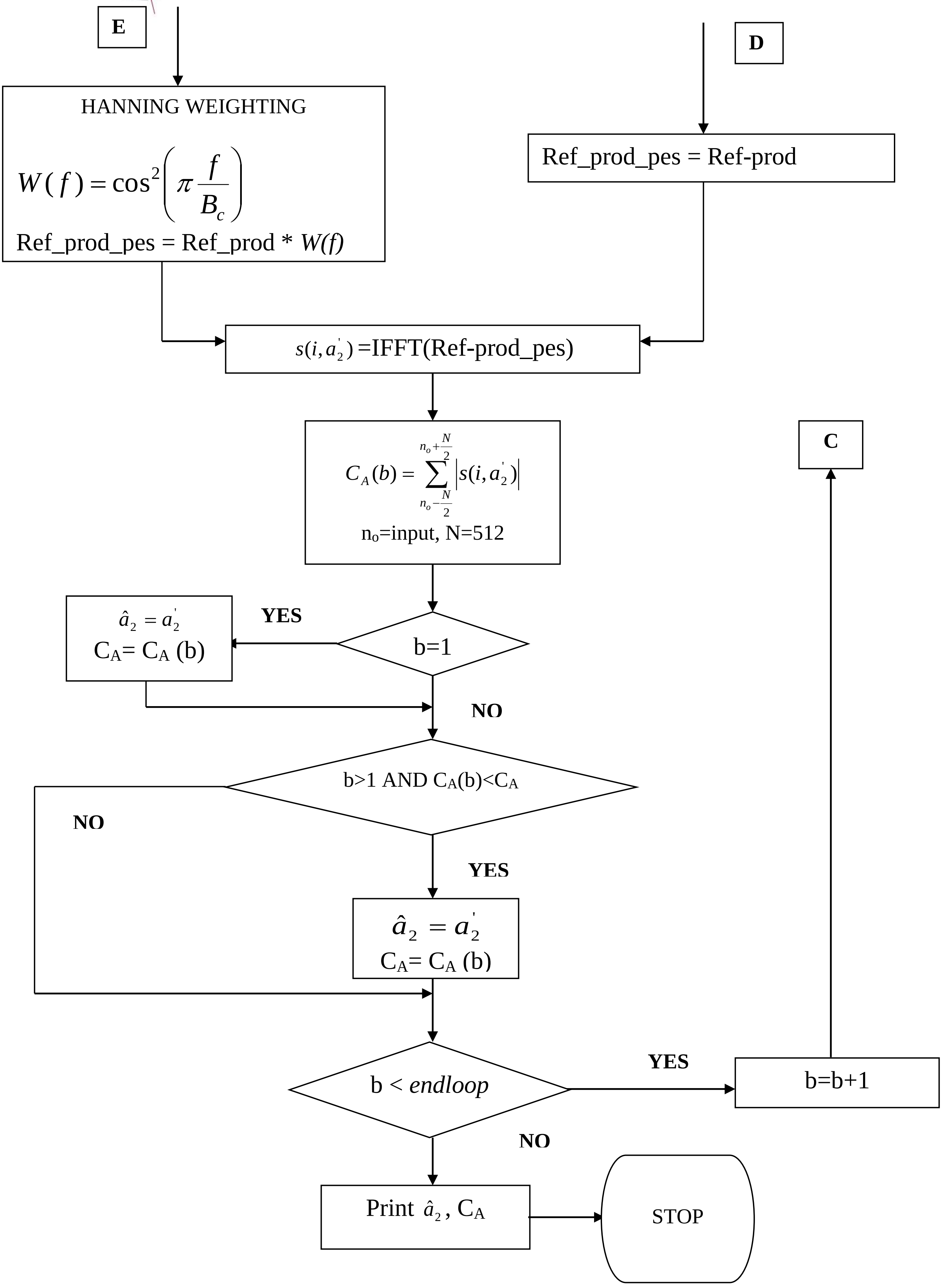
$$\Delta\phi(f) = a_2(f - f_0)^2 + a_3(f - f_0)^3 + a_4(f - f_0)^4$$

f= array_512 elements f=-fs/2+fs/512+f_0 ,…….,fs/2+f_0
step=fs/512


Definizione Funzione di Riferimento:
Ref_fun
$k = \sqrt{\frac{4B_c}{T}}$
$h(t_i) = \frac{k}{2}\exp\left[j\frac{\pi B_c}{T}t_i^2\right]$
t_i = -125*10^{-6}+1/fs,..-125*10^{-6}+i*1/fs,.. 125*10^{-6}
W(t)=cos^2(πt_i/T)
$h(t_i)=h(t_i)W(t)$
NO
Fresnel Ripple Extraction?
YES
Ref_fun=H(f)=FFT{h(t_i)}
Ref_fun_def(f)=Ref_fun(f)
f= f_0-f_d/2,…….., f_0+ f_d /2
Ref_fun_def(f)= Ref_fun_def(f_0)
f= f_0-f_s/2,…….., f_0- f_d /2- f_s/512
f= f_0+ f_d /2+ f_s/512,……, f_0+f_s/2
step = f_s/512
Inizio ciclo Contrasto
YES
Open Loop?
NO
b=1
C
Best Fitting: Levenberg-Macquardt
B= 1 MHz
$\Delta\phi'(f) = a'_{2bf}(f-f_o)^2 + a'_{3bf}(f-f_o)^3 + a'_{4bf}(f-f_0)^4$
$a'_2 = a'_{2-in} + (b - \frac{endloop}{2})\Delta a_2$
A
B


A
Ref-cor= $\exp(-j\Delta\phi'(f))$
Ref-prod = SD/ Ref-cor*((Ref-fun-def)*)
HANNING WEIGHTING
$W(f) = \cos^2\left(\pi \frac{f}{B_c}\right)$
Ref_prod_pes = Ref_prod * *W(f)*
STOP


B
Formule Modificate?
YES
NO
Formule Modificate
IF f_0 = 1.8 MHz
f_{01}=1.4 MHz τ_{01}=700 μsec
$a_3' = \frac{a_2'}{f_{o1}}\left(1 - \frac{a_2' f_{o1}}{\pi \tau_{o1}}\right)$;
$a_4' = -\left[\left(\frac{a_2'}{1.1 f_{01}}\right)\left(1 - \frac{a_2'(1.1 f_{01})}{0.5\pi\tau_{01}}\right)\right] / f_{01}$
IF f_0 = 5 MHz
f_{01}=2.8 MHz τ_{01}=1600 μsec
$a_3' = \frac{a_2'}{f_{o1}}\left(1 - \frac{a_2' f_{o1}}{\pi \tau_{o1}}\right)$;
$a_4' = -\left[\left(\frac{a_2'}{0.95 f_{01}}\right)\left(1 - \frac{a_2'(0.95 f_{01})}{0.5\pi(0.7\tau_{01})}\right)\right] / f_{01}$
Vecchie Formule
$a_3' = \frac{a_2'}{f_0}\left(1 - \frac{a_2' f_0}{\pi \tau_o}\right)$;
$a_4' = -\left[\left(\frac{a_2'}{f_0}\right)\left(1 - \frac{a_2' f_0}{\pi \tau_0}\right)\right] / f_0$
$\Delta\phi'(f) = a_2'(f - f_o)^2 + a_3'(f - f_o)^3 + a_4'(f - f_0)^4$
Ref-cor= $\exp(-j\Delta\phi'(f))$
Fresnel Ripple extracted?
YES
NO
Ref-prod = SD*Ref-cor* Ref-fun-def(f_0)/ (Ref-fun-def(f))
Ref-prod = SD*Ref-cor* (Ref-fun-def(f))*
E
D


E
D
HANNING WEIGHTING
$W(f) = \cos^2\left(\pi \frac{f}{B_c}\right)$
Ref_prod_pes = Ref_prod * W(f)
Ref_prod_pes = Ref-prod
$s(i, a_2^{'})$ =IFFT(Ref-prod_pes)
$C_A(b) = \sum_{n_o - \frac{N}{2}}^{n_o + \frac{N}{2}} |s(i, a_2^{'})|$
n_o=input, N=512
C
$\hat{a}_2 = a_2^{'}$
$C_A = C_A$ (b)
YES
b=1
NO
b>1 AND C_A(b)<C_A
NO
YES
$\hat{a}_2 = a_2^{'}$
$C_A = C_A$ (b)
b < *endloop*
YES
b=b+1
NO
Print $\hat{a}_2$, C_A
STOP

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