TITLE: # Metrics for the quantization error and definition of a "fictious" σ/q

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CHANGE RECORD

Issue	Date	Sheet	Description of Change	Release
0	2008 Aug 11		Starting of the document	0
1	2008 Aug ??		First Issue	0

1 SCOPE

This document describes the problem of properly expressing the REBA processing error and how to relate to the obsolete σ/q index quoted in old documents.

1.1 LIMITS OF APPLICABILITY

This document has been issued in the framework of CSL tests.

2 APPLICABLE/REFERENCE DOCUMENTS

2.1 APPLICABLE DOCUMENTS

[AD-1] Planck – LFI – Characterization of the onboard processing parameters
M.Maris
PL-LFI-OAT-TN-030, Issue 0, April 2004

2.2 REFERENCE DOCUMENTS

[RD–1] Reconfiguration for LFI on-board data processing and scientific telemetry
M. Miccolis, A. Mennella, M. Bersanelli, M. Maris
PL-LFI-PST-TN-037, Issue 1.0, March 2003

2.3 ACRONYMS LIST

3 THE REQUIRED σ/Q > 2

In many documents within the Planck mission it is stated that the target compression rate $C_{r,tgt}=2.4$ has to be obtained while keeping the $\sigma/q>2$. What does it mean? The σ/q is a metric to describe the quantization error induced by processing. Before the change of baseline in 2003 [RD-1] the idea was to directly quantize the onboard differentiated signal with a quantization step q, the processing error was proportional to q so that the ratio of the processing error to the noise RMS, σ , where inversely proportional to σ/q .

The consideration behind the $\sigma/q>2$ specification has been simply the need to properly sample the histogram of noise for repeated sampling of the same region of sky.

It is obvious that for $\sigma/q<2$ (as an example $\sigma/q\approx 1$) the noise distribution would be heavily undersampled. A simple analogy with sampling of a signal based on Nyquist theorem, leads to the need to have at least two quantization steps for each σ, so that the normal noise distribution would be spreaded over at least 12 quantization steps.

In addition, the quantization error introduced by processing in the old case is

$$\epsilon_q=\frac{q}{\sqrt{12}} \quad ; \qquad (1)$$

so that the error relative to the signal RMS is nothing else than

$$\frac{\epsilon_q}{\sigma}=\frac{1}{\sqrt{12}}\frac{1}{\sigma/q} \quad . \qquad (2)$$

Hence the required $\sigma/q>2$ is equivalent to ask for $\epsilon_q/\sigma<0.14$ not too far from the 10% limit for a systematic to be worth of a detailed analysis (in general a systematic is important if it is larger than 10% of the noise).

With the introduction of the new baseline [RD-1] this requirement becomes a source of confusion causing miss leading conclusions if not properly interpreted and corrected.

In the new baseline what is quantized are the two independent linear combinations

$$P_1=V_{sky}-r_1V_{load}; \quad (3a)$$
$$P_2=V_{sky}-r_2V_{load}; \quad (3b)$$

which are then used to regenerate sky and load at ground from which differentiated data are produced.

So when looking at σ/q of which σ are we speaking? We have now σ for $P_{1,}P_{2,}V_{sky},V_{load}$ and for the differentiated data, and the same holds for the processing errors. However, let to be driven again by the Nyquist criterion. In that case σ could be the RMS of either P_1 or P_2 or the smaller of the two. When looking at the processing errors it is easy to see that

$$\epsilon_{q,sky}=\frac{q}{\sqrt{1\,2}}\frac{\sqrt{r_2^2+r_1^2}}{|r_2-r_1|}; \quad (6a)$$

$$\epsilon_{q,load}=\frac{q}{\sqrt{1\,2}}\frac{\sqrt{2}}{|r_2-r_1|}; \quad (6b)$$

So that q is no longer then only parameter determining the processing error. I.e. we may have σ/q as large as we want, having at the same time the processing error much larger than what could be expected looking just at the σ/q ratio. In addition, the compression rate is not just a function of q but also of r_1 and r_2.

For all of these reasons the σ/q ratio can not be considered a significant metrics of the quality of the onboard processing.

4 PROPOSAL FOR NEW METRIC AND A "FICTIOUS" σ/ Q

A much convenient metric is to report directly the three relative processing errors:

$$\frac{\epsilon_{q,sky}}{\sigma_{sky}}, \frac{\epsilon_{q,load}}{\sigma_{load}}, \frac{\epsilon_{q,diff}}{\sigma_{diff}}; \quad (7)$$

asking they are not too large than 0.1.

In order not to lost completely the connection with old documents, it could be convenient in some case to express the ratios in Eq. (7) as a "fictious" σ/q defined as

$$\left(\frac{\sigma}{q}\right)_x = \frac{\sigma_x}{\sqrt{1\,2}\epsilon_{q,x}}; \quad (8)$$

where x = *sky*, *load* and *diff*.