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ORIGINAL ARTICLE

Height bias and scale effect induced by antenna gravitational deformations in geodetic VLBI data analysis

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Abstract The impact of signal path variations (SPVs) caused by antenna gravitational deformations on geodetic very long baseline interferometry (VLBI) results is evaluated for the first time. Elevation-dependent models of SPV for Medicina and Noto (Italy) telescopes were derived from a combination of terrestrial surveying methods to account for gravitational deformations. After applying these models in geodetic VLBI data analysis, estimates of the antenna reference point positions are shifted upward by 8.9 and 6.7 mm, respectively. The impact on other parameters is negligible. To simulate the impact of antenna gravitational deformations on the entire VLBI network, lacking measurements for other telescopes, we rescaled the SPV models of Medicina and Noto for other antennas according to their size. The effects of the simulations are changes in VLBI heights in the range [−3, 73] mm and a net scale increase of 0.3–0.8 ppb. The height bias is larger than random errors of VLBI position estimates, implying the possibility of significant scale distortions related to antenna gravitational deformations. This demonstrates the need to precisely measure gravitational deformations of other VLBI telescopes, to derive their precise SPV models and to apply them in routine geodetic data analysis.

Keywords VLBI · ITRF · Reference frames · Signal path variation · Antenna gravitational deformation

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1 Introduction

Very long baseline interferometry (VLBI) and global positioning system (GPS) techniques are used for establishing global and permanent geodetic networks whose fundamental stations are distributed over the entire Earth and whose coordinates are derived with formal errors <1 mm. In many areas of geophysics, astronomy and geodesy the knowledge of fundamental station positions is required with ever increasing accuracy. A comprehensive discussion on position accuracy requirements related to a variety of societal applications, Earth observations, natural hazards mitigation, Earth and planetary science are discussed by Gross et al. (2009) for instance. Evaluation of realistic errors and mitigation of observation biases are therefore vital for the many applications that depend on the global and permanent geodetic networks.

Since the very beginning of geodetic VLBI observations, gravitational deformation of VLBI telescopes was recognized as a potentially important error source. A remarkable example on this matter was offered by the prescient work of Carter et al. (1980) written 30 years ago. Comparing results obtained from ground surveys and VLBI data analysis, the authors found a 19 mm difference in the vertical component of the 1.24 km-long Haystack–Westford baseline. Most of this difference, 13 mm, was ascribed to the gravitational flexure of the 37-m Haystack telescope structure which was found to vary as the sine of the observation elevation angle. The signature of this error aliases indistinguishably into the estimated VLBI height, thus biasing the relative VLBI antenna reference point position. The authors could not rule out the existence of additional gravitational deformation effects that might account for the residual discrepancy with the survey result. Little further work has been done in the three decades since Carter et al. (1980) to

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correct or mitigate VLBI height errors due to antenna gravitational deformations. A detailed theoretical study for a VLBI antenna in Alaska was carried out by Clark and Thomsen (1988).

Gravitational deformations affecting the 32-m AZ-EL mount VLBI telescopes at Medicina and Noto (Italy) were recently investigated in detail by Sarti et al. (2009a,b) with a combination of terrestrial surveying techniques. They quantified the signal path variation (SPV) traveled by the radio signal in the near-field of the radio telescopes due to the elevation-dependent action of gravity on the structures of VLBI antennas. The SPV is retrieved by determining and combining the deformations of the primary mirror, the motions of the whole reflecting system and the quadripod. The variation depends on the pointing elevation e of the radio telescope. For a VLBI antenna with receivers located in primary focus, it can be expressed as

$$\Delta L(e) = \alpha_F \Delta F(e) + \alpha_V \Delta V(e) + \alpha_R \Delta R(e) \quad (1)$$

where ΔF represents the focal length variation of the primary reflector, ΔV is the vertex displacement along the line of sight, ΔR is the feed horn's phase centre displacement along the same direction. The coefficients $(\alpha_F, \alpha_V, \alpha_R)$ relate the previous quantities to the change in signal path ΔL. Details on the derivation of Eq. (1), the role of the three terms at the right hand side on the total SPV as well as the adopted methods for determining the value of the three Δ terms can be found in Clark and Thomsen (1988) and Sarti et al. (2009a). Figure 1 shows a schematic representation of a radio telescope structure and how its primary reflector, its quadripod and its vertex are affected by the gravitational deformation. Elevation-dependent motions or deformations of these elements along the line of sight (i.e. pointing direction) cause SPV that affects the VLBI delay observations. If left unmodelled, the SPV will systematically bias geodetic estimates of the antenna reference point.

Alternatively to space geodesy, RP coordinates of space geodetic (SG) instruments can be estimated with terrestrial observations adopting different terrestrial surveying methods (e.g. Sarti et al. 2004; Richter et al. 2005; Dawson et al. 2007). The relative position vector between reference points of co-located SG instruments is thereafter called tie vector. A data analysis procedure that combines observations from two or more SG techniques at co-located sites along with accurate tie vectors (1 mm or better), has significant advantages with respect to single technique solutions. A clear advantage resides in the comparison of reference point positions from each individual SG technique expressed in ITRF with *independently measured* tie vectors. It can provide a quantitative measure of disagreement between results of SG techniques and spotlight the presence of technique-dependent systematic errors.

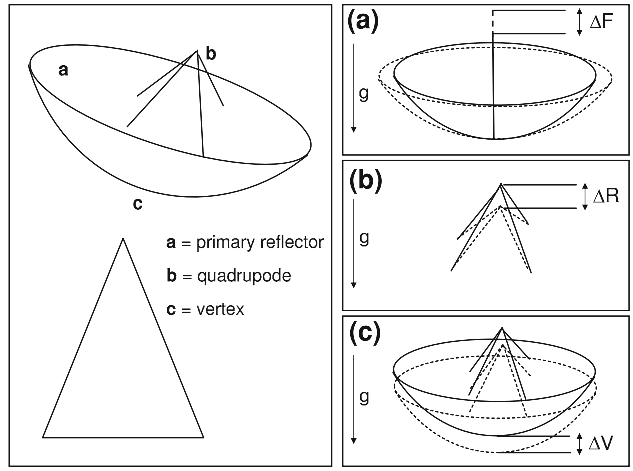

Fig. 1 Deformations induced by gravity g of **a** the primary mirror, **b** the quadripod and **c** the reflecting system (whose position is represented by the vertex) of the generic VLBI radio telescope sketched on the *left*. **a** Shows how gravity force g folds outward and flattens the primary mirror as the telescope pointing elevation increases toward zenith. Accordingly, the focal length of the paraboloid varies with the elevation as $\Delta F(e)$ and it is longer at higher elevations. **b** Shows how gravity g acts on the quadripod b and sags it by a quantity $\Delta R(e)$ which is maximal at zenith. **c** The whole system is dragged down by g by a quantity $\Delta V(e)$ proportional to the projection of g along the pointing elevation e

Ray and Altamimi (2005) analyze the VLBI–GPS tie vector at several co-location sites and show that the discrepancies of the tie vectors derived from the SG techniques and from the ground surveys are in the 10–20 mm range, while the formal errors of each estimation are below 1 mm. We can identify four potential reasons of these discrepancies: *(i)* inconsistencies in SG data analysis techniques; *(ii)* biases in ground survey measurements or errors in their analysis; *(iii)* GPS specific errors; *(iv)* VLBI specific errors. To mitigate the effects *(i)* , several groups have developed software to uniformly model and process geodetic observables of different techniques (see e.g. Thaller et al. 2007; Gambis et al. 2009; Tesmer et al. 2009). Though consistent reprocessing did not eliminate the discrepancy. Concerning *(ii)* , errors in terrestrial surveying were initially suspected to be a major source of disagreement. For this reason, tie vectors were measured and re-measured at many sites with ever increasing precision, refining surveying approaches, adopting sophisticated geometric models and applying more robust computational methods. From the pressure put on terrestrial surveying, we are now able to accurately and repeatedly determine positions for SG instruments reference point to the 1 mm level (see e.g. Dawson et al. 2007). Though refined approaches to tie vector surveying and computation did not eliminate the discrepancy. We concentrate here on *(iv)* , specifically, on evaluating the biases introduced by gravitational deformations of VLBI telescope structures, not previously considered in routine VLBI data analysis.

Table 1 Variations of the distance between the elevation axis and *(i)* the receiver (ΔR, estimated via terrestrial triangulation and trilateration), *(ii)* the vertex of the paraboloid (ΔV, computed with finite element model); ΔF is the focal length variation of the primary reflector, estimated with laser scanner

Elevation (°)	Medicina						Noto					
	ΔR	$\sigma_{\Delta R}$	ΔV	$\sigma_{\Delta V}$	ΔF	$\sigma_{\Delta F}$	ΔR	$\sigma_{\Delta R}$	ΔV	$\sigma_{\Delta V}$	ΔF	$\sigma_{\Delta F}$
5	−1.0	1.6	–	–	–	–	−1.3	1.7	–	–	–	–
10	−2.1	1.7	–	–	–	–	−2.3	1.6	–	–	–	–
15	–	–	−1.3	–	12.5	0.1	–	–	−1.3	–	6.6	0.1
20	−4.0	1.7	–	–	–	–	−3.4	1.7	–	–	–	–
30	−5.4	1.7	−2.3	–	22.4	0.1	−5.1	1.6	−2.3	–	14.7	0.1
40	−7.4	1.7	–	–	–	–	−6.5	1.7	–	–	–	–
45	–	–	−3.4	–	30.2	0.1	–	–	−3.4	–	15.9	0.1
50	−8.0	1.7	–	–	–	–	−8.4	1.7	–	–	–	–
60	−9.3	1.7	−4.3	–	34.1	0.1	−9.3	1.8	−4.3	–	21.5	0.1
70	−10.2	1.8	–	–	–	–	−10.0	1.6	–	–	–	–
75	–	–	−5.1	–	37.0	0.1	–	–	−5.1	–	24.1	0.1
80	−10.8	1.8	–	–	–	–	−10.8	1.7	–	–	–	–
90	−11.3	1.6	−5.7	–	36.3	0.1	−10.3	1.7	−5.7	–	24.4	0.1

The standard deviations σ_V cannot be computed with the finite element model and are not given. All variations are referred to a minimum value set to be 0 mm at 0° elevation. Values taken from Abbondanza and Sarti (2010)

Realizing the importance of modelling antenna deformations, we have developed a novel approach to measure elevation-dependent radio telescope gravitational deformations, and we have derived the SPV models for the Medicina and Noto VLBI antennas (Sarti et al. 2009a). Hereafter, we describe several VLBI data reductions obtained with and without SPV corrections for Medicina and Noto. We also simulate the net geodetic impact of gravitational deformations of all other VLBI antennas. A rough model is developed for each VLBI antenna by scaling SPVs at Medicina or Noto to any other antenna according to its size and its focal configuration (primary or secondary).

2 Data analysis

The elevation-dependent deformations caused by gravity on the structure and primary mirror of the Medicina and Noto VLBI antennas were determined by Sarti et al. (2009a,b) with different terrestrial surveying techniques. As previously mentioned, deformation of the paraboloidal shape of the primary mirror maps into a change in its focal length ΔF (see Fig. 1). Analogously, deformations of the quadripod causes motion ΔR of the feed horn phase centre (if the receiver is located at the primary focus) or the sub-reflector (if the receiver is located at the secondary focus position). Finally, the motion of the paraboloid vertex ΔV represents the displacement of the entire reflecting system with respect to the incoming planar wave front. The values of the deformation patterns measured at Medicina and Noto are reported in Table 1.

These values are interpolated with an elevation-dependent second-order polynomial function, whose coefficients are reported in Table 2.

The combination of these functions results in the elevation-dependent SPV along the line of sight as expressed by Eq. (1). A comprehensive discussion on the computation of coefficients ($\alpha_F, \alpha_V, \alpha_R$) and the total signal path ΔL for Medicina and Noto telescopes is presented in Abbondanza and Sarti (2010). In that manuscript, in contrast to the works of Clark and Thomsen (1988) and Sarti et al. (2009a) where constant illumination functions were used, the authors show that the use of a more realistic illumination function in the form of Gaussian or polynomial functions (see e.g. Baars 2007) is crucial to compute accurate α coefficients and to determine improved ΔL. According to Abbondanza and Sarti (2010), the path variation for station Medicina is

$$\Delta L_{\mathrm{Md}} = 0.22 \cdot e - 0.0012 \cdot e^2 \tag{2}$$

where ΔL_{Md} is expressed in millimeters and the pointing elevation e in degrees. The rms of ΔL_{Md} is 0.2 mm. Following the same procedure, the signal path correction for the telescope in Noto can be determined as

$$\Delta L_{\mathrm{Nt}} = 0.12 \cdot e - 0.00045 \cdot e^2 \tag{3}$$

The rms of ΔL_{Nt} is 0.5 mm. It is easy to verify that for Eqs. (2) and (3) $\underset{e\in[0,90]}{\arg\max}\, \Delta L_{\mathrm{Md}} = 10.1$ mm and

Table 2 Coefficients of the second-order interpolating polynomials $a_0 + a_1 \cdot e + a_2 \cdot e^2$ for ΔR, ΔV and ΔF as function of the elevation e. $e \in [0, 90]$ is expressed in (°) and the interpolating polynomials are referred to length units in (mm)

Values taken from Abbondanza and Sarti (2010)

Medicina ΔR	ΔV	ΔF	Noto ΔR	ΔV	ΔF
a_0					
0.0	0.0	0.0	0.0	0.0	0.0
a_1					
-2.2×10^{-1}	-8.6×10^{-2}	9.2×10^{-1}	-2.1×10^{-1}	-8.6×10^{-2}	5.1×10^{-1}
a_2					
1.0×10^{-3}	2.5×10^{-4}	-5.8×10^{-3}	1.0×10^{-3}	2.5×10^{-4}	-2.7×10^{-3}

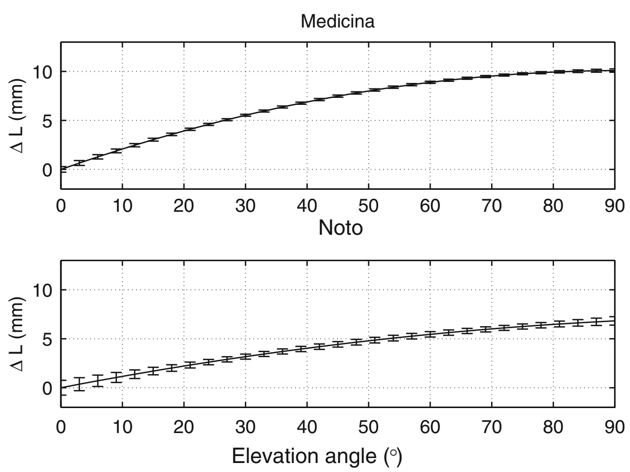

Fig. 2 Signal path variations for Medicina [Eq. (2)] and Noto [Eq. (3)] telescopes. The *error bars* represent the standard deviations (see Abbondanza and Sarti 2010)

$\arg\max_{e\in[0,90]} \Delta L_{\mathrm{Nt}} = 7.2$ mm at zenith. Medicina and Noto telescopes have the same dimensions, the same original mechanical design but, according to Eqs. (2) and (3), do not equally deform (see Fig. 2). The main difference resides in the terms ΔF (cf. Table 1). In fact, the focal lengths F of the two telescopes were measured at six pointing elevations (15°, 30°, 45°, 60°, 75° and 90°) with laser scanning surveys of the primary reflectors (Sarti et al. 2009b). The focal length values of the two telescopes were found to be different and to vary differently with the elevation as shown in Table 1. This is most probably due to the increased rigidity of the Noto primary reflector after the upgrade to an adaptive surface that took place in 2001 (Orfei et al. 2004). The corners of the panels that realize the primary reflector were connected one another by actuators. They can move the panels with the purpose of balancing the deformation effect of gravity on the primary reflector. The actuators exert a mechanical constraint on the surface thus making it less deformable than the Medicina primary reflector.

SPVs in the form of Eqs. (2) and (3) are uniquely available for Medicina and Noto telescopes. These were incorporated in the software VTD for computing the theoretical VLBI path delay (Petrov 2008). In order to assess the impact of gravitational deformations on routine VLBI data analysis, several VLBI solutions were computed adopting the same parameterization and similar models. We processed all geodetic VLBI observations available at the IVS Data Center (Schlüter and Behrend 2007), from April 1980 through August 2009, including 345 24-h sessions with participation of Medicina, and 150 sessions with Noto. A set of global solutions using all 7.08 million observations was reduced with analysis software VTD/Solve, a modern extension of the popular CALC/SOLVE package developed at NASA since the 1970s. Estimated parameters were mean site positions, linear velocities, harmonic site position variations, harmonic position variations for some sites represented with a B-spline; source coordinates, Earth orientation parameters; zenith path delay in the neutral atmosphere and station clock functions represented with B-splines. No-net-rotation and no-net-translation constraints were applied for 46 stations, *excluding* Medicina and Noto. A detailed description of a reduction model, parameterization and constraints can be found in Petrov et al. (2009). In the reference solutions $R1$ we did not apply the SPV model compensating for gravitational deformations. In solution $A1$ the Medicina and Noto SPV models were applied using the files provided as supplementary material of this paper. They contain the values of the SPV as a function of the antenna pointing elevation expressed by Eqs. (2) and (3). Any difference in the results of $R1$ and $A1$ can be directly assigned to the effect of ΔL_{Md} and ΔL_{Nt}.

It should be stressed that this is the first time that antenna gravitational deformation models have been used in the analysis of the global VLBI data set. All previous global VLBI solutions, including those used for computing ITRF2005 (Vennebusch et al. 2007) as well as the forthcoming ITRF2008 (Böckmann et al. 2010), have ignored VLBI gravitational deformations.

3 Results

Comparing solutions $A1$ and $R1$ shows that the only noticeable impact of applying the gravitational deformation model

Table 3 Differences of local geodetic coordinates of some selected European VLBI stations obtained as $A1$ (gravitational deformation modelled) $-R1$ (gravitational deformation not modelled)

Station	ΔU (mm)	ΔE (mm)	ΔN (mm)	# Sess
DSS65	0.0	0.0	0.0	86
MATERA	0.0	0.0	0.0	632
MEDICINA	8.9	0.0	0.0	345
NOTO	6.7	0.0	0.0	150
NYALES20	0.0	0.0	0.0	912
ONSALA60	0.0	0.0	0.0	632
WETTZELL	0.0	0.0	0.0	2,612

The model uniquely impacts on the Up component of the stations for which it was applied

Table 4 Estimates of the axis offsets for MEDICINA and NOTO

Site name	Solution A2 Axis offset (mm)	σ (mm)	Solution R2 Axis offset (mm)	σ (mm)
MEDICINA	1,828.53	0.312	1,828.06	0.312
NOTO	1,829.14	0.844	1,830.36	0.844

The second column contains the axis offset values in (mm) obtained applying the SPV models (solution A2). The fourth column reports the axis offset values estimated not applying SPV models (solution R2)

is an upward shift of the Medicina and Noto height estimates. As shown in Table 3, the effect on the Medicina height component is 8.9 mm, while it is 6.7 mm for Noto. The formal uncertainties of the estimates of the vertical coordinate of Medicina and Noto are 0.5 and 1.5 mm, respectively. The SPV model affected all other station positions by <0.05 mm.

The wrms of postfit residual variation due to the use of the SPV is <0.001 ps, which is completely negligible. The impact on zenith atmospheric path delay estimates was not more than 0.5 mm, which is very small.

We produced a second set of solutions $R2$ and $A2$ which use the same setup as $R1$ and $A1$ but additionally estimate the antenna axis offsets for all the antennas—the axis offset being the distance between the fixed and the moving axes of the telescope. The difference between $A2$ and $R2$ in the estimated heights is 9.5 and 4.9 mm for Medicina and Noto, respectively. The change of Medicina and Noto heights corresponds to the scale change of their position vector by 1.5 and 0.8 ppb, respectively. Again, the estimate of positions and axis offsets of other stations of the global network are not affected. The estimates of the axis offsets obtained for Medicina and Noto telescopes with solutions $A2$ and $R2$ are provided in Table 4.

The use of SPV models for Medicina and Noto does not affect the estimate of the axis offsets of other VLBI telescopes. The variation of the axis offset between $A2$ and $R2$ for Medicina is <0.5 mm and for Noto is ≈ -1.2 mm.

To understand these results, notice that SPVs in Eqs. (2) and (3) can be approximated by

$$\Delta L(e) = a + b \cdot \sin e + c \cdot \cos e \tag{4}$$

where e is the pointing elevation. The parameters a, b, c can be fitted using the LSQ for the elevation range $[10°, 90°]$. The results of the fit are $\Delta L_{\mathrm{Md}} = -0.15 + 10.13 \sin e + 0.63 \cos e$ for Medicina and $\Delta L_{\mathrm{Nt}} = 1.39 + 5.34 \sin e - 1.08 \cos e$ for Noto. ΔL is expressed in millimeters. The rms of the fit is 0.12 mm for Medicina and 0.07 mm for Noto. The computation of another fit, excluding the cosine term, results in $\Delta L_{\mathrm{Md}} = 0.71 + 9.44 \sin e$ and $\Delta L_{\mathrm{Nt}} = -0.08 + 6.53 \sin e$.

The partial derivative of the VLBI delay with respect to the up component of the position vector is proportional to $-\sin(e)$. With relative accuracy better than 10^{-4}, the path delay between the arrival of the signal at antenna #1 (t_1) and the arrival at antenna #2 (t_2) can be expressed as $\tau = (t_1 - t_g) - (t_2 - t_g)$, where t_g is the arrival time of the wavefront at the geocentre. With the same accuracy, $(t_1 - t_g) \approx -\frac{1}{c}\mathbf{r}_1 \cdot \mathbf{s} \approx -\frac{1}{c}|r| \sin e$, where $\mathbf{r}_1$ is the position vector of the first antenna of the baseline relative to the Earth's centre and $\mathbf{s}$ is the vector of source positions, as seen from the antenna.

Similarly, the partial derivative of the path delay with respect to the axis offset length is proportional to $\cos e$ for azimuth-elevation antennas. Rigorous derivation can be found in Sovers et al. (1998), but we can notice that the antenna offset vector lies in the same plane as the station position and source position vectors, and always perpendicular to the site position vector. Since the partial derivative with respect to the length of the position vector is proportional to $-\sin(e)$, the partial derivative with respect to the length of the vector perpendicular to it is proportional to $-\sin(e + 90°) = \cos(e)$.

If some delay contribution has a form of Eq. (4), then, unaccounted, the first term will be absorbed by clock estimates, the second term will be absorbed by estimates of the vertical site position and the third term will be absorbed by estimate of the axis offset length. Thus, the unaccounted sine term will bias height estimates and the cosine term will bias the axis offset length, but they will not affect the rms of the fit. Therefore, it is impossible to discover these biases using only VLBI observations.

4 Effect of gravitational deformation modeling on estimates of VLBI positions

So far, precise measurements of antenna gravitational deformations were performed uniquely on the two Italian antennas and their SPVs were determined with an accuracy better than 0.5 mm (see Fig. 2). What would be the effect of applying corrections for the whole VLBI network, given SPV models for each VLBI station?

We showed that, due to gravitational deformations, the path delay at Medicina and Noto increases as the pointing elevation increases toward zenith. The sign of the VLBI path delay is opposite to the sign of the height bias (cf. Sect. 3). In our case, the observed VLBI delay is reduced by the application of the SPV model and we can expect an increase of the height component. If unaccounted deformations of the other telescopes systematically bias the height position of VLBI stations, the scale factor of the VLBI network will be unavoidably affected.

In absence of measurements concerning gravitational deformations of all the VLBI telescopes except for Medicina and Noto, we cannot provide a precise estimate of this effect on the entire VLBI network. Nevertheless, a coarse estimate of the order of magnitude of SPV models can be derived with a simplified assumption. All VLBI telescopes are assumed to be identical to Medicina and Noto in terms of shape of the antenna structure. Therefore, for an antenna of diameter D_a, the size of each element can be scaled by D_a/D_{Md} or by D_a/D_{Nt}, where D_{Md} and D_{Nt}, respectively, are the diameter of Medicina and Noto. In order to derive the dependency of the antenna gravitational deformation on its dimension, we can consider that each element of the antenna deforms according to the Hooke's law. Its deformation Δx is described by

$$\Delta x = \frac{\rho V g L_0}{A E}, \tag{5}$$

where ρ is the density of the material, V—volume of the element, L_0—size of the element, A—the cross-section of the element and E—the Young's modulus. We can notice that by increasing all dimensions of an element by γ times, the element's deformation will increase by γ^2. Under the assumption that the shape of the antenna dishes is identical to that of Medicina (or Noto), the SPV model of an antenna a with a diameter D_a can be expressed as $\mathrm{SPV}_a = (D_a/D_{\mathrm{Md}})^2 \cdot \mathrm{SPV}_{\mathrm{Md}}$ or $\mathrm{SPV}_a = (D_a/D_{\mathrm{Nt}})^2 \cdot \mathrm{SPV}_{\mathrm{Nt}}$.

We can draw the same conclusion about the dependence of deformations with antenna size by invoking structural mechanics considerations. Let us assume the structures of the telescopes behave as a circular plate subjected to a uniformly distributed load. Under this circumstance, the deflection Δw of the circular plate (i.e. the deformation in the direction orthogonal to the plate) can be expressed as (see e.g. Selvadurai 2000)

$$\Delta w = \frac{k q l^4}{M} \tag{6}$$

where k—a dimensionless number related to the mechanical constraints applied to the structure. We may assume it is constant and not varying with the dimensions of the structure; q—distributed load (force per unit surface) linearly dependent on the dimension of the structure; l—linear dimension of the structure; M—flexural rigidity of the structure varying with its volume. Therefore, the deflection Δw varies as l^2, i.e. exhibits a dependency on the area of the structure. As shown earlier, the SPV model for an antenna a of diameter D_a must be scaled according to the same factor $s = (D_a/D_{\mathrm{Md}})^2$ or $s = (D_a/D_{\mathrm{Nt}})^2$.

The S/X receivers of Medicina and Noto telescopes are located at the primary focus. For the antennas having geodetic receivers located at the secondary focus, the same scaling factor s was applied to the SPV specifically computed by Abbondanza and Sarti (2010) for Medicina and Noto antennas assuming the S/X receivers were located at the Cassegrain focus. In particular, the elevation-dependent SPV model for Cassegrain antennas is [cf. Eq. (1)]

$$\Delta L'(e) = \alpha'_F \Delta F(e) + \alpha'_V \Delta V(e) + 2\,\alpha'_R \Delta R(e) \tag{7}$$

Note that the linear coefficients α'_F, α'_V and α'_R differ from the analogous coefficients in expression (1) and the displacement of the sub-reflector ΔR (the receiver in primary focus configuration) is accounted twice (see e.g. Cha 1987; Abbondanza and Sarti 2010). Evaluation of expression (7) for Medicina and Noto highlights that $0 < \Delta L'(e) < \Delta L(e)$. The term $2\,\alpha'_R \Delta R(e)$ effectively counterbalances $\alpha'_F \Delta F(e)$, being opposite in sign, thus decreasing the net SPV due to gravitational deformations.

We computed a set of global VLBI solutions applying this gravitational deformation model to all antennas in the global VLBI network. Neither reference solutions $R1$ and $R2$ considered deformation but were without and with axis offset estimation for all antennas, respectively. No-net-translation and no-net-rotation constraints were applied to all stations, excluding Medicina and Noto. Solutions $A3$ and $A4$, respectively, use gravitational deformation models for all antennas derived from the Medicina [Eq. (2)] and Noto [Eq. (3)] SPV models but do not estimate the antenna axis offsets. Solutions $A5$ and $A6$, respectively, correspond to solutions $A3$ and $A4$ and include additionally the estimation of antenna axis offsets.

Applying the model of antenna gravitational deformations caused changes in height estimates in the range [−3, 73] mm. We computed Helmert transformation parameters between solutions $A3$, $A4$ and $R1$, as well as between solutions $A5$, $A6$ and $R2$. Results are presented in Table 5. Estimates of the Helmert parameters not shown in Table 5 do not significantly change.

5 Discussions and conclusions

This study shows the importance of investigating and eventually modelling gravity-induced SPVs within VLBI antenna structures. The effects on parameters estimated in geodetic

Table 5 The scale factors (SF) and shifts along the Z-axis (ΔZ) of the global VLBI network obtained with antenna gravitational deformation models derived from the SPVs of Medicina and Noto

Solution	Ref model	SF (ppb)	ΔZ (mm)
A3–R1	Md	0.67 ± 0.05	-2.4 ± 0.4
A4–R1	Nt	0.25 ± 0.06	-1.4 ± 0.3
A5–R2	Md	0.77 ± 0.05	-2.7 ± 0.4
A6–R2	Nt	0.29 ± 0.03	-0.9 ± 0.2

The first two rows correspond to the case when antenna axis offsets were not estimated

VLBI data processing have been quantified and shown to be substantial only for VLBI height determinations, depending directly on the magnitude of the SPV.

The SPV and the resulting bias were detected using alternative and independent measuring techniques, such as terrestrial surveying. We have to emphasize that this is the only practical means of developing suitable SPV models for data analysis. The bias cannot be determined by relying on VLBI data alone as its effect is fully absorbed into the estimated station height and antenna axis offset. Path delay variation cannot be characterized by "flexibility" or "stiffness" of the antenna structure: it depends on the linear combination of ΔF, ΔR, and ΔV. Even for flexible antennas, this combination may result in SPV which remains close to zero on the whole $[0°, 90°]$ elevation interval.

Unaccounted SPVs cause systematic errors in antenna axis offset estimates derived from VLBI solutions, due to the presence of $\cos(e)$ term in SPV model. Therefore, the axis offset should always be double-checked with terrestrial measurements and considered as an additional quantity to be measured along with ΔF, ΔR, ΔV and the position of the antenna reference point. In fact, terrestrial surveys are the only way to determine axis offset free of any hypothesis.

The net effect of unaccounted gravitational deformation was simulated applying a scale factor to the SPV models of Medicina and Noto. The result is a network scale distortion of 0.3–0.8 ppb and a systematic shift of 1–3 mm along the radial direction from the geocentre (cf. Table 5). This estimate is not intended for being used in data reduction but it shows that a distortion of the scale factor is *possible*.

Our results imply the uncertainty in the VLBI frame scale can be as large as 0.8 ppb due to the lack of knowledge of real antenna gravitational deformations. Measurements made at all stations with the accuracy achieved in our study for Medicina and Noto (0.2 and 0.5 mm, respectively) would reduce the uncertainty in the scale factor due to gravitational deformations to <0.05 ppb.

Any net shift in VLBI station height estimates affects the scale of the station position catalogue ITRFyy, since it is usually defined using only VLBI and satellite laser ranging observations. Though, only VLBI was used in ITRF2005 (Altamimi et al. 2007). Consequently, all site position estimates will be affected, including position estimates of GPS stations in the case if the a priori positions from the station position catalogue ITRFyy are used in the data analysis procedure.

We have presented evidences that a millimetre accuracy level for VLBI station positions cannot be achieved without modelling the antenna gravitational deformations derived from the dedicated terrestrial survey measurements.

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