Publication Year	2009
Acceptance in OA@INAF	2024-02-02T14:01:26Z
Title	Local effects of redundant terrestrial and GPS-based tie vectors in ITRF-like combinations
Authors	Abbondanza, C.; Altamimi, Z.; SARTI, PIERGUIDO; NEGUSINI, MONIA; Vittuari, L.
DOI	10.1007/s00190-009-0321-6
Handle	http://hdl.handle.net/20.500.12386/34694
Journal	JOURNAL OF GEODESY
Number	83

J Geod (2009) 83:1031–1040
DOI 10.1007/s00190-009-0321-6

ORIGINAL ARTICLE

Local effects of redundant terrestrial and GPS-based tie vectors in ITRF-like combinations

Claudio Abbondanza · Zuheir Altamimi ·
Pierguido Sarti · Monia Negusini · Luca Vittuari

Received: 10 March 2009 / Accepted: 13 May 2009 / Published online: 4 June 2009

Abstract Tie vectors (TVs) between co-located space geodetic instruments are essential for combining terrestrial reference frames (TRFs) realised using different techniques. They provide relative positioning between instrumental reference points (RPs) which are part of a global geodetic network such as the international terrestrial reference frame (ITRF). This paper gathers the set of very long baseline interferometry (VLBI)–global positioning system (GPS) local ties performed at the observatory of Medicina (Northern Italy) during the years 2001–2006 and discusses some important aspects related to the usage of co-location ties in the combinations of TRFs. Two measurement approaches of local survey are considered here: a GPS-based approach and a classical approach based on terrestrial observations (i.e. angles, distances and height differences). The behaviour of terrestrial local ties, which routinely join combinations of space geodetic solutions, is compared to that of GPS-based local ties. In particular, we have performed and analysed different combinations of satellite laser ranging (SLR), VLBI and GPS long term solutions in order to (i) evaluate the local effects of the insertion of the series of TVs computed at Medicina, (ii) investigate the consistency of GPS-based TVs with respect to space geodetic solutions, (iii) discuss the effects of an imprecise alignment of TVs from a local to a global reference frame. Results of ITRF-like combinations show that terrestrial TVs originate the smallest residuals in all the three components. In most cases, GPS-based TVs fit space geodetic solutions very well, especially in the horizontal components (N, E). On the contrary, the estimation of the VLBI RP Up component through GPS technique appears to be awkward, since the corresponding post fit residuals are considerably larger. Besides, combination tests including multi-temporal TVs display local effects of residual redistribution, when compared to those solutions where Medicina TVs are added one at a time. Finally, the combination of TRFs turns out to be sensitive to the orientation of the local tie into the global frame.

Keywords Local ties · Co-locations · Tie vector · VLBI · GPS · ITRF · Combination

C. Abbondanza (✉) · M. Negusini · P. Sarti
Istituto di Radioastronomia (IRA),
Istituto Nazionale di Astrofisica (INAF),
Via P. Gobetti, 101, 40129 Bologna, Italy
e-mail: c.abbondanza@ira.inaf.it

M. Negusini
e-mail: negusini@ira.inaf.it

P. Sarti
e-mail: p.sarti@ira.inaf.it

Z. Altamimi
Institut Géographique National (IGN),
Ecole Nationale des Sciences Géographiques (ENSG),
6 et 8 Avenue Blaise Pascal, 77455 Marne-la-Vallée
Cedex 2, Paris, France
e-mail: altamimi@ensg.ign.fr

L. Vittuari
DISTART, Area Geodesia e Topografia,
Universitá di Bologna, Viale Risorgimento, 2,
40136 Bologna, Italy
e-mail: luca.vittuari@mail.ing.unibo.it

1 Introduction

The term *local tie* designates the entire process of surveying, theoretical modelling and statistical estimation which results in 3D baselines (i.e. TVs) linking the ITRF tracking points at co-located sites. Local ties should be provided with a complete variance–covariance (VC) matrix in Software INdependent EXchange (SINEX) format. They are essential to connect TRFs stemming from diverse space geodetic techniques (Altamimi et al. 2002, 2007)

Previous geodetic research has focussed on two fundamental aspects related to co-location ties issues. On the one hand, a methodological discussion concerning procedures used for the estimation of ITRF tracking points and aimed at supplying very precise TVs with full VC matrices (Dawson et al. 2007; Sarti et al. 2004; Sarti and Angermann 2005). On the other hand a computational aspect strictly related to combinations: the handling of TVs (Altamimi 2005), the selection of optimal co-location sites for the realization of consistent joint reference frames and the attempt to formulate qualitative criteria in order to isolate erroneous ties (e.g. Ray and Altamimi 2005; Krügel and Angermann 2005).

Usually determined via terrestrial observations of a local ground control network (LGCN), TVs are in principle referred to a local topocentric frame. Conversely, space geodetic solutions of ITRF stations are expressed with respect to a global reference frame. This raises a problem of frame consistency which is addressed by aligning TVs from a local to a global reference frame (see Sect. 3.4). In this respect, the investigation on the sensitivity of a combination to imprecise TV alignments (*ill-alignments*) is of particular importance.

In this study, the results of geodetic activities undertaken at the observatory of Medicina (Northern Italy) are presented. Five different TVs between the very long baseline interferometry (VLBI) and the global positioning system (GPS) in the years 2001, 2002, 2003, 2006 were realised (see Sects. 2 and 3). Their insertion into ITRF-like combinations of VLBI, GPS and SLR long-term solutions is discussed (see Sect. 4).

Medicina TVs have been estimated with an *indirect approach* (Sarti and Angermann 2005), which requires a geometrical modelling to relate network observations to the ITRF tracking points (see Sect. 3). Local ties have been measured by following two different procedures: (i) observations of angles, distances and height differences (classical or terrestrial local ties) and (ii) GPS observations (GPS-based local ties). The former is a well-established approach for estimating very precise TVs, which are routinely used in inter-technique combinations of TRFs. The latter approach has been experimented at the observatory of Medicina in 2002 and 2006 and its behaviour is here tested for the first time in ITRF-like combinations (see Sect. 4.5).

In addition, realistic ill-alignments have been simulated through the rotation of 2006 TV about the three ITRF coordinate axes, in order to investigate the effects on the combinations (see Sect. 5).

Therefore this work aims to address the following questions:

- Can GPS-based local ties be introduced as an alternative to classical tie information in inter-technique combinations of TRFs?
- How do GPS-based local ties generally behave into ITRF-like combinations, when compared to classical local ties?
- What is the global behaviour of an ITRF-like combination when inserting multiple realizations of the same TV?
- At what magnitude does the effect of an ill-alignment in the tie orientation impact on the estimation of a final joint reference frame?

2 The Medicina VLBI-GPS co-location: site's peculiarities and working datasets

The observatory of Medicina hosts a co-location between a GPS receiver and a VLBI radiotelescope; both are part of the network which realises the ITRF. The first rigorous determination of the VLBI–GPS local tie at the observatory dates back to 2001 and it has been remeasured almost yearly with GPS and/or terrestrial observations of the LGCN (Vittuari et al. 2005). Table 1 reports the DOMES numbering of fundamental geodetic markers of Medicina LGCN and their description.

No official guidelines have been yet established by geodetic community concerning the approaches to be applied when the estimation of space geodetic instrument's RPs is required. Nonetheless the literature includes at least three different methods for estimating instrumental RPs, namely the direct, hybrid and indirect approaches (Sarti and Angermann 2005). All the TVs considered in this study were estimated via an indirect approach. Classical local ties were measured through terrestrial observations of the LGCN in 2001, 2002 and 2003 whereas GPS-based local ties were obtained in 2002 and 2006 campaigns.

Table 1 IERS DOMES numbers, local identifiers (LI) and their description in the Medicina LGCN

DOMES #	LI	Description
12711M004	P1	Forced centering brass marker dug on top of a concrete pillar
12711M005	P2	Forced centering brass marker dug on top of a concrete pillar
12711M006	P3	Forced centering brass marker dug on top of a concrete pillar
12711M007	G7	Forced centering device on top of SLR pillar(C)/Top and centre of the screwed adapter
12711M003	GPS-MEDI	GPS Tracking Point
12711S003	VLBI	VLBI Tracking Point

3 Local tie estimation procedure step by step

Whenever a local tie is estimated through an indirect approach, the following fundamental stages can be identified:

1. Survey of LGCN and data reduction.
2. Network solution and datum definition.
3. Application of a geometrical model and estimation of the TV.
4. Local tie alignment.
5. Production of a SINEX file.

Stages 1, 2, 3 and 5 are common to any local tie; conversely stage 4 is peculiar only to terrestrial local ties. In the analysis of steps 1, 2, 3 and 5 the differences between the two surveying approaches will be pointed out, as described in the following sections.

3.1 Survey of LGCN and data reduction

In the case of Medicina the ITRF tracking points are the antenna reference point (ARP) of GPS-MEDI and the invariant point (IP) of VLBI (see Table 1). Therefore, the local tie surveying has to deal with peculiar networks: vertexes can be either **M**-type tracking points, which are physically monumented or **S**-type instrumental RPs, which are immaterial and not directly accessible. This originates part of the problem in a local tie estimation. Indirect observations must be adopted when surveying **S**-type tracking points and might be helpful on **M**-type tracking points also. Specifically, whenever the observations of **M**-type markers require a removal of a permanent tracking system (e.g. IGS stations), they can be used to observe the position of the tracking point without interfering with the space geodetic observations. This approach has always been successfully applied to GPS-MEDI (Sarti et al. 2004).

3.1.1 Terrestrial surveying

Trilaterations and triangulations were extensively performed on retro-reflecting prisms installed in the LGCN and on the VLBI telescope's structure. Redundant measurements were performed with a TDA5005 (0.15 mgon, 1 mm + 2 ppm) and a TC2003 (0.15 mgon, 1 mm + 1 ppm). Total stations' standpoints were realised on all ground markers, i.e. on concrete pillars (points **P1**, **P2**, **P3**, see Table 1) and tripods which were temporarily installed. Markers on pillars were 2D self-centering devices on which a direct instrumental height reading can be performed. Details about the terrestrial survey procedures can be found in Vittuari et al. (2005). To date, indirect methods have proved to be effective and consistent in providing high precise TVs with a full VC matrix (Johnston and Dawson 2004a,b; Johnston et al. 2004; Sarti et al. 2004; Dawson et al. 2007). Table 2 contains the number of observations collected during the three terrestrial surveys along with the overall number of network points. The number of observations considerably increases when passing from the first survey in 2001 to the last in 2003: this testifies the network geometry has been improved and strengthened. Terrestrial data adjustment has been performed by means of STAR*NET v6.0.24 (Sawyer 2001) so as to allow a datum definition for classical local ties.

3.1.2 GPS survey

GPS-based surveying may be efficiently applied to VLBI–GPS eccentricities, as well as at all co-location sites where ITRF tracking points can be measured through GPS technique. This approach offers some advantages with respect to that terrestrial, since it is faster, it does not require particular surveying abilities, it is semi-automatic and time-saving, thus reducing the downtime of the VLBI telescope during the measurement sessions. In both GPS-based ties, two GPS receivers were installed on the external part of the VLBI dish using L-shaped devices in symmetric positions parallel to the elevation axis (see Fig. 1). These stainless steel supports assured the two GPS antennas remained pointing vertical as the VLBI telescope was steered in elevation. Phase observations were acquired for each GPS antenna with a 5 s sampling rate at different telescope's elevation and azimuth positions, based on a rapid-static approach: in 2002 the roving GPS antennas acquired observable standing still for 15 min in each position, whereas in 2006 the static interval was extended to 30 min. The schedule of 2002 survey was partially completed owing to an interruption in the power supply. Two different kinds of antenna were used: $TRM22020.00 + GP$ in 2002 and $LEIAT504$ Choke Ring in 2006. The LGCN pillars **P1**, **P3**, **G7** were surveyed in both campaigns, whereas in

Table 2 Characteristics of the terrestrial surveys, in terms of observations and unknowns, performed during 2001, 2002, 2003 for estimating the VLBI–GPS eccentricity at Medicina

Site name	# Observations			# Points	Epoch (yy:ddd)
	Azimuth ang	Zenith ang	Distances		
Medicina	297	288	271	106	01:174
Medicina	308	339	327	105	02:252
Medicina	550	554	561	236	03:274

Fig. 1 Positions of the two GPS antennas installed on the VLBI telescope

2006 two further receivers were installed on **P2** and on a temporary tripod. A three step procedure was used for GPS data computation. In the first step, the data collected by the ground GPS antennas located at the observatory were analysed together with a selected subset of European IGS stations (MEDI, CAGL, GRAS, GRAZ, MATE, NOT1, WTZR, ZIMM) in order to compute a priori coordinates and tropospheric parameters. These latter were applied to the roving GPS antennas located on the VLBI dish by accounting for the relative height differences. Data analysis was performed with Bernese GPS Software v5.0 (Dach et al. 2007); IGS final orbits and Earth rotation parameters were used together with the absolute antenna phase centre variations (PCV) and offsets (Schmid et al. 2005). Elevation cut-off angle was set to 10° and an ambiguity fixed solution was computed. In the second step, baselines between the roving antennas and the ground stations were formed depending on the availability and quality of observations. $L1$ carrier frequency was analysed using an a priori ionosphere model; in both steps daily normal equations (NEQs) were stacked. In the last step, local and regional NEQs were combined together in order to estimate precise coordinates and their full VC matrix for ground and roving GPS systems.

3.2 Network solution and datum definition

Datum definition for a LGCN requires the definition of seven parameters, if its evolution in time is neglected: they correspond to three translations (origin), three rotations (orientation) and a homothety factor (scale) whose choice is, to a certain extent, conventional.

Terrestrial ties rely on the acquisition of measurements made by survey instruments, whose optics and mechanics are sensitive to local gravity field. Measurements are thus intrinsically referred to the normal to the local geoid. As a consequence, four out of seven parameters defining the Datum have to be estimated in case of terrestrial ties: these are an axial rotation about this reference direction and three translations. No homothety factor is required since range measurements define the scale. Datum for terrestrial ties is thus local and topocentric. In particular, within the terrestrial data adjustment, (i) horizontal coordinates of the geodetic marker set up on **P3** have been constrained to 0; (ii) height of **G7** has been constrained to 0; (iii) the direction linking **P3** to **P1** has been constrained for orienting the local network.

In case of GPS-based local ties, GPS networks present three degrees of freedom. In particular, within the GPS data processing, datum was defined via the application of a no net translation condition (NNT, see Dach et al. 2007) acting over a subset of IGS stations (CAGL, GRAS, GRAZ, MATE, NOT1). Datum defined for GPS-based local ties is global and VLBI IP is consistently expressed with respect to ITRF2000.

3.3 Application of a geometrical model and estimation of the TV

The application of the indirect surveying approach allows to recover the two rotational axes of the VLBI telescope and, accordingly, the IP through the definition of planes and spheres. The GPS ARP is recovered by triangulating fictitious points, which are symmetrically coupled on the GPS antenna structure. A detailed description of the fundamentals of the basic geometrical model used for the tracking points' estimation can be found in Sarti et al. (2004). This model has been implemented with further geometrical conditions:

- *Intra-group parallelism of elevation and azimuth planes* (IGP), i.e. the groups of planes containing the targets/GPS receivers during the antenna rotations have to be parallel.
- *Inter-axial orthogonality* (IAO), which imposes the elevation and the fixed axes of the VLBI telescope to be orthogonal.
- *Axis offset estimate* (AOE), which allows the estimation of the offset between the VLBI fixed and moving axis.

The degree of geometric conditioning can be varied and different geometrical scenarios have been used for the local ties' estimation at Medicina. Loosely conditioned solution (LCS) corresponds to the basic geometrical model detailed in Sarti et al. (2004). In the following are indicated the various geometrical configurations:

- **C1**: LCS
- **C2**: LCS + IGP
- **C3**: LCS + IGP + IAO
- **C4**: LCS + IGP + IAO+AOE

Table 3 Set of local ties tested in the ITRF-like combinations

Site name	DOMES #	From	To	**dX**	**dY**	**dZ**	Epoch (yy:ddd)	GS	Kind	Tie ID
Medicina	12711	M003	*S*001	-30.9062	3.3976	54.5221	01:174	**C2**	C	2001-C
Medicina	12711	M003	*S*001	-30.9065	3.3968	54.5251	02:252	**C3**	C	2002-C
Medicina	12711	M003	*S*001	-30.9041	3.3956	54.5243	03:274	**C1**	C	2003-C
Medicina	12711	M003	*S*001	-30.9040	3.4013	54.5289	02:253	**C1**	G	2002-G
Medicina	12711	M003	*S*001	-30.9099	3.3907	54.5235	06:194	**C2**	G	2006-G

Vector components are expressed in (m) w.r.t. ITRF2000
The column GS indicates the Geometrical Scenario applied when estimating the eccentricity whereas the column Kind indicates the surveying approach used (C classical, G GPS-based)

The four configurations have been applied on the whole set of Medicina ties. A more detailed analysis on the effects of the application of a geometrical modelling on the terrestrial and the GPS-based ties can be found in (Dawson et al. 2007; Abbondanza et al. 2009). Solutions characterised by a minimum variance on the magnitude of the TV have been selected for being inserted into ITRF-like combinations. Table 3 reports the geocentric components of each TV, expressed into ITRF2000, along with the surveying epoch, the geometrical configuration applied and the measurement strategy used.

3.4 Local tie alignment

This phase corresponds to the transformation of the TV from a local topocentric frame to an ITRFyy. It is required whenever the local tie is surveyed through terrestrial observations. As mentioned in Sect. 1, a local to global mapping makes terrestrial ties consistent with space geodetic solutions. On the contrary, GPS-based ties are intrinsically inserted into an ITRFyy and this makes the alignment unnecessary. The transformation is a 3D isometry applied to the tracking points connected by the tie: the vector is thus translated and rotated keeping its magnitude unaltered. This alignment is routinely performed with the estimation of a 3D isometry which relies and depends on the availability and the number of tie points within the co-located site. Tie points are surveyed with both terrestrial and GPS in order to estimate coordinates in local and global frames. Terrestrial local ties at Medicina have been all aligned on the basis of 2006 LGCN GPS survey.

4 Local ties and interactions with an ITRF-like combination procedure

The effects of the introduction of local ties into ITRF-like combinations have been tested under various processing options supported by the combination software. SINEX files of SLR, VLBI and GPS long term solutions have been combined with TVs in order to compute a joint reference frame.

4.1 Combination and analysis of terrestrial reference frames

ITRF combinations are performed by means of Combinations and Analysis of Terrestrial REference Frames (CATREF). It is a scientific software developed at Institut Géographique National (IGN), France, for the ITRF combination activities. It combines SINEX files of different space geodetic solutions and permits an accurate datum definition according to the well acknowledged approach of minimal constraints (Altamimi et al. 2007).

4.2 Handling of local ties in ITRF-like combinations

Two possible strategies exist for inserting TVs into CATREF:

- First strategy: through their SINEX files.
- Second strategy: through their vector components.

In the first strategy, CATREF estimates the similarity transformation parameters (plus their rates) for every SINEX introduced in the combination, except for the tie SINEX files. These latter dispose of much less observations and therefore the estimation of all the tie specific transformation parameters cannot be performed. These are thus reduced to the three translations which the TV can undergo during the combination. Neither tie-specific homotethy factor nor rotation angles are reckoned: as a consequence, each local tie must be inserted into the combination with a reliable orientation, which is held fixed in the computation.

In the second strategy, the insertion into the combination is led through the components of the TVs (Altamimi 2005). Also in this context, TVs have to be aligned in a global frame before the combination is carried out, thus making their integration with space geodetic solutions fully consistent. In this case, no estimation of tie-specific similarity parameters is performed: the TV is neither rotated nor translated and scaled and the comprehensive estimation of datum definition parameters is limited to space geodetic solutions. Particularly, two possible stochastic models associated with the local ties can be chosen: (i) a VC matrix with only diagonal terms

Table 4 Description of the strategies used for introducing TVs in CATREF and their identifiers adopted in the text (Cod in the last column)

Strategy	Description	StMod	Cod
First	TV SINEX	FVC	**S#**
Second	TV components	ODT	**S#a**
Second	TV components	BDT	**S#b**

FVC full VC matrix, *ODT* only diagonal terms, *BDT* block diagonal terms

\# in the last column refers to the number of solution (*S*) introduced in Table 6. StMod in the third column indicates the stochastic models related to the VC matrix structure

Table 5 Space geodetic solutions included into the ITRF-like combinations: characteristics of the SINEX files in terms of stations contained, number of solutions considered and observing span time

	# Stations	# Solutions	Obs. span time
GPS	45	103	1996:2006
SLR	48	55	1992:2005
VLBI	48	63	1979:2006
COMB	141	221	1979:2006

(approach *a*, see Table 4); (ii) a block-diagonal VC matrix (approach *b*, see Table 4).

4.3 Input data for the ITRF-like combinations

The three SINEX files reproducing long term solutions of TRFs related to VLBI, SLR and GPS have been used as input datasets; 43 co-location sites have been selected for joining the technique specific frames. 27 sites are characterised by a double instrument co-location, 8 sites by a triple co-location, 6 sites host a quadri co-location and 2 a co-location between 5 space geodetic instruments. The total number of baselines (linking the ITRF tracking points) included into the combinations is 69. For each of the three SINEX files, the long term solutions contain positions and velocities and represent an extraction from the overall amount of stations used for the ITRF2005 computation.

Table 5 reports the characteristics of the SINEX files containing the space geodetic solutions and the final combined frame. Since temporal breaks were introduced in the observing history of most of the stations, the number of solutions computed for each technique is greater than the number of stations included.

4.4 An insight on the ITRF-like combinations

The combination tests performed with CATREF are coded in Table 6. Except for the solution **S0**, all the combinations are highly redundant since they are realised using the total amount of 69 TVs plus those listed in Table 6. Specifically **S0** identifies a low redundancy solution. Due to the scarce number of tie vectors introduced, this combination is evidently not optimal and *a posteriori* residuals are not shown here. Only formal errors related to the VLBI–GPS transformation parameters are discussed (see Table 9); the general behaviour of low redundancy VLBI–GPS combinations is thoroughly investigated in Ray and Altamimi (2005). Solutions from **S1** to **S5** are executed by adding to the group of 69 TVs the five tie realizations at Medicina, one at a time. **S6** and **S7** combine space geodetic solutions using redundant realizations of the Medicina local tie: in particular **S6** includes the two GPS-based local ties, whereas **S7** includes the three terrestrial local ties altogether.

Table 6 Combination tests performed with CATREF, along with the additional Medicina local ties included in the working dataset

S	DOMES #	From	To	Epoch (yy:ddd)	Kind
0	12711	M003	S001	02:253	G
	12717	M004	S001	03:176	C
	50116	M004	S002	02:081	C
1	12711	M003	S001	01:174	C
2	12711	M003	S001	02:252	C
3	12711	M003	S001	02:253	G
4	12711	M003	S001	03:274	C
5	12711	M003	S001	06:194	G
6	12711	M003	S001	02:253	G
	12711	M003	S001	06:194	G
7	12711	M003	S001	01:174	C
	12711	M003	S001	02:252	C
	12711	M003	S001	03:274	C

S in the first column identifies the code assigned to the combination

All the combinations were performed according to the following settings of CATREF: (i) GPS TRF: no similarity transformation parameter was estimated for mapping the GPS solution on the final combined reference frame; GPS was thus assumed to be the reference solution. (ii) VLBI and SLR TRFs: all the 14 transformation parameters were estimated. (iii) Local tie weighting strategy: VC scaling factors bigger than 1 were applied to all TVs whose normalised residual exceeded the threshold 4 in any of its components. (iv) Solutions including redundant Medicina TV realisations (see **S6**, **S7** in Table 6): no discontinuities were introduced in the positions of stations 12711M003 and 12711S003. (v) It was assumed that the VLBI and GPS velocities were the same at the co-location sites.

4.5 Assessment of the results

For each of the combinations reported in Table 6, both the strategies have been tested (see Sect. 4.2). Results are discussed considering the *a posteriori* residuals between local ties and the combined solution, which are expressed in a geodetic local frame. Residuals characterise the agreement between space geodetic observations and local ties and express the consistency of the joint reference frame.

Considering the first strategy, residuals of **S1**, **S2**, **S4** reported in Table 7 highlight that the consistency between the GPS and VLBI reference frames considerably improves, when passing from **S1** to **S4**: this latter attains residuals' values which are under the millimeter in all the three components. This shows that the increased robustness of the local network's geometry (see Table 2) and the application of a varied geometrical conditioning positively affect the final combination.

In examining the behaviour of GPS-based local ties added one at a time in the combinations (solutions **S3**, **S5**), it can be observed that, when passing from combination **S3** to **S5**, the agreement between the two frames improves. This might also be due to the major geometrical robustness of 2006-G with respect to that of 2002-G. In fact, as mentioned in Sect. 3.1.2, schedule of 2002-G local tie was partially completed owing to a technical failure. This caused the reduction of the number of surfaces used in the geometrical modelling for the VLBI RP estimation. In particular, the improvement in the VLBI–GPS frames' consistency is clearly evident for the residuals in the horizontal components (N, E), whose values are comparable in magnitude to those terrestrial. Both the TVs show a considerable residual in the Up component which slightly diminishes when passing from **S3** (**dU** = 6.7 mm) to **S5** (**dU** = 5.7 mm). Therefore GPS-based local ties tend to bias the global estimations in the Up component much more than those terrestrials. This effect is probably due to (i) the intrinsic higher uncertainty in the Up component of the GPS data analysis and to (ii) gravity acting on the edge of the VLBI dish where the GPS roving antennas were mounted. In particular, previous experiments show that gravity induces a downward flattening of the edge of the primary reflector (Sarti et al. 2009) that biases the estimate of VLBI IP Up component. This reflects on the combination of frames and thus any attempt to recover the VLBI IP with indirect methods should not disregard gravity deformations of the telescope structures.

Table 7 Residuals for local ties from VLBI–GPS–SLR combinations (values are expressed in mm w.r.t. to the geodetic local frame) for the different combinations performed

S	DOMES	**dN**	**dE**	**dU**	Epoch	Kind
1	12711	2.2	−2.3	1.8	01:174	C
1a	12711	0.8	−1.0	1.5	01:174	C
1b	12711	1.1	−1.2	1.3	01:174	C
2	12711	0.1	−1.1	1.0	02:252	C
2a	12711	0.0	−0.5	0.9	02:252	C
2b	12711	0.0	−0.5	0.9	02:252	C
3	12711	1.5	−3.5	6.7	02:253	G
3a	12711	0.2	−2.3	5.0	02:253	G
3b	12711	0.2	−2.3	5.0	02:253	G
4	12711	0.2	0.6	0.6	03:274	C
4a	12711	0.1	0.0	0.3	03:274	C
4b	12711	0.1	0.0	0.4	03:274	C
5	12711	−0.8	1.6	5.7	06:194	G
5a	12711	−0.3	−0.6	1.9	06:194	G
5b	12711	0.6	−1.7	12.4	06:194	G
6	12711	1.3	−5.0	−2.8	02:253	G
	12711	−0.4	4.2	6.6	06:194	G
6a	12711	1.1	−9.3	−7.3	02:253	G
	12711	−0.6	−0.1	2.2	06:194	G
6b	12711	1.8	−9.4	−5.6	02:253	G
	12711	0.1	−0.2	3.8	06:194	G
7	12711	1.1	−1.3	1.9	01:174	C
	12711	−1.3	−0.6	0.1	02:252	C
	12711	0.7	1.1	−0.7	03:274	C
7a	12711	1.0	−2.0	2.2	01:174	C
	12711	−1.5	−1.3	0.4	02:252	C
	12711	0.6	0.4	−0.4	03:274	C
7b	12711	0.9	−2.1	2.1	01:174	C
	12711	−1.5	−1.4	0.3	02:252	C
	12711	0.5	0.2	−0.5	03:274	C

Solutions codified with just a number in the column *S* refer to first strategy whereas those codified with letters *a* and *b*, respectively, refer to second strategy where VC matrix has *only diagonal terms* and *block-diagonal terms*. Epochs are expressed in (yy:ddd); in the column Kind, *G* indicates the GPS-based local ties whereas *C* indicates those classical

Solutions **S6** and **S7** combine the realizations of the Medicina local tie computed at different epochs. Since no discontinuities have been introduced in the stations' positions, all the tie realizations have been applied to the same temporal window of VLBI and GPS data so as to increase the total redundancy of the solution. Hence, the combination acts over the different tie realizations by averaging their contribution according to the weights introduced. It is worth highlighting that the residuals of all ties are modified with respect to the corresponding residuals obtained introducing one tie at a time. Solutions denoted with *a* and *b* in Table 7 refer to the results of combinations obtained inserting local ties through their vector components (second strategy, see Sect. 4.2). The residuals are all modified with respect to those obtained with the first strategy. In particular single-tie solutions **S**1a, **S**1b, **S**2a, **S**2b, **S**3a, **S**3b, **S**4a, **S**4b have smaller residuals and thus are characterised by smaller discrepancies in all the three components. This does not hold for solution **S**5b, whose Up component residual exceeds 1 cm and is, so far, the largest detected among all the combinations performed. This might be an evidence of inconsistencies introduced by the off-diagonal terms of the VC matrix associated with the 2006-G TV.

Table 8 Differences of residuals (mm) w.r.t. geodetic local frame between the first strategy and second strategy (approaches *a* and *b*)

S	DOMES	δ**dN**	δ**dE**	δ**dU**	Epoch	Kind
1						
1-1a	12711	1.4	−1.3	0.3	01:174	C
1-1b	12711	1.1	−1.1	0.5	01:174	C
2						
2-2a	12711	0.1	−0.6	0.1	02:252	C
2-2b	12711	0.1	−0.6	0.1	02:252	C
3						
3-3a	12711	1.3	−1.2	1.7	02:253	G
3-3b	12711	1.3	−1.2	1.7	02:253	G
4						
4-4a	12711	0.1	0.6	0.2	03:274	C
4-4b	12711	0.1	0.6	0.2	03:274	C
5						
5-5a	12711	−0.5	4.4	2.8	06:194	G
5-5b	12711	−1.4	3.3	−6.7	06:194	G
6						
6-6a	12711	0.2	4.3	4.5	02:253	G
	12711	0.2	4.3	4.4	06:194	G
6-6b	12711	−0.5	4.4	2.8	02:253	G
	12711	−0.5	4.4	2.8	06:194	G
7						
7-7a	12711	0.1	0.7	−0.3	01:174	C
	12711	0.2	0.7	−0.3	02:252	C
	12711	0.1	0.7	−0.3	03:274	C
7-7b	12711	0.2	0.8	−0.2	01:174	C
	12711	0.2	0.8	−0.2	02:252	C
	12711	0.2	0.9	−0.2	03:274	C

Epochs are expressed in (yy:ddd)

In fact, the solution **S**5a (only diagonal terms) has smaller post fit residuals.

Table 8 shows the differences between residuals of the first and second strategy (approaches *a* and *b*). The intercomparison of the residuals (see Table 8) points out how the information carried on by the local ties is handled in the combination. In particular, the different strategies originate different residuals and thus the introduction of VC information directly affects the final combined frame. In some cases, approaches *a* and *b* have the same impact on the combination; this holds for **S2**, **S3**, **S4**. When redundant information is introduced in the combination (**S6**, **S7**), it can be noticed that the residuals differences of all TVs between the first and the second strategy (approaches *a* and *b*) are identical. This basically means that the same rotation is applied to all the group of TVs and the rotation depends on the combination strategy.

Table 9 gathers the estimated VLBI–GPS transformation parameters along with their standard deviations for all the combinations performed. These results allow an evaluation of the quality of the combined reference frames throughout all the solutions. Considering **S0**, rotation parameters are much more affected by the low degree of redundancy: the uncertainties in R_x and R_z can attain values up to ten times greater than the redundant combinations. Concerning these latter, it has to be noticed that better results are always achieved when using the first strategy. The uncertainties related to the three translation parameters are in the order of 1 mm. The maximum uncertainty for the homothety factor corresponds to solution **S6** (second strategy), and it attains the value of 0.24 ppb (1.5 mm).

The maximum value of the rotation parameters' uncertainties again relates to solution **S6** (second strategy) and it attains the value of 0.053 mas (1.6 mm). Therefore, joint VLBI–GPS reference frames can be established with 1-mm datum consistency throughout these ITRF-like combinations. This datum consistency is preserved when including the GPS-based local ties (see **S3** and **S5**), thus suggesting that this approach is potentially reliable and effective. Nevertheless further investigations are needed to assess the repeatability.

5 Ill-alignment scenarios

Simulations of realistic ill-alignments of the same 2006-G TV were produced applying subsequent rotations about the three axes of a local reference frame, whose orientation is consistent with the ITRF2000 and whose origin has been translated in the VLBI IP. Angular values of these rotations are chosen to be directly proportional to the TV's uncertainty, suitably represented by the three rotation angles $(\omega, \lambda, \vartheta)$ (see Table 10). As mentioned in Sect. 3, the alignment of a terrestrial local tie is purely statistical. Thus its orientation cannot be univocally assigned, since the same local tie can originate vectors with the same magnitude but different components. The purpose is to quantify how the combination reacts to the introduction of an eccentricity undergoing slight differential rotations. Rotations of 3, 6 and 10 times the uncertainties of 2006 local ties were applied to 2006-G TV; three tie SINEX files were obtained and introduced in the combinations of the 69 TVs, one at a time. Table 10 shows the vector components of these ties along with the differential 3D rotations applied.

Table 11 reports the residuals (Space Geodesy-Tie) of the three combinations obtained with the first strategy. Despite of the fact that the three TVs have the same magnitude, the residuals are different in the three combinations and the discrepancies tend to augment as the misalignment angles increase. **dN** and **dE** residuals do not exceed a few millimetre while **dU** can be as large as 12.2 mm (see **Sim3**). North component

Table 9 Estimated transformation parameters from VLBI to GPS solution along with their standard deviations (reported in brackets) for the various combinations performed

S	T_x (cm)	T_y (cm)	T_z (cm)	λ (10^{-8})	R_x (mas)	R_y (mas)	R_z (mas)
0	1.32 (0.50)	−0.16 (0.20)	−0.45 (0.50)	−0.040 (0.042)	0.977 (0.437)	−0.237 (0.129)	0.749 (0.417)
1	0.19 (0.11)	0.34 (0.10)	0.22 (0.10)	−0.043 (0.017)	0.122 (0.038)	−0.095 (0.033)	−0.076 (0.032)
1a	0.27 (0.14)	0.27 (0.12)	0.18 (0.12)	−0.026 (0.021)	0.124 (0.046)	−0.158 (0.045)	−0.100 (0.040)
1b	0.27 (0.14)	0.27 (0.12)	0.17 (0.12)	−0.025 (0.021)	0.122 (0.046)	−0.156 (0.045)	−0.097 (0.040)
2	0.18 (0.10)	0.31 (0.10)	0.20 (0.09)	−0.045 (0.017)	0.122 (0.038)	−0.086 (0.033)	−0.088 (0.031)
2a	0.26 (0.14)	0.27 (0.11)	0.12 (0.12)	−0.025 (0.021)	0.118 (0.046)	−0.130 (0.044)	−0.100 (0.039)
2b	0.25 (0.14)	0.27 (0.11)	0.12 (0.12)	−0.026 (0.021)	0.117 (0.046)	−0.130 (0.043)	−0.099 (0.039)
3	0.01 (0.11)	0.25 (0.10)	0.07 (0.10)	−0.075 (0.017)	0.120 (0.038)	−0.026 (0.033)	−0.125 (0.031)
3a	0.08 (0.14)	0.22 (0.11)	0.09 (0.12)	−0.054 (0.022)	0.126 (0.046)	−0.099 (0.047)	−0.134 (0.039)
3b	0.08 (0.14)	0.21 (0.11)	0.05 (0.12)	−0.053 (0.022)	0.125 (0.046)	−0.083 (0.045)	−0.140 (0.039)
4	0.16 (0.10)	0.32 (0.10)	0.26 (0.09)	−0.051 (0.017)	0.124 (0.038)	−0.117 (0.032)	−0.085 (0.031)
4a	0.23 (0.14)	0.31 (0.11)	0.18 (0.12)	−0.032 (0.021)	0.119 (0.046)	−0.159 (0.043)	−0.080 (0.038)
4b	0.23 (0.14)	0.31 (0.11)	0.18 (0.12)	−0.032 (0.021)	0.119 (0.046)	−0.160 (0.043)	−0.080 (0.038)
5	0.20 (0.10)	0.41 (0.09)	0.18 (0.09)	−0.045 (0.017)	0.116 (0.037)	−0.075 (0.032)	−0.040 (0.031)
5a	0.31 (0.14)	0.43 (0.11)	0.10 (0.12)	−0.015 (0.021)	0.090 (0.045)	−0.128 (0.043)	−0.022 (0.037)
5b	0.28 (0.14)	0.43 (0.11)	0.07 (0.12)	−0.018 (0.021)	0.088 (0.045)	−0.113 (0.042)	−0.023 (0.037)
6	0.19 (0.11)	0.34 (0.10)	0.15 (0.10)	−0.044 (0.018)	0.115 (0.040)	−0.064 (0.035)	−0.079 (0.033)
6a	0.29 (0.16)	0.42 (0.13)	0.09 (0.14)	−0.013 (0.024)	0.091 (0.053)	−0.126 (0.051)	−0.027 (0.044)
6b	0.26 (0.16)	0.42 (0.13)	0.06 (0.14)	−0.017 (0.024)	0.088 (0.053)	−0.110 (0.051)	−0.029 (0.044)
7	0.18 (0.11)	0.30 (0.10)	0.25 (0.10)	−0.047 (0.017)	0.124 (0.039)	−0.112 (0.033)	−0.091 (0.031)
7a	0.25 (0.14)	0.30 (0.12)	0.17 (0.12)	−0.028 (0.021)	0.119 (0.047)	−0.155 (0.044)	−0.086 (0.039)
7b	0.25 (0.14)	0.30 (0.12)	0.17 (0.12)	−0.028 (0.021)	0.119 (0.047)	−0.156 (0.044)	−0.084 (0.039)

S in the first column identifies the code assigned to each combination

Table 10 Ill-alignment configurations of 2006 GPS-based local ties generated by progressively misaligning the TV

DOMES	GPS VLBI	**dX**	**dY**	**dZ**	Epoch (yy:ddd)	Modulus (m)	Rotations (as)		
							dθ (X)	**d**ω (Y)	**d**λ (Z)
12711	M003 S001	−30.9099	3.3907	54.5235	06:194	62.7673	0	0	0
12711	M003 S001	−30.9144	3.3894	54.5210	06:194	62.7673	19.32	9.81	1.65
12711	M003 S001	−30.9190	3.3881	54.5185	06:194	62.7673	38.64	19.62	3.30
12711	M003 S001	−30.9251	3.3864	54.5152	06:194	62.7673	64.40	32.70	5.50

The angles (ω, λ, ϑ) designates rotations about the three axes of a local frame oriented as ITRF2000 and whose origin is located in the VLBI IP. 1 s of arc (as) rotation applied to a TV of 63 m approximately equals to a normal displacement of 0.3 mm

Table 11 Residuals for local ties from VLBI–GPS–SLR combination

Sim	DOMES	**dN**	**dE**	**dU**	Epoch	Kind
1	12711	−1.2	1.7	7.7	06:194	G
2	12711	−1.7	1.8	9.6	06:194	G
3	12711	−2.4	1.9	12.2	06:194	G

Values are expressed in mm w.r.t. to geodetic local frame for the different combinations performed. Epochs are expressed in (yy:ddd) and correspond to the last three lines of Table 10

doubles its residual when passing from **Sim1** to **Sim3** solution whereas the East component does not significantly vary. Therefore the 3D rotation applied to the 2006-G TV affects more the Up and North components of the joint reference frame.

This simulation points out that the ill-alignment of a TV can distort the estimation of a combined reference frame. Such an aspect highlights that providing the local ties with a reliable orientation plays a key role for combining reference frames with a high degree of consistency.

6 Conclusions

Our tests of ITRF-like combinations remark the effectiveness of an enforced network's geometry and the impact of the geometric model implemented for the indirect approach. There is evidence of an improvement when the local network is enforced both for terrestrial (see combinations **S1**, **S2** and **S4** in Table 7) and GPS-based TVs (see combinations **S3**, **S5** Table 7). Due to their behaviour in the combination and to the easiness in the surveying approach, GPS-based ties appear to be particularly promising, while still at an experimental stage. Results showed that local effects of GPS-based TVs, when considered one at a time, are comparable, in terms of residuals, with classical local ties, especially in the horizontal (N, E) components, while the Up-component appears to be weaker.

On the whole, combinations with redundant information (i.e. solutions **S6**, **S7**) display an effect of residual redistribution when compared to solutions **S1**, **S2**, **S3**, **S4**, **S5**. This suggests that the joint reference frame tends to locally adjust according to the redundant information conveyed by the TVs, as if they would have been averaged. Finally, varying degrees of TV misalignments were tested; this confirmed that the final orientation of the TV is crucial and that the combination is potentially sensitive to any misalignment. This equally holds for terrestrial ties, which initially do not contain any information about the global frame and that must be necessarily transformed from the local frame to the ITRF with a process which could potentially alias their intrinsic accuracy.

Acknowledgments This study was conducted during the period that C Abbondanza spent as a visiting Ph.D. student at the Laboratoire des Récherches en Géodésie (Marne La Vallée, France). The author would like to thank LAREG for their gracious hospitality and for having provided the combination dataset and X Collilieux for his friendly and fruitful support. This work is based on observations with the Medicina telescope operated by INAF - Istituto di Radioastronomia.

References

Abbondanza C, Negusini M, Sarti P, Vittuari L (2009) VLBI–GPS eccentricity vectors at Medicina's observatory via GPS surveys: reproducibility, reliability and quality assessment of the results. In: Drewes H (ed) IAG springer series 134, International IAG symposium, Geodetic reference frames, Munich, 9–14 October 2006, ISBN: 978-3-642-00859-7

Altamimi Z (2005) ITRF and co-location sites. IERS Tech Note 33:8–15

Altamimi Z, Sillard P, Boucher C (2002) ITRF2000: a new release of the International Terrestrial Reference Frame for Earth science applications. J Geophys Res 107(B10):2114–2133

Altamimi Z, Collilieux X, Legrand J, Garayt B, Boucher C (2007) ITRF2005: a new release of the International Terrestrial Reference Frame based on time series of station positions and Earth Orientation Parameters. J Geophys Res 112(B09):401. doi:10.1029/2007JB004949

Dach R, Hugentobler U, Fridez P, Meindl M (2007) Bernese GPS Software Version 5.0. Astronomical Institute, University of Bern

Dawson J, Sarti P, Johnston G, Vittuari L (2007) Indirect approach to invariant point determination for SLR and VLBI systems: an assessment. J Geod 81(6–8):433–441

Johnston G, Dawson J (2004a) The 2002 Mount Pleasant (Hobart) radio telescope Local Tie Survey. Tech. rep., Geoscience Australia Record 2004/21

Johnston G, Dawson J (2004b) The 2003 Yarragadee (Moblas 5) Local Tie Survey. Tech. rep., Geoscience Australia

Johnston G, Dawson J, Naebkhil S (2004) The 2003 Mount Stromlo Local Tie Survey. Tech. rep., Geoscience Australia Record 2004/20

Krügel M, Angermann D (2005) Frontiers in the combination of space geodetic techniques. In: Tregonin P, Rizos C (eds) IAG Springer series. International IAG symposium. Dynamic planet: monitoring and understanding a dynamic planet with geodetic and oceanographic tools, Cairns, Australia, 22–26 August 2005

Ray J, Altamimi Z (2005) Evaluation of co-locations ties relating the VLBI and GPS reference frames. J Geod 79(4–5):189–195

Sarti P, Angermann D (2005) Terrestrial data analysis and SINEX Generation. IERS Tech Note 33:118–127

Sarti P, Sillard P, Vittuari L (2004) Surveying co-located space geodetic instruments for ITRF computation. J Geod 78(3):210–222

Sarti P, Vittuari L, Abbondanza C (2009) Laser scanner and terrestrial surveying applied to gravitational deformation monitoring of large VLBI telescopes' primary reflector. J Surv Eng. http://dx.doi.org/10.1061/(ASCE)SU.1943-5428.0000008

Sawyer R (2001) STAR*NET-PRO V6 least-squares survey network adjustment program reference manual. STARPLUS SOFTWARE, INC, Oakland

Schmid R, Rothacher M, Thaller D, Steigenberger P (2005) Absolute phase center corrections of satellite and receiver antennas. GPS Solut 9(4):283–293

Vittuari L, Sarti P, Sillard P, Tomasi P, Negusini M (2005) Surveying the GPS-VLBI eccentricity at Medicina: methodological aspects and practilities. IERS Tech Note 33:38–48