An Analysis of Local Tie Vectors' Temporal Evolution and Site Stability at Medicina Observatory through Terrestrial and GPS-based Observations

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Abstract. The observatory of Medicina (Italy) hosts a co-location between GPS and VLBI, whose eccentricity vector has been re-measured several times once since 2001.
As a result, the co-located site is now provided with a series of nearly annual local tie vectors produced by means of terrestrial and GPS-based observations.

This work aims at summarising the results of a systematic analysis of the whole set of local ties produced so far and highlighting at the same time their temporal evolution.

As such, the application of algebraic tools focussed on deformation detections can benefit the analysis of the local tie temporal variations and at the same time that one of the "geodynamical" stability of the co-located site.

Nevertheless, just like in any other problem of deformation detection, such an analysis would ask for a reproducible local datum, according to which the relative displacements among the points have to be referred to.

The application of algebraic projection theory to the local tie oriented-networks will be carried out in order to select a "common and fully reproducible" datum onto which the network solutions will be projected and discuss possible displacements within the re-measured networks.

Keywords. Local ties**,** Co-location, GPS, VLBI, Inner constraint solution.

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1 Introduction

Since 2001 a research project at Medicina Observatory has been undertaken with the aim to measure and compute the eccentricity between VLBI and GPS instruments. As a result, the co-located site is now endowed with a series of nearly yearly VLBI-GPS local tie vectors, measured *via* a classical terrestrial approach or a wholly GPS-based one.

This investigation will sum up the results of a consistent analysis of the entire set of local ties performed up to now at Medicina by highlighting their temporal evolution.

In fact, a proper temporal analysis of such eccentricities along with all the network information extracted from the SINEX files might point out possible displacements of the ground control points belonging to the local network, as well as meaningful variations in the Reference Points of the VLBI and GPS techniques.

Nonetheless this investigation is not straightforward at all and it is further complicated by the difficulties in reproducing - epoch by epoch- a stable and consistent local reference frame which the displacements ought to be referred to.

That's the reason why such analysis necessarily have to cope with the attempt of "projecting" all the different realizations of the local reference frame coming from different surveys onto the same space with respect to that they'll be all consistent and comparable.

Once this has been done, an evaluation of displacements in terms of difference of coordinates will be theoretically possible and meaningful.

2 An overview of Inter-Technique eccentricities performed at Medicina

As well known, results in space geodesy claim the necessity of inter-technique eccentricities at 1 mm-level when the definition of a multitechnique Terrestrial Reference Frame with 1 mm consistency is needed (Ray and Altamimi 2005). In this regard, during the last years, different surveys at the observatory of Medicina were performed aiming at the computation of high precision eccentricities.

2.1 Surveying strategies of tie vectors and connections with the local geodetic network's robustness.

Considering the overall set of VLBI-GPS eccentricities of Medicina, two different groups of local ties can be elected according to the way they were surveyed and the geodetic network's geometry designed as a basis for their processing . The 2 local ties of 2002 and 2006 are completely *GPS-derived*, whereas the 3 local ties of 2001, 2002 and 2003 are computed through *terrestrial* observations.

Terrestrial local ties are typically performed measuring a local geodetic network in the co-located site, according to a scheme of redundant multiple intersections (angles and distances) along with spirit levelling measurements (for further details see Vittuari et al. 2005).

Table 1. Geometrical features of the data sets used: number of azimuthal and elevational surfaces exploited in the local tie computation.

Survey	Azimuthal Surfaces (Nr)	Elevational Surfaces (Nr)
TER 2001	8	3
TER 2002	8	4
TER 2003	10	11
ABS 2002	4	4
ABS 2006	8	14

On the contrary, the basic observables at the root of the GPS derived local ties' computation are represented by baselines bridging the ground fixed points to those traced by moving GPS receivers attached to the VLBI dish. The survey of the VLBI RP (Reference Point) is based on a rapid static approach experienced at Medicina in 2002, whose details are thoroughly discussed in Tomasi et al. (2003).

In both cases (terrestrial and GPS-based) through the observation of the positions of moving targets or GPS receivers fixed on the structure of the VLBI antenna, it is possible to recover circles which are geometrically connected to the VLBI IP.

As showed above, Table 1 details the number of surfaces recovered while the VLBI antenna rotates in azimuth and elevation, during the various surveys.

It's worth noticing that a correlation between the local geodetic network's robustness and the VLBI IP estimation exists : looking at Table 1, a comparison between the ABS 2006 and the ABS 2002 solutions immediately testifies a worst definition of the VLBI IP in the last case.

In spite of the differences in magnitude, an akin behaviour can be inferred from the comparison between TERR 2001 (worst definition) and TERR 2003 (better definition) formal errors in the three geocentric components. In fact a relationship exists between the number of surfaces involved (Table 1) in the VLBI IP geometrical modelling and its accuracy: the higher is the geometrical robustness of the network (number of surfaces exploited), the more accurate is the IP estimation.

2.2 Geometrical and Statistical Indicators

Five GPS-VLBI eccentricities have been computed throughout the period 2001-2006, whose results in terms of moduli are summarized in Table 2; as widely discussed in Sarti et al. (2004) and Dawson et al. (2006), the estimation of each of these local tie vectors relies on an indirect approach entailing a proper geometrical model. Final computation of each eccentricity, provided in a SINEX format, gathers the estimations of the Invariant Points (IP), the ensemble of the points belonging to the local network along with the entire VCV matrix.

Table 2. Moduli and formal errors of the eccentricity vector (values in m).

TER 2001	TER 2002	TER 2003	ABS 2002	ABS 2006
62.7646 (0.0007)	62.7673 (0.0004)	62.7654 (0.0003)	62.7691 (0.0020)	62.7673 (0.0003)

As testified in Table 2, the formal errors (1-σ level) for the local tie moduli can attain submillimetrical values. Analogously, Table 3 displays the formal errors of the VLBI IP coordinates for the same ties, thus bearing out the effectiveness of the indirect approach for the eccentricity computation.

Moreover it's worth noticing that, apart from the 2002 GPS local tie, the accuracies of that one of 2006 are strikingly comparable with those terrestrial; they're even better if compared to the terrestrial eccentricities of 2001. As it's been proved in Dawson et al. (2006) for simulated datasets, such results further confirm that a suitable geometrical modelling can achieve a good level in filtering the noise affecting much more GPS data than the terrestrial ones.

Table 3. Formal errors of the VLBI IP coordinates expressed into ITRF2000 for all the local ties (values in mm)

	TER 2001	TER 2002	TER 2003	ABS 2002	ABS 2006
σ_x	0.45	0.35	0.29	5.27	0.35
σ_y	0.25	0.21	0.15	1.13	0.21
σ_z	0.45	0.36	0.29	5.13	0.35

2.3 The relationship between GPS derived and terrestrial local ties

From an operational point of view, the datum definition for local ties aims at establishing a suitable frame according to which the points of the local networks and the IP may be consistently expressed.

Algebraically speaking, datum definition requires the resolution of the rank deficiency stemming from observations.

Terrestrial tridimensional networks measured with multiple intersections of angles and distances display 4 feasible degrees of freedom: one rotational defect about the Up-axis and three translations which define the frame's origin.

Such a rank-defect is reduced *via* a loosely minimal constrained solution, by fixing three coordinates with a standard deviation of 1 meter and a bearing with a standard deviation of 1.5 gon.

As a consequence of the terrestrial survey procedures, the eccentricity will be always framed into a local reference frame which is topocentric (LTF) and geoid-dependent (Up component normal to the local geoid).

Figure 1 displays the local network of Medicina with the point-to-point connections; in this case the local reference frame, which should be reproduced each time the network is re-measured, originates –for terrestrial eccentricities- in the concrete pillar P3 with orientation along the connection P1-P3.

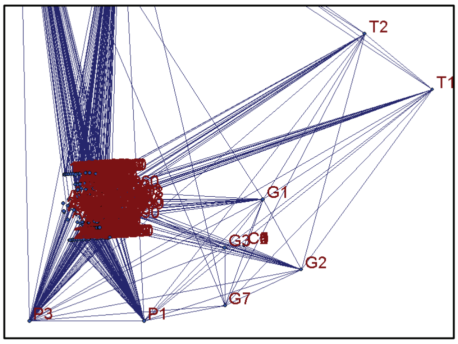

Fig. 1 Terrestrial local geodetic network at Medicina (2001)

If a final alignment into an ITRFyy realization is needed, each terrestrial local tie must be necessarily framed into such a global frame. The passage from LTF to a global frame is typically worked out by means of a rigid transformation (Rototranslation) whose 6 parameters will be stochastically estimated in a Least Square context (Sarti and Angermann 2005).

Such an estimation requires the knowledge of a set of tie points which link the LTF to the ITRFyy: coordinates coming from the GPS-based surveys can be used for this; hence this makes terrestrial eccentricities GPS-dependent as far as the alignment into an ITRFyy is concerned.

3 Is it possible to detect relative site-dependent displacements from multi-temporal realizations of tie vectors?

Despite the fact that local tie vectors are generally regarded as time invariant entities into a terrestrial reference frame combination, the possibility of performing surveys repeated time by time might point out an eccentricity's variation due to local instability related to geophysical phenomena within the co-located site. A correct evaluation of these relative displacements should be taken into account in order to establish the phenomenon's magnitude and to compare it with the linear temporal evolution impressed to the ITRS realizations.

The local tie surveying strategy aside, each eccentricity we can work out is the result of the processing of a local geodetic network, set up within the co-located site; hence also the adjusted coordinates of reference points, besides the IP involved in the eccentricity, are available whenever the network is re-measured.

For instance, in the case of Medicina, we're provided with multi-temporal network solutions of at least *five points*: three geodetic markers placed on concrete pillars (P1, P3 and G7) and two instrumental reference points. Thus, by exploiting network's solutions, potential relative displacements can be investigated.

3.1 Local datum reproducibility as basic assumption for detecting relative displacements.

Deformations we are going to investigate can be thoroughly defined as a function of the network markers' displacements.

In the case of Medicina, local tie vectors are determined by means of a "relative" geodetic network whose points are assumed to wholly belong to a deformable "area"; thus no point of the network can be theoretically regarded as stable and served as a reference point which the absolute displacements have to be referred to; this makes the geodetic network at a certain extent "relative".

Let $\underline{x}_i(t)$ be the time-dependent position vector of the *i-th* point belonging the local geodetic network expressed with respect to a three dimensional frame; then the displacement undergone by such a point from epoch t_1 to epoch t_2 is itself a 3d vector which can be expressed by differencing:

$$\delta_i = \underline{x}_i(t_2) - \underline{x}_i(t_1) \quad (1)$$

Equation (1) clearly shows that the modulus of the point displacement $\underline{\delta_i}$ in a local network is independent from the reference frame definition, providing *it is possible* to define a *datum consistent and repeatable epoch by epoch*.

In fact, if a datum is badly reproducible when passing from epoch t_1 to t_2, such a displacement δ can embed false movements which tend to mask the actual network deformations.

Badly reproducible datum may arise from inconsistencies related to the network repeated in time owing to different reasons; for instance

- changes in network's topology due to the *addition or removal of some points*, which can affect the application of minimal constrained solutions at different epochs;
- a modification in the observation scheme when passing from an epoch to the other (this is the case for our local tie-oriented surveys), which still produces a change in the network's shape.

In all these cases, a projection of the network coordinates onto a new datum free from any error and reference effect (Sillard and Boucher 2001) is mandatory and it represents a valuable tool in order to make the different realizations of our networks comparable. This can be achieved by exploiting the algebraic theory of projectors.

3.2 Datum definition and *Inner constraint solution* for a terrestrial based local tie and its underlying network

When dealing with least square solutions which involve a datum definition problem, the linear system characterizing the observations

$$\underset{m\cdot n}{\mathbf{A}} \cdot \underset{n\cdot 1}{\underline{x}} = \underset{m\cdot 1}{\mathbf{Y}} \quad (2)$$

is obviously *inconsistent* (the solution is not unique) and *not of full rank*; this is formally equivalent to say that:

$$\begin{cases} m > n \\ \rho(\mathbf{A}) < \min\{m, n\} = n \end{cases} \quad (3)$$

where m and n respectively denote the number of observations and the unknown parameters and ρ the rank of $\mathbf{A}$. Being $\mathbf{A}$ a rank deficient matrix, the normal system is not invertible in a classical sense. As it can be inferred by reading Sillard and Boucher (2001), if $\mathbf{W}$ denotes the metric tensor for the dot product onto $\mathbb{R}^m$, each possible least square solution of the not full-rank problem

$$\left(\underset{n\cdot m}{\mathbf{A}^t} \cdot \underset{m\cdot m}{\mathbf{W}} \cdot \underset{m\cdot n}{\mathbf{A}}\right) \cdot \underset{n\cdot 1}{\underline{x}} = \underset{n\cdot m}{\mathbf{A}^t} \cdot \underset{m\cdot m}{\mathbf{W}} \cdot \underset{m\cdot 1}{\mathbf{Y}} \quad (4)$$

lies onto the affine space

$$\Psi = (\underset{n\cdot m}{\mathbf{A}^t} \cdot \underset{m\cdot m}{\mathbf{W}} \cdot \underset{m\cdot n}{\mathbf{A}})^{-} (\underset{n\cdot m}{\mathbf{A}^t} \cdot \underset{m\cdot m}{\mathbf{W}} \cdot \underset{m\cdot 1}{\mathbf{Y}}) + Ker(\underset{m\cdot n}{\mathbf{A}}) \quad (5)$$

where $(\mathbf{H})^{-}$ indicates a generalized inverse for **H** and $Ker(\mathbf{A})$ the null space of **A**.

Whatever the least square solution is, it is unique its projection along the orthogonal direction to Ψ: such solution we're searching for is the inner constraint solution.

As such, the application of projection theory to the local tie-oriented geodetic network allows to define a new datum into which all the different realizations of eccentricities will be framed. From an analytical point of view, let $\underline{x}_1$ be a generic realization of a local tie vector issuing from the network observations at epoch t.

Such realization can be connected to another one at the same epoch by means of an equivalence which expresses a similarity transformation:

$$\underset{3n_p}{\underline{x}_1} = \underset{3n_p}{\underline{x}_0} + \underset{3n_p\cdot t}{\mathbf{D}} \cdot \underset{t\cdot 1}{\theta} \quad (6)$$

where the vector θ contains the t transformation parameters and **D** is a matrix based on the linearization of the similarity. Such a matrix accounts for the network datum defect: it's got as much columns as the possible degrees of freedom of the network and a number of lines equal to $3\cdot n_p$ (number of network's points).

Terrestrial geodetic networks underlying eccentricities show a datum defect of 4: 3 translations and 1 rotation about the Up-axis, whose direction is fully determined through the gravity field. In this case the matrix **D** has the following structure:

$$\underset{3n_p\cdot 4}{\mathbf{D}} = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & -y_i^0 \\ 0 & 1 & 0 & x_i^0 \\ 0 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \quad i = 1,..,n_P \quad (7)$$

where the first three columns pertain to the 3 translations and the last one to the unique rotation; the couple (x_i^0, y_i^0) represents the *a priori* coordinates for the *i-th* point.

The problem of projecting the frame realization $\underline{x}_1$ in order to obtain an inner constraint solution $\underline{x}_0$ was widely discussed in geodetic literature; hereafter we propose the solution presented by Sillard and Boucher (2001) relying on the concept of projection onto orthogonal complementary vector spaces. By solving equation (6) in a least square sense with respect to the parameters contained in θ, one can get

$$\begin{cases} \underset{3n_p\cdot 1}{\underline{x}_0} = (\underset{3n_p\cdot 3n_p}{\mathbf{I}} - \underset{3n_p\cdot t}{\mathbf{D}} (\underset{t\cdot 3n_p}{\mathbf{D}^t} \underset{3n_p\cdot 3n_p}{\mathbf{W}} \underset{3n_p\cdot t}{\mathbf{D}})^{-1} \underset{t\cdot 3n_p}{\mathbf{D}^t} \underset{3n_p\cdot 3n_p}{\mathbf{W}}) \cdot \underset{3n_p\cdot 1}{\underline{x}_1} \\ \mathbf{D}\theta = (\mathbf{D}(\mathbf{D}^t\mathbf{W}\mathbf{D})^{-1}\mathbf{D}^t\mathbf{W}) \cdot \underline{x}_1 \end{cases} \quad (8)$$

where $\mathbf{Q} = \mathbf{D}(\mathbf{D}^t\mathbf{W}\mathbf{D})^{-1}\mathbf{D}^t\mathbf{W}$ is, from an algebraic point of view, a projection matrix.

4 Displacement computation and discussion of the results

According to equations (8) projections of each terrestrial network solution onto the normal direction to Ψ have been performed by means of the projector Q, in order to select the inner constraint solution.

At this stage, the approach has been applied only to the terrestrial local ties in such a way that two displacement fields can be evaluated: the first one relevant to the epochs 2001 and 2002, the second one to the epochs 2001 and 2003.

It's worth stressing that, if a network solution was computed by tightly constraining some points to a given value, such constraints have to be removed both from the adjusted coordinates $\underline{x}_1$ and the VCV matrix before the estimation of the inner solution $\underline{x}_0$ (Panafadina et al. 2006)

Conversely, if a loosely constrained solution was performed as in this case, the removal can be limited just to the *a posteriori* VCV network matrix.

The following Table 4 summarizes the results for the two groups of displacements, computed by analysing the 2001, 2002, 2003 terrestrial geodetic networks at Medicina. The first group is relevant to the displacements during the period 2002-2001 whereas the second one to those of 2003-2001; an incremental effect is evident when passing from the first to the second group. Each of these displacements is obviously consistent with the modulus of the

VLBI-GPS eccentricity expressed in the initial realization $\underline{x}_I$ of the local frame: this happens because the inner constraint solution leaves unaltered the geometrical invariants of the network.

Table 4. Displacements for the terrestrial network points related to the *time lag 2002-2001 and 2003-2001*. Shown are the values in mm of the three vector components expressed wrt to the *inner constrained solution*; ***mod*** lines contain the *moduli* of displacements (mm)

		P1	**P3**	**G7**	**GPS ARP**	**VLBI IP**
2002-2001	X	-1.87	-1.87	3.17	-1.51	2.08
	Y	1.57	-2.85	0.01	-0.81	2.09
	Z	0.57	-2.38	-0.53	2.48	-0.13
	mod	**2.51**	**4.16**	**3.22**	**3.01**	**2.95**
2003-2001	X	-6.32	-6.32	6.45	-3.51	9.69
	Y	4.73	-0.32	0.68	-4.32	-0.77
	Z	0.89	-3.14	0.84	1.83	-0.42
	mod	**7.94**	**7.06**	**6.54**	**5.86**	**9.73**

All the displacements are statistically meaningful: their magnitude is bigger than their formal errors at 1σ value (according to the *VCV* matrix expressed with respect to the inner constraint solution).

Moreover the results deduced by Table 4 suggest relative effects in the displacements among the network's points; in facts P3 and P1, which concur to define the origin and the orientation in the original frame, undergo differential displacements into the new one with respect to all the three components. The point G7 is, on the contrary, fairly stable in the Z and Y components, but shows a significant migration along the X component of about 6 mm. In addition it has to be stressed that the VLBI IP attains the maximum value of local displacement.

5 Conclusions

The inner constraint approach proves to be an effective and highly flexible tool also for tackling the issue of local displacement evaluation within a co-located site.

On the whole such an approach, applied to the terrestrial derived local ties, is capable to highlight local displacements and differential effects within the network which can integrate the description of the co-located site yielded at a global scale by the ITRS realizations.

Concerning local tie vectors, the application of projection theory bears out the idea that an eccentricity cannot be in general regarded as a time invariant entity, since in the case of Medicina its reference points may undergo relative displacements up to 10 mm.

Finally, in order to better describe the magnitude of the phenomenon and to wholly detect eventual deformational patterns, this approach suggests that it is worth widening the investigations by including all the terrestrial surveys executed so far together with the GPS-based solutions.

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