TITLE:	**Planck LFI – Characterization of the Onboard Processing Parameters**
DOC. TYPE:	**TECHNICAL NOTE**
PROJECT REF.:	**PL-LFI-OAT-TN–030**
PAGE:	**I of IV, 00**
ISSUE/REV.:	**0.0**
DATE:	**12 Mar 2004**

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CHANGE RECORD

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1 SCOPE

A generalized discussion of processing in view of the new baseline is given.

This document is important both for L1 DPC as for the determination of efficiency of onboard compression schemes reported, as an example, in [RD-3].

1.1 LIMITS OF APPLICABILITY

This document represents preparatory work for the documents that will be issued by LABEN toward IAC-E for the validation of the onboard compressor for Planck/LFI and for the setting up of onboard processing.

2 APPLICABLE/REFERENCE DOCUMENTS

2.1 APPLICABLE DOCUMENTS

[AD-1] Reconfiguration for LFI on-board data processing and scientific telemetry
M. Miccolis, A. Mennella, M. Bersanelli, M. Maris
PL-LFI-PST-TN-037, Issue 1.0, March 2003

[AD-2] Planck-LFI Communications, ICD,
M. Miccolis
PL-LFI-PST-ID –013, Version 3.0, January 2004

2.2 REFERENCE DOCUMENTS

[RD-1] Compression Paper

[RD-2] Quantization Paper

[RD-3] Planck LFI – Characterization of the Compression Rate for the New Baseline for the Scientific Data Streams Coding
M. Maris
PL-LFI-OAT-TN-029, Ver. 0.0, Mar 2004

2.3 ACRONYMS LIST

FP	Floating Point

3 INTRODUCTION

The new baseline for download of processing in Planck/LFI consist of sending couples of independent linear combinations of Sky and Load samples taken at the same times from each radiometric chain [AD-1, AD2].

The linear combinations allow to:

i. equalize the statistics of samples sent to the compressor,
ii. reduce the impact of 1/f noise and other non idealities as non gaussianities,

all of this in order to minimize the impact of the new baseline over a compression system already designed and tested for the old baseline, for which differences of Sky and Load samples would have had to be downloaded to ground [RD-1, RD-2].

In the present version of the baseline [AD-1] couples of signals both sampled at time *t* and introduced in the data streams are formed according to the formula:

$$\begin{pmatrix} T_{1,t} \\ T_{2,t} \end{pmatrix} = \begin{pmatrix} 1 & -r_1 \\ 1 & -r_2 \end{pmatrix} \begin{pmatrix} T_{\mathrm{sky},t} \\ T_{\mathrm{load},t} \end{pmatrix}, \qquad [3.1]$$

where r1 and r2 are positive constants, Tsky and Tload are sampled from the same radiometric chain.

Three are three clasees of problems to be solved in order to accept definetly this strategy:

Optimality

One may wonder wether [3.1] may represents the best choice for such transformations.

Optimizzation

Parameters r1 and r2 have to be choosen in order to optimize the compression rate and to minimize the truncation error when, at ground, the reverse of [3.1] will be applied to recover Tsky and Tload.

Determination of T1, T2 statistics

The determination of software quantization parameters as of the maximum compression rate requires to infer the statistical properties of T1 and T2 from the statistical properties of Tsky and Tload and the transformation matrix.

In this contribution we will face on with these problems starting with a generalized formalism which will include [3.1] as a sub case.

4 GENERAL FORMALISM

In this section the general formalism is introduced and general results are described.

4.1 MIXING, DEMIXING AND NOTATION

Let be to indicate with (T_α); $\alpha = \mathrm{s}, \mathrm{l}$; the couple os Sky and Load samples to be coded with *s* standing for Sky and *l* for Load. While with (T_i); $i = 1,2$ the samples of independent linear combinations. Moreover, we will assume that *any greek index* (as: α, β, γ, …) will indicate a couple of Sky, Load signals, while *any latin index* will indicate a couple of linear combination signals [1]. With this convention any transform of the kind [3.1] will be represented by:

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} M_{1,\mathrm{sky}} & M_{1,\mathrm{load}} \\ M_{2,\mathrm{sky}} & M_{2,\mathrm{load}} \end{pmatrix} \begin{pmatrix} T_{\mathrm{sky}} \\ T_{\mathrm{load}} \end{pmatrix}, \qquad [4.1.1a]$$

or shortly by

$$T_i = M_{i,\alpha} T_\alpha \,, \qquad [4.1.1b]$$

where as usual repeated indexes on the same side represents summations [2].

At the same manner the reverse of [4.1.1] will be represented by

$$T_\alpha = M_{\alpha,i} T_i \,, \qquad [4.1.2]$$

i.e. the interchange of greek and latin indexes will denote the reversing of the operation so that

$$\begin{aligned} M_{\alpha,i} M_{i,\beta} &= \delta_{\alpha,\beta} \,, \\ M_{i,\alpha} M_{\alpha,j} &= \delta_{i,j,} \end{aligned} \qquad [4.1.3]$$

with $\delta_{\alpha,\beta}$, $\delta_{i,j}$ the usual Kronecher symbols.

The matrix $\mathrm{M} = (M_{i,\alpha})$ in [4.1.1] and its inverse $\mathrm{M}^{-1} = (M_{\alpha,i})$ in [4.1.2] are named respectively *mixing matrix* and *demixing matrix*.

The structure of the (de)mixing matrix determines the features of the coding strategy. According to constraints on the coding strategy a particular family may be more effective than the others.

[1] So T_α, T_β, T_γ, … will represents independent measures of Sky and Load, while T_i, T_j, T_k, … independent samples of linear combinations.

[2] So as an example shall read as $T_i = \sum_{\alpha=\mathrm{s}}^{\mathrm{l}} M_{i,\alpha} T_\alpha = M_{i,\mathrm{s}} T_\mathrm{s} + M_{i,\mathrm{l}} T_\mathrm{l}$.

4.2 FIRST CONSTRAINT: REVERSIBILITY

The first constraint on $\mathrm{M} = \left(M_{i,\alpha}\right)$ is that its inverse exists this implies

$$|\mathbf{M}| = M_{1,\mathrm{S}} M_{2,\mathrm{L}} - M_{1,\mathrm{L}} M_{2,\mathrm{S}} \neq 0 \quad , \qquad [4.2.1a]$$

which is equivalent to

$$\frac{M_{1,\mathrm{S}}}{M_{1,\mathrm{L}}} \neq \frac{M_{2,\mathrm{S}}}{M_{2,\mathrm{L}}} . \qquad [4.2.1b]$$

4.3 SECOND CONSTRAINT: STATISTICS

We define the expectation and the covariance matrix of T_i in terms of corresponding quantities for T_α and M.

$$\begin{aligned} &\mathrm{E}[T_i] = M_{i,\alpha} \mathrm{E}[T_\alpha], \\ &\mathbf{C}_{i,j} = \mathrm{cov}[T_i, T_j] = M_{i,\alpha} M_{j,\beta} \,\mathrm{cov}[T_\alpha, T_\beta] = M_{i,\alpha} M_{j,\beta} \mathbf{C}_{\alpha,\beta} . \end{aligned} \qquad [4.3.1]$$

Then it is straightforward to obtain for the T_i variances $\sigma_i^2 = \mathrm{var}[T_i] = \mathbf{C}_{i,i}$

$$\sigma_i^2 = M_{i,\alpha} M_{i,\beta} \mathbf{C}_{\alpha,\beta} = .M_{i,s}^2 \sigma_s^2 + M_{i,l}^2 \sigma_l^2 + 2 M_{i,s} M_{i,l} \sigma_{s,l} , \qquad [4.3.2]$$

for the covariance $\forall\, i \neq j$,

$$\sigma_{i,j} = \mathrm{cov}[T_i, T_j] = \mathbf{C}_{i,j} = M_{i,\alpha} M_{j,\beta} \mathbf{C}_{\alpha,\beta} = M_{i,s} M_{j,s} \sigma_s^2 + M_{i,l} M_{j,l} \sigma_l^2 + 2 M_{i,s} M_{j,l} \sigma_{s,l} , \qquad [4.3.3]$$

and for the cross-correlation

[4.3.4]

$$\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j} = \frac{M_{i,s} M_{j,s} \dfrac{\sigma_s}{\sigma_l} + M_{i,l} M_{j,l} \dfrac{\sigma_l}{\sigma_s} + 2 M_{i,s} M_{j,l} \rho_{s,l}}{\sqrt{M_{1,s} M_{1,s} \dfrac{\sigma_s}{\sigma_l} + M_{1,l} M_{1,l} \dfrac{\sigma_l}{\sigma_s} + 2 M_{1,s} M_{1,l} \rho_{s,l}} \sqrt{M_{2,s} M_{2,s} \dfrac{\sigma_s}{\sigma_l} + M_{2,l} M_{2,l} \dfrac{\sigma_l}{\sigma_s} + 2 M_{2,s} M_{2,l} \rho_{s,l}}} ,$$

where σ_s, σ_l, $\sigma_{s,l}$ and $\rho_{s,l}$ denotes variances, co-variances and correlation coefficients for sky and load.

It is important to note that, in general, output of mixing results in correlated signals if inputs are uncorrelated. While if input signals are correlated, output may be uncorrelated, depending on the values of the components of the mixing parameters.

Statistical constrains are defined asking for identical moments of the distributions of T_i up to a given order. If T_i are normal distributed it is sufficient to assure equalization of mean and variance of T_i i.e.

$$\mathrm{E}[T_1] = \mathrm{E}[T_2]\,, \qquad [4.3.5a]$$

$$\mathrm{var}[T_1] = \mathrm{var}[T_2]\,, \qquad [4.3.5b]$$

which translates into two independent conditions on the components of $\mathrm{M} = (M_{i,\alpha})$.

To recover these conditions let us to define

$$\Delta = \mathrm{E}[T_2] - \mathrm{E}[T_1]\,, \qquad [4.3.6a]$$

$$\Delta_{\mathrm{var}} = \mathrm{var}[T_2] - \mathrm{var}[T_1]\,, \qquad [4.3.6b]$$

then from [4.3.1]

$$\Delta = [M_{2,\alpha} - M_{1,\alpha}]\mathrm{E}[T_\alpha]\,, \qquad [4.3.9]$$

and the constraint on the first moment $\Delta = 0$ results in

$$\frac{[M_{2,s} - M_{1,s}]}{[M_{2,l} - M_{1,l}]} = -\frac{\mathrm{E}[T_l]}{\mathrm{E}[T_s]}, \qquad [4.3.10]$$

while the constraint on the second moment $\Delta_{\mathrm{var}} = 0$ results in the condition

$$\left[M_{2,\alpha}M_{2,\beta} - M_{1,\alpha}M_{1,\beta}\right]\mathbf{C}_{\alpha,\beta} = 0\,. \qquad [4.3.11]$$

This reduces to a very simple constrain if sky and load are not correlated

$$\frac{.(M_{2,s}^2 - M_{1,s}^2)}{(M_{2,l}^2 - M_{1,l}^2)} = .-\frac{\sigma_l^2}{\sigma_s^2}\,. \qquad [4.3.12]$$

A further simplification may be obtained taking in account that

$$\sigma_\alpha = \frac{\mathrm{E}[T_\alpha]}{\sqrt{\beta}}\,, \qquad [4.3.13]$$

with β a constant factor. Combining this with [4.3.10] the condition [4.3.12] becomes

$$\frac{(M_{2,s}+M_{1,s})}{(M_{2,l}+M_{1,l})} = \cdot\frac{\mathrm{E}[T_l]}{\mathrm{E}[T_s]}. \qquad [4.3.14]$$

These conditions allows to determine two components of the mixing matrix when the statistics of the input signal is known and given the other two components.

4.3.1 THEOREM OF EXCLUSION

Equations [4.3.10] and [4.3.14] allows to demonstrate the following theorem on the form of the mixing matrix:

Mixing matrices with

$$M_{2,s} = M_{1,s} \text{ and / or } M_{2,l} = M_{1,l},$$

violates the statistical constrain and shall not be used.

In case of uncorrelated input mixing matrices with

$$M_{2,s} = -M_{1,s} \text{ and / or } M_{2,l} = -M_{1,l},$$

violates the statistical constrain and shall not be used.

Note that while first statement does not depend on the statistics of the input signal, the second statement depends on it, so that in principle it would be possible to violate [4.3.14], but this may result in a not robust system.

4.4 PACKET SIZE AND VARIABILITY OF THE MIXING MATRIX

It shall be noted that in case of a true signal, the size of a packet introduces a natural maximum time scale over which to evaluate correlations between data. Sources of non – stationarities or randomness inducing correlations in sky and load on time scales longer than this scale may be neglected, provided the components of the mixing matrix may be adapted.

4.5 QUANTIZATION

The effect of software quantization shall be evaluated both for the stationary case as for the unstable case.

Quantization acts since it introduces a truncation after the application of the mixing matrix, so that demixing will be no more exact.

Following [RD-1] and [RD-2] we define a quantized and reconstructed mixed temperature as

$$T_i^{RQ} = q\breve{Q}\left(\frac{T_i}{q}\right), \qquad [4.5.1]$$

with $\breve{Q}(.)$ the quantization operator, here assumed to be the round(.) operator, and q the quantization step.

The quantization error is defined as

$$\delta_\alpha^q = q\breve{Q}\left(\frac{T_i}{q}\right) - T_i \,. \qquad [4.5.2]$$

After demixing reconstructed sky and load will be

$$T_\alpha^{RQ} = M_{\alpha,i} q\breve{Q}\left(\frac{T_i}{q}\right) = M_{\alpha,i} q\breve{Q}\left(\frac{M_{i,\alpha} T_\alpha}{q}\right), \qquad [4.5.3]$$

and the quantization error is

$$\delta_\alpha^q = M_{\alpha,i} q\breve{Q}\left(\frac{T_i}{q}\right) - T_\alpha = M_{\alpha,i}\delta_i^q \,, \qquad [4.5.4]$$

it is immediately evident that in case the mixing is nearly singular the quantization error will be large.

Given that the quantization operator has no bias:

$$E[\delta_\alpha^q] = 0 \,,$$

$$\mathrm{cov}[\delta_\alpha^q, \delta_b^q] = M_{\alpha,i} M_{\beta,j}\, \mathrm{cov}[\delta_i^q, \delta_j^q] \,,$$

depending on the level of covariance between the unquantized mixed quantities, the quantization may both increase or decrease the covariance.

The covariance of the quantization error is

4.6 NOT STATIONARITIES AND PECULIAR CASES

In the previous sections we assumed that the signal is completely random and stationary. To study the goodness of a particular mixing scheme, it is important to consider the case of not-stationarities or the occurrence of peculiar cases as spikes or drifts in the signal. The following cases are relevant:

spikes in sky or load, linear drifts, and sinusoidal fluctuations as the cosmological dipole.

4.7 ZERO-POINT LINEAR DRIFTS

A zero-point linear drift in sky and load has the form

$$T_\alpha^D = T_{0,\alpha} + D_\alpha t \,, \qquad [4.7.1]$$

where $T_{0,\alpha}$ is the zero-point of the drift and D_α is the drift rate. It is straightforward to recover the equation for the drift in T_i

$$T_i^D = M_{i,\alpha} T_\alpha^D = M_{i,\alpha} T_{0,\alpha} + M_{i,\alpha} D_\alpha t \,, \qquad [4.7.2]$$

from which the drift takes the form

$$\begin{aligned} T_i^D &= T_{0,i} + D_i t, \\ T_{0,i} &= M_{i,\alpha} T_{0,\alpha}, \\ D_i &= M_{i,\alpha} D_\alpha, \end{aligned} \qquad [4.7.3]$$

The third of these equations allows to define the condition on the mixing matrix which allows to cancel out the drift:

$$M_{i,\alpha} D_\alpha = 0 \,, \qquad [4.7.4]$$

which for a drift with identical drift rates for sky and load reduces to

$$M_{1,s} + M_{1,l} = 0 \,, \qquad [4.7.5a]$$

$$M_{2,s} + M_{2,l} = 0 \,, \qquad [4.7.5b]$$

A drift will have affect both averages and variances calculated over a packet of time length Δt_{pck} and starting point t_{pck} . In particular it is important to estimate the effect on the differences of averages and variances.

The moments calculated over a packet for the linear drift are

$$\begin{aligned}
&\mathrm{E}_{\mathrm{pck}}[T_i^D] = T_{0,i} + D_i\left(t_{\mathrm{pck}} + \frac{\Delta t_{\mathrm{pck}}}{2}\right),\\
&\mathbf{C}_{i,j}^{\mathrm{pck}} = D_i D_j \frac{\Delta t_{\mathrm{pck}}^2}{12},\\
&\mathrm{var}_{\mathrm{pck}}[T_i^D] = D_i^2 \frac{\Delta t_{\mathrm{pck}}^2}{12},\\
&\rho_{i,j}^{\mathrm{pck}} = 1.
\end{aligned} \qquad [4.7.6]$$

If so the effect on the difference of the averages and variances is

$$\begin{aligned}
&\Delta_{\mathrm{pck}}[T_i^D] = (D_2 - D_1)\left(t_{\mathrm{pck}} + \frac{\Delta t_{\mathrm{pck}}}{2}\right),\\
&\mathrm{var}_{\mathrm{pck}}[T_2^D] - \mathrm{var}_{\mathrm{pck}}[T_1^D] = (D_2^2 - D_1^2)\frac{\Delta t_{\mathrm{pck}}^2}{12}.
\end{aligned} \qquad [4.7.7]$$

The condition to avoid drifts in the differences of averages and variances is

$$(D_2 - D_1) = 0, \qquad [4.7.8a]$$

$$(D_2 - D_1)(D_2 + D_1) = 0. \qquad [4.7.8b]$$

Note that if the first is satisfied both of them are satisfied and both variances as differences in zero points do not drift in time. The opposite is not true, the lack of changes in the difference of variances does not guarantee that there are not drifts in the differences of zero points. From the definition of drifts

$$(M_{2,s} - M_{1,s})D_s + (M_{2,l} - M_{1,l})D_l = 0, \qquad [4.7.9a]$$

$$(M_{2,s} + M_{1,s})D_s + (M_{2,l} + M_{1,l})D_l = 0, \qquad [4.7.9b]$$

note that when [4.7.11a] is satisfied it is possible to equalize both mean and variance, while when only [4.7.12b] is respected the equalization of the variances only are allowed. This is important since these two conditions are, obviously, mutually exclusive in many cases.

We may now consider some special sub cases:

1. even drift, $D_s = D_l$,
2. odd drift, $D_s = -D_l$,
3. completely asymmetrical drift, $D_s = 0 \vee D_l = 0$,

In the case of an even drift [4.7.10a] and [4.7.10b] will be always satisfied provided

$$\left(M_{2,s}-M_{1,s}\right)=-\left(M_{2,l}-M_{1,l}\right), \qquad [4.7.10a]$$

note that this is in conflict with [4.3.11] whatever the matrix is unless $\mathrm{E}[T_l]=\mathrm{E}[T_s]$. Which is not the case in Planck/LFI. Condition [4.7.11b] is satisfied provided

$$\left(M_{2,s}+M_{1,s}\right)=-\left(M_{2,l}+M_{1,l}\right), \qquad [4.7.10b]$$

this may be satisfied without to infringe [4.3.11] only if noises are correlated and only for special cases. If not [4.3.11] will collapse into [4.3.12] which will be respected in case [4.7.10b] is true only if $\mathrm{E}[T_l]=\mathrm{E}[T_s]$.

In the case of an odd drift [4.7.12a] and [4.7.12b] reverse they signs.

In the case of a completely asymmetrical drift (quite unlikely in Planck/LFI) [4.7.10] becomes

$$\left(M_{2,s}-M_{1,s}\right)=0, \text{if } D_l=0, \qquad [4.7.11a]$$

$$\left(M_{2,l}-M_{1,l}\right)=0, \text{if } D_s=0, \qquad [4.7.11b]$$

both of them would violate [4.3.10] and likely [4.3.11].

It is evident how in case of drift it is not possible to remove it with a mixing of the kind $|M_{1,s}|=|M_{1,s}|$ or of the kind $|M_{2,s}|=|M_{2,s}|$.

More over it is not possible to be able to have an equalized average and variance if a significant even or odd or completely asymmetrical drift is present.

In addition in has to be considered that in all cases a drift will be present it will introduce a change in the zero point of the averaged mixed signals which will change in time, this even if the averages coincides at beginning of the packet.

4.8 SINUSOIDAL DRIFT

A zero-point sinusoidal drift in sky and load has the form

$$T_\alpha^S=\left[A_\alpha \sin\left(\omega_\alpha t+\varphi_\alpha\right)\right], \qquad [4.8.1]$$

where A_α, ω_α, φ_α are the amplitudes, frequencies and phases. Note that here summation for terms in the squared bracket over the index α is not carried on. It is straightforward to recover the equation for the drift in T_i

$$T_i^S = M_{i,\alpha}\left[A_\alpha \sin(\omega_\alpha t + \varphi_\alpha)\right]. \quad [4.8.2]$$

However it is uneasy to work out general solutions for this case, instead since the most important sinusoidal drift is due to the cosmological dipole affecting just the sky signal, the most important case is the total asymmetrical case.

4.8.1 TOTAL ASYMMETRICAL CASE

In case the sinusoidal signal is just on the sky channel [4.8.1] and [4.8.2] reduces to

$$T_\alpha^S = \begin{bmatrix} A_s \sin(\omega_s t + \varphi_s) \\ 0 \end{bmatrix}, \quad [4.8.1.1]$$

$$T_i^S = M_{i,s} A_s \sin(\omega_s t + \varphi_s), \quad [4.8.1.2]$$

the expectations and the variances over an interval will depend on the phase and period of the signal, as on the time in which the interval is evaluated and on its length. Formulae may be complex, but in any case:

$$T_\alpha^S = \begin{bmatrix} A_s \sin(\omega_s t + \varphi_s) \\ 0 \end{bmatrix}, \quad [4.8.1.1]$$

$$T_i^S = M_{i,s} A_s \sin(\omega_s t + \varphi_s), \quad [4.8.1.2]$$

resulting in a very simple scaling

$$\begin{aligned} &\mathrm{E}_{\mathrm{pck}}[T_1^S]/M_{1,s} = \mathrm{E}_{\mathrm{pck}}[T_2^S]/M_{2,s} = \mathrm{E}_{\mathrm{pck}}[T_s^S], \\ &\mathrm{var}_{\mathrm{pck}}[T_1^S]/M_{1,s}^2 = \mathrm{var}_{\mathrm{pck}}[T_2^S]/M_{2,s}^2 = \mathrm{var}_{\mathrm{pck}}[T_s^S] = 0. \end{aligned} \quad [4.8.1.3]$$

So that

$$\begin{aligned} &\Delta_{\mathrm{pck}}[T_i^S] = \left[M_{2,s} - M_{1,s}\right]\mathrm{E}_{\mathrm{pck}}[T_s^S], \\ &\mathrm{var}_{\mathrm{pck}}[T_2^S] - \mathrm{var}_{\mathrm{pck}}[T_1^S] = .\left[M_{2,s}^2 - M_{1,s}^2\right]\mathrm{var}_{\mathrm{pck}}[T_s^S]. \end{aligned} \quad [4.8.1.4]$$

In conclusion this kind of drift will not affect average differences and variances if $M_{2,s} = M_{1,s}$.

4.9 SPIKES

Spikes in sky or load radiometers may be due to bright sources or fast transients.

5 FAMILIES OF MIXING MATRICES

From Section 4 it is evident how the choice of a mixing matrix has to be a compromise between the need to equalize various features of the signal. However in case a signal is stationary just conditions [4.2.1], [4.3.10] and [4.3.12] are important. They introduces just two constraints, leaving two degrees of freedom. According to the way these degree of freedom are arranged within the matrix we may define many families of mixing matrices. Of them, one is the current baseline and will be studied first.

5.1 $M_{1,s} = M_{2,s} = 1$

The current baseline in Planck/LFI for the mixing matrix is [AD-1]

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 1 & -r_1 \\ 1 & -r_2 \end{pmatrix} \begin{pmatrix} T_{\rm sky} \\ T_{\rm load} \end{pmatrix}, \qquad [5.1.1]$$

with $r_1 > 0,\ r_2 > 0$, equivalent to

$$\begin{aligned} M_{1,s} &= M_{2,s} = 1, \\ M_{1,l} &= -r_1, \\ M_{2,l} &= -r_2, \end{aligned} \qquad [5.1.2]$$

the reversibility condition [4.2.1] asks for

$$r_1 \neq r_2 . \qquad [5.1.3a]$$

The condition [4.3.10] can not be satisfied at all, since it is equivalent to $0 = [r_2 - r_1]\,\mathrm{E}[T_l]/\mathrm{E}[T_s]$ asking for $r_2 = r_1$ at odd with [5.1.3a].

The condition [4.3.11] (taking in account of [5.1.3a]) becomes

$$(r_2 + r_1)\sigma_s^2 - 2\sigma_{s,l} = 0 , \qquad [5.1.3b]$$

note that if the input data are not correlated it is not possible to equalize the variances unless $r_1 < 0 < r_2$ or $r_2 < 0 < r_1$.

This scheme the dipole will cause changes in the mean values, but not in their difference or variance.

5.2 $M_{1,s} = M_{2,l} = 1$

An alterative scheme is

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 1 & -r_1 \\ -r_2 & 1 \end{pmatrix} \begin{pmatrix} T_{\text{sky}} \\ T_{\text{load}} \end{pmatrix}, \quad [5.2.1]$$

with $r_1 > 0,\ r_2 > 0,$ equivalent to

$$\begin{aligned} M_{1,s} &= M_{2,l} = 1, \\ M_{1,l} &= -r_1, \\ M_{2,s} &= -r_2, \end{aligned} \quad [5.2.2]$$

the reversibility condition [4.2.1] asks for

$$1 \neq r_1 r_2 . \quad [5.2.3a]$$

The complementar family $M_{2,s} = M_{1,l} = 1$ of course has similar properties and is not studied here.

The condition [4.3.10] can now be satisfied provided that

$$\frac{1+r_2}{1+r_1} = \frac{\mathrm{E}[T_l]}{\mathrm{E}[T_s]}, \quad [5.2.3b]$$

which does not conflict with [5.2.3a].

The condition [4.3.11] becomes

$$\left(1-r_2^2\right)\sigma_s^2 - \left(1-r_1^2\right)\sigma_l^2 + 2\left(r_2 - r_1\right)\sigma_{s,l} = 0, \quad [5.2.3c]$$

which potentially does not conflict with [5.2.3a] and [5.2.3b]. In particular if sky and load are not correlated

$$\frac{\left(1-r_2\right)}{\left(1-r_1\right)} = \frac{\mathrm{E}[T_l]}{\mathrm{E}[T_s]}, \quad [5.2.3d]$$

which may be satisfied if and only if $r_2 \neq 1$ and $r_1 \neq 1$.

In the case of an even-drift [4.7.10a] and [4.7.10b] becomes $r_2 = r_1$, so even in this case it is not possible to remove the drift equalizing the variances. The same is true for an odd drift.

In this scheme the dipole will cause changes in the mean values as in their difference and variances. In particular the ratio between the two variances will be r_2^2 which will not depend on any feature of the dipole and sampling method.