Publication Year	2015
Acceptance in OA	2020-03-23T16:47:05Z
Title	Linear spectropolarimetry: a new diagnostic tool for the classification and characterization of asteroids
Authors	Bagnulo, S., CELLINO, Alberto, Sterzik, M. F.
Publisher's version (DOI)	10.1093/mnrasl/slu154
Handle	http://hdl.handle.net/20.500.12386/23485
Journal	MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY
Volume	446

Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

MNRAS **446,** L11–L15 (2015) doi:10.1093/mnrasl/slu154

Linear spectropolarimetry: a new diagnostic tool for the classification and characterization of asteroids

S. Bagnulo,[1]★ A. Cellino[2] and M. F. Sterzik[3]

[1]*Armagh Observatory, College Hill, Armagh BT61 9DG, UK*
[2]*INAF–Osservatorio Astrofisico di Torino, I-10025 Pino Torinese, Italy*
[3]*European Southern Observatory, Karl-Schwarzschild-Strasse 2, D-85748 Garching, Germany*

Accepted 2014 September 14. Received 2014 September 8; in original form 2014 July 24

ABSTRACT

We explore the use of spectropolarimetry as a remote sensing tool for asteroids in addition to traditional reflectance measurements. In particular, we are interested in possible relationships between the wavelength-dependent variation of linear polarization and the properties of the surfaces, including albedo and composition. We have obtained optical spectropolarimetric measurements of a dozen asteroids of different albedo and taxonomic classes and of two small regions at the limb of the Moon. We found that objects with marginally different relative reflectance spectra (in the optical) may have totally different polarization spectra. This suggests that spectropolarimetry may be used to refine the classification of asteroids. We also found that in some cases the Umov law may be violated, that is, in contrast to what is expected from basic physical considerations, the fraction of linear polarization and the reflectance may be positively correlated. In agreement with a few previous studies based on multicolour broad-band polarimetry, we found that the variation of linear polarization with wavelength and with phase-angle is correlated with the albedo and taxonomic class of the objects. Finally, we have serendipitously discovered that spinel-rich asteroid (599) Luisa, located very close to the Watsonia family, is a member of the rare class of Barbarian asteroids. We suggest that future modelling attempts of the surface structure of asteroids should be aimed at explaining both reflectance and polarization spectra.

Key words: polarization – minor planets, asteroids: general – Moon.

Downloaded from https://academic.oup.com/mnrasl/article-abstract/446/1/L11/1051874 by guest on 17 March 2020

1 INTRODUCTION

Light scattered by surfaces is polarized. This may be intuitively understood by thinking that an electron sitting in a planar surface and hit by an electromagnetic wave is more free to oscillate in the direction parallel to the surface itself rather than perpendicular to it. Accordingly, the radiation re-emitted by the electron is partially linearly polarized in the direction parallel to the surface and perpendicular to the scattering plane (i.e. the plane containing the incident and the scattered light beams). Since the radiation produced by the oscillations of an electron moving up and down through the surface is more efficiently damped by a darker surface than by a brighter one, one can expect that the light reflected by a darker surface is more polarized than the light reflected by a brighter surface. The state of the polarization of the scattered radiation depends on the structure and composition of the reflecting surface and on the scattering angle, and its measurement may reveal information about the physical properties of the reflecting surface.

Broad-band linear polarization (BBLP) measurements have long been used as a remote sensing tool for the characterization of the objects of our Solar system. BBLP measurements in the standard optical filters are usually plotted as a function of the phase-angle (the angle between the Sun and the observer as seen from the target object) and the morphology of the resulting *phase-polarization curves* may be used for the purposes of albedo determinations (see Cellino et al. 2012, and references therein), and for asteroid classification (Penttilä et al. 2005). Since main-belt asteroids orbit at a significantly longer distance from the Sun than Earth, the phase-angles at which they may be observed are restricted to a small interval, typically ∼0°–30°. In the case of near-Earth objects, the maximum attainable phase-angle can be higher, well above 40°. Perhaps the most surprising feature of asteroid polarimetric properties is that at small phase-angles the plane of linear polarization is parallel to the scattering plane, in contrast to the simple scattering mechanism sketched out above. This phenomenon, which is

---

★ E-mail: sba@arm.ac.uk

© 2014 The Authors
Published by Oxford University Press on behalf of the Royal Astronomical Society

traditionally referred to with the somehow confusing term of *negative polarization*, is normally seen in the 0°–20° phase-angle range (usually referred to as the *negative branch* of the phase-polarization curve) and may be explained in terms of coherent backscattering (Muinonen et al. 2002).

A widely adopted remote sensing tool for the physical characterization of small Solar system bodies is spectroscopy. Similarly to what happens in stellar spectroscopy, asteroid reflectance spectra are classified into distinct taxonomic classes. Taxonomy based on multiband optical photometry was first developed by several authors in the 70s, and culminated in the classical work by Tholen (1984). More recently, broad-band photometry has evolved in full-fledged spectroscopy using spectrographs equipped with CCDs. A commonly adopted taxonomic classification based on spectra at visible wavelengths was published by Bus & Binzel (2002), and an extension to the near-IR region was more recently proposed by DeMeo et al. (2009).

In this Letter, we want to assess whether spectropolarimetry may be used to complement and refine the observing techniques of spectroscopy and broad-band polarimetry, that so far have been only *separately* considered. For this reason, we have started a survey of spectropolarimetry of asteroids, to our knowledge the first of its kind.

The taxonomic classifications of reflectance spectra by Tholen (1984) and Bus & Binzel (2002) were based on principal component analysis of hundreds of objects. So far, our spectropolarimetric data set is far too small to allow us any systematic classification. This Letter presents therefore the results of a pilot project aimed at assessing the usefulness of further investigations using this technique.

2 OBSERVATIONS

We have obtained spectropolarimetric measurements of a sample of asteroids using the FORS2 instrument (Appenzeller et al. 1998) of the European Southern Observatory (ESO) Very Large Telescope (VLT), and the ISIS instrument of the William Herschel Telescope (WHT) of the Isaac Newton Group of Telescopes. During an earlier VLT-FORS visitor mode run dedicated to the observations of the Earthshine (Sterzik, Bagnulo & Pallé 2012), we have also observed the sunlit limb of the Moon.

The instruments employed in our measurements are slit-fed and are equipped with similar polarimetric optics, consisting of a retarder waveplate and a beam-splitter polarizer: a Wollaston prism in case of FORS2, and a Savart plate in case of ISIS. The retarder waveplates may be set at fixed position angles, allowing one to exploit the advantages of the 'beam-swapping' technique (Bagnulo et al. 2009). Thanks to the beam-swapping technique, to the fact that both instruments are slit-fed, and that the light reflected by the target reaches the polarimetric optics without oblique reflections, we were able to obtain very accurate measurements of the continuum polarization. Observations with the FORS instrument were obtained using grism 300V with and without order-sorting filter *GG*435, covering the wavelength range 435–930 and 390–930 nm, respectively. ISIS observations were obtained using grism R158R and order-sorting filter *GG*495, covering the spectral range 480–975 nm.

Reductions of FORS data were performed with the aid of the ESO FORS pipeline (Izzo et al. 2010), and dedicated FORTRAN routines. Spectra obtained with ISIS were extracted then wavelength calibrated using IRAF routines, and then combined with FORTRAN routines. Throughout this Letter, we will refer to the reduced Stokes parameter $P_Q(\lambda) = Q/I$ representing the flux perpendicular to the plane Sun–Object–Earth (the scattering plane) minus the flux parallel to that plane, divided by the sum of the two fluxes. For symmetric reasons, Stokes U is expected to be zero. From the spectropolarimetric data, we calculated synthetic BBLP values (see Table 1). Approximate reflectance spectra $r(\lambda)$ were obtained by dividing the intensity spectra by the spectrum of solar analogue HD 30246 observed on 2014 January 30, but without taking into account

Table 1. BBLP values in the Bessel *VRI* filters from P_Q spectra. Photon-noise is negligible, and accuracy is limited by instrumental polarization, which we estimate ≤0.1 per cent. The double taxonomy classification given in column 2 are from Tholen (1984, left) and Bus & Binzel (2002, right). Asteroid observations were obtained from 2013 September to 2014 March. (1) Ceres was observed with ISIS, all the remaining targets with FORS. The Moon was observed with FORS in 2011 April and June.

Object	Class	α	V (%)	R (%)	I (%)
(1) Ceres	G/C	22°.4	1.17	1.21	1.25
(2) Pallas	B/B	27°.5	2.25	2.29	2.33
		22°.9	0.99	1.00	1.03
(7) Iris	S/S	26°.9	0.58	0.52	0.48
		27°.5	0.68	0.62	0.56
		28°.2	0.75	0.68	0.64
(8) Flora	S/S	28°.4	0.78	0.68	0.60
(21) Lutetia	M/Xk	14°.6	−1.19	−1.23	−1.23
(24) Themis	C/B	14°.0	−1.23	−1.18	−1.12
(44) Nysa	E/Xc	9°.1	−0.27	−0.30	−0.32
		24°.2	0.23	0.24	0.25
(51) Nemausa	CU/Ch	15°.7	−1.11	−1.10	−1.06
(208) Lacrimosa	S/Sk	13°.7	−0.46	−0.47	−0.50
(236) Honoria	S/L	7°.1	−1.00	−1.08	−1.17
(433) Eros	S/S	42°.0	1.99	1.87	1.86
(599) Luisa	S/K	26°.9	−0.39	−0.30	−0.16
Moon E	n.a.	81°.7	9.86	8.28	7.07
Moon M	n.a.	78°.3	5.81	4.99	4.36

Downloaded from https://academic.oup.com/mnrasl/article-abstract/446/1/L11/1051874 by guest on 17 March 2020

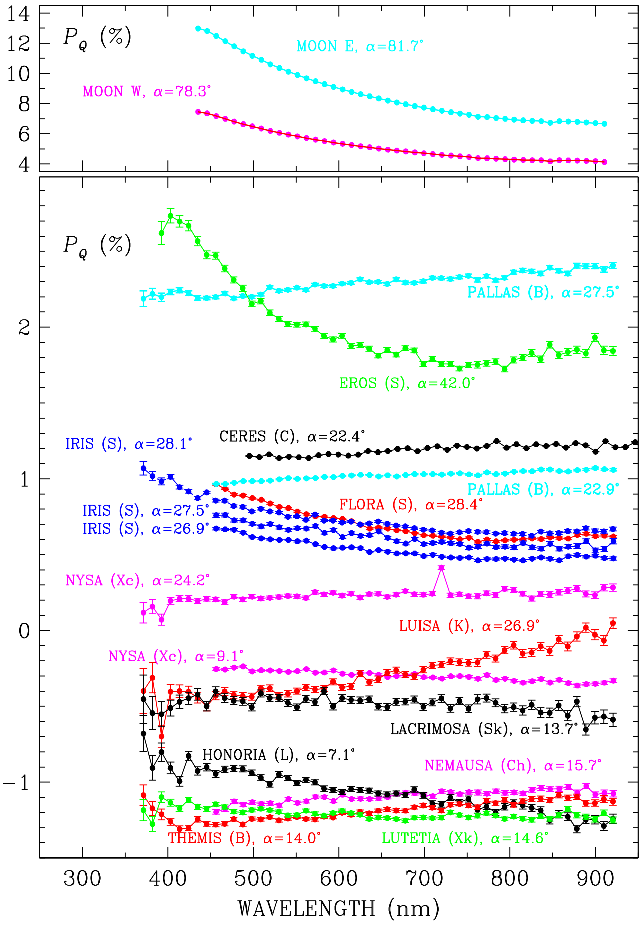

Figure 1. Polarization spectra of 12 asteroids (bottom panel) and of two regions of the Moon (top panel).

wavelength-dependent slit losses, and then normalized to $\lambda = 550$ nm. Data were rebinned to a spectral bin of ∼11 nm.

Polarization spectra of our targets are shown in Fig. 1. As expected, we found positive polarization (i.e. perpendicular to the scattering plane) at phase-angles $\alpha \gtrsim 20^\circ$, and negative polarization (i.e. parallel to the scattering plane) at phase-angles $\alpha \lesssim 20^\circ$. Remarkably, there is one exception: in spite of having been observed at a phase-angle as large as ∼27°, asteroid (599) Luisa exhibits a negative polarization. This makes it a new member of the class of the so-called Barbarians (Cellino et al. 2006), i.e. asteroids displaying an anomalous phase-polarization curve, characterized by a very wide negative polarization branch, extending up to $\alpha \sim 30^\circ$.

3 DISCUSSION

To discuss the diagnostic power of spectropolarimetry, we are going to address the following inter-related questions.

(i) Do polarization spectra depend on the phase-angle?

(ii) Do asteroids of a given taxonomic class have identical polarization spectra?

(iii) Do asteroids of different taxonomic classes have different polarization spectra?

(iv) What is the relationship between polarization spectra and reflectance spectra?

Firm answers require observations of several asteroids per taxonomic class with a homogeneous sampling of the phase-angle range. However, even our limited data set suggests some tentative answers, and, most importantly, guides us on how to refine the strategy for future observations.

We already know from classical BBLP measurements that the fraction of linear polarization does depend on phase-angle. In this analysis, however, we are more interested in the *shape* of the polarization spectra. Observations of (2) Pallas and (7) Iris suggest that in the positive branch, at least within limited phase-angle ranges, the shape of the P_Q spectra does not change, although observations of (44) Nysa suggest that the shape of the P_Q spectra obtained in the positive branch may differ from that obtained in the negative branch. We therefore introduce the polarization spectra normalized to the value at $\lambda = 550$ nm:

$$p_q(\lambda, \alpha) = \frac{P_Q(\lambda, \alpha)}{P_Q(\lambda = 550\,\mathrm{nm}, \alpha)}\,.$$

The introduction of this new quantity allows us to compare data of different objects obtained at different phase-angles under the approximation that, at least to first-order, the dependence of the polarization upon phase-angle may be separated from the dependence upon wavelength, in which case we have $p_q(\lambda, \alpha) \simeq p_q(\lambda)$. We note that unless the P_Q spectra cross the zero, p_q is always positive.

Answering questions (ii) and (iii) is equivalent to explicitly addressing the issue of whether spectropolarimetry brings additional information than spectroscopy. Fig. 1 suggests that in the specific case of asteroids (7) Iris and (8) Flora – both S-class in the Bus & Binzel (2002) system, and both observed at $\alpha \sim 28^\circ$ – the answer to question (ii) is yes. To address question (iii), we may consider that asteroids (2) Pallas (B-class), (7) Iris (S-class), and (599) Luisa (K-class) which were all observed close to $\alpha \sim 27^\circ$, show rather different P_Q spectra. To proceed further, we can only compare observations of different asteroids obtained at different phase-angles, making use of the normalized polarization spectra p_q introduced above.

The top panel of Fig. 2 shows the p_q spectra of B- and C-type asteroids. Asteroids (2) Pallas and (1) Ceres are both observed in the positive branch, and share similar p_q spectra. Asteroids (21) Themis and (51) Nemausa are both observed in the negative branch, and also share similar p_q spectra. In Fig. 1, we see that the P_Q spectra of B- and C-type asteroids always have a negative wavelength gradient. We note that since in the negative branch the gradients of p_q and P_Q spectra have opposite sign, B- and C-type asteroids have $\mathrm{d}p_q/\mathrm{d}\lambda < 0$ in the negative branch, and $\mathrm{d}p_q/\mathrm{d}\lambda > 0$ in the positive branch (see Fig. 2).

The mid panel shows the p_q spectra of four S-type asteroids: (7) Iris (observed three times around $\alpha \sim 28^\circ$), (433) Eros (a near-Earth asteroid observed at $\alpha = 42\overset{\circ}{.}0$), (8) Flora (observed at $\alpha = 28\overset{\circ}{.}4$), and (208) Lacrimosa (observed in the negative branch at $\alpha = 13\overset{\circ}{.}7$). The three p_q spectra of (7) Iris overlap each other well. The p_q spectrum of (433) Eros exhibits a marginally more pronounced concavity than that of (8) Flora and (7) Iris, but we are not able to say whether this (small) difference comes from the fact that Eros observations were obtained at a quite different phase-angle (∼14° larger than those of Flora and Iris), or because we are observing objects with different surface structure. We note that the P_Q spectra obtained in the positive branch have a negative gradient. The P_Q spectrum of (208) Lacrimosa, the only S-class asteroid observed in the negative branch, also has a negative gradient (which corresponds to a positive gradient for p_q). We therefore conclude that the intermediate albedo S-class asteroids exhibit a polarimetric

Downloaded from https://academic.oup.com/mnrasl/article-abstract/446/1/L11/1051874 by guest on 17 March 2020

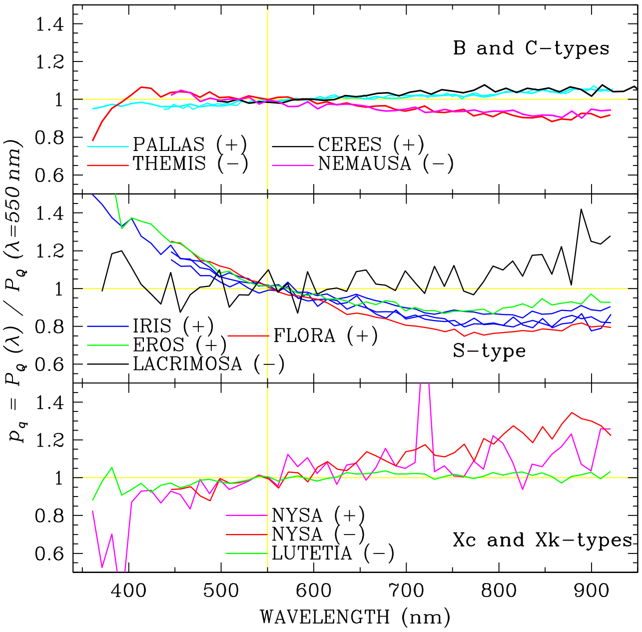

Figure 2. p_q spectra of asteroids (i.e. P_Q spectra normalized to the value at $\lambda = 550$ nm). The symbol (+) means that the spectrum was obtained in the positive branch, while the symbol (−) means that it was obtained in the negative branch.

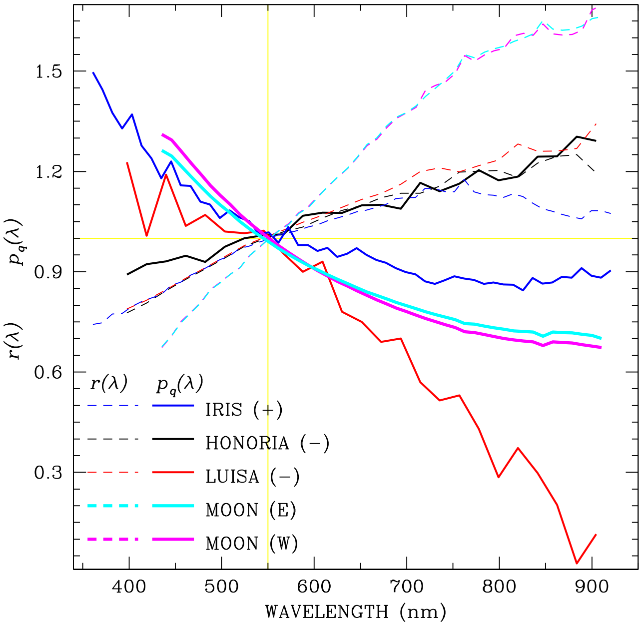

Figure 3. Normalized polarization spectra p_q (thick solid lines) and reflectance r spectra (thin dashed lines) of two regions of the Moon and of three asteroids of different taxonomic classes.

behaviour opposite to that of low-albedo B- and C-class asteroids, i.e. the gradient of their P_Q spectra is always negative.

The bottom panel of Fig. 2 shows that the p_q spectra of high-albedo Xc-class asteroid (44) Nysa are somewhat similar both in the negative and in the positive branch, and similar to the other Xk-class asteroid (21) Lutetia. The slope of the P_Q spectra of (44) Nysa is negative in the negative branch and positive in the positive branch, therefore it must change its sign somewhere around the inversion angle. We may speculate that this feature is common to all high-albedo asteroids, but more data are needed to confirm this.

We now consider the polarization spectra of two regions at the limb of the Moon, one close to the Grimaldi crater and one close to the Mare Crisium, which are plotted in the top panel of Fig. 1. Due to the high phase-angle value, both lunar P_Q spectra have a much higher amplitude than that observed for asteroids. Compared among themselves, the two lunar spectra show a similar trend but, due to the different phase-angle at which they were obtained, have a quite different amplitude. Once they are normalized, they nearly overlap each other, as shown with the light blue and magenta solid lines in Fig. 3.

Fig. 3 allows us to compare spectropolarimetric data of asteroids of different taxonomic classes and of the Moon, and also to make some considerations about the reflectance spectra, that are shown with dashed lines (normalized to $\lambda = 550$ nm).

First, we consider the three asteroids (7) Iris, (236) Honoria, and (599) Luisa, which, although belonging to different classes in the Bus & Binzel (2002) system, were all classified as S-type in the Tholen system. It is remarkable that, while their reflectance spectra appear similar to each other (which explains their common classification in the Tholen system), the p_q spectra appear completely different from each other. In particular, both (236) Honoria and (599) Luisa were observed in the negative branch, but display polarization spectra with opposite gradients: the p_q spectrum of (599) Luisa (observed at $\alpha \sim 27°$) has a strong negative gradient, i.e. the absolute value of the polarization decreases with wavelength; vice versa, the absolute value of the polarization of (236) Honoria (observed at $\alpha \sim 7°$) strongly increases with wavelength. The p_q spectra of the Moon are steeper than that of S-type asteroid (7) Iris (blue solid lines), but less steep than K-type asteroid (599) Luisa (red solid line). It is likely that the difference in slope is determined by a remarkable diversity of the surface structures and compositions.

In general, we expect that higher albedo corresponds to smaller polarization and lower albedo to higher polarization. Indeed, the behaviour of lunar spectra confirms the results by Dollfus, Bowell & Titulaer (1971) that for lunar regions, the polarization and reflectance spectra obey to the Umov law (Umov 1905), i.e. $P_Q(\lambda) \propto 1/r(\lambda)$. The Umov law is also valid for asteroids (7) Iris and (599) Luisa, but in the case of asteroid (236) Honoria, both the absolute value of the polarization and the reflectance have a positive gradient, i.e. both polarization and albedo increase with wavelength. A similar behaviour is exhibited by (21) Lutetia (not shown in the figure), that in the Tholen system had been classified as M-type. This is another aspect of the phenomenon discussed by Belskaya et al. (2009), who discovered that in M-type and S-type asteroids, which have higher albedo in the red than in the blue, the minimum of the polarization curves is deeper in the red than in the blue. The Umov law is rooted on the basic mechanism described by the Fresnel laws. Perhaps it is not surprising that it is violated in those conditions when Fresnel laws cannot even explain the orientation of the polarization. This phenomenon deserves further observational and theoretical investigation. In particular, it would be interesting to assess if it manifests itself only at small phase-angles, when the polarization is parallel to the scattering plane (being perhaps linked to the coherent backscattering mechanism), or if it may be observed also at large phase-angles.

(236) Honoria is a known Barbarian. Our discovery that also (599) Luisa is a Barbarian is particularly interesting. In the space of orbital proper elements, this asteroid is located in a high-inclination region where other Barbarians are also present, i.e. (387) Aquitania, (980) Anacostia, and (729) Watsonia. The latter is the lowest-

Downloaded from https://academic.oup.com/mnrasl/article-abstract/446/1/L11/1051874 by guest on 17 March 2020

numbered member of a dynamical family (Novaković, Cellino & Knezević 2011; Milani et al. 2014) which has been found by Cellino et al. (2014) to be a reservoir of small Barbarians. Moreover, spectroscopic data show that (387) Aquitania, (980) Anacostia and (599) Luisa have peculiarly high abundances of the spinel mineral, up to 30 per cent (Sunshine et al. 2008). The link between the Barbarian polarimetric behaviour and a composition rich in spinel is therefore further confirmed by our discovery that the spinel-rich asteroid (599) Luisa is also a Barbarian. We remind that (236) Honoria and (599) Luisa display opposite polarimetric gradients (see Fig. 3). Perhaps these differences are due to the large gap in phase-angle at which the observations were taken (though both in the negative branch), which would imply that the wavelength gradient of the polarization changes its sign in the negative branch. If instead the difference of the polarization spectra reflects a difference in structure surface and composition, we may have found a hint to the existence of different categories of Barbarians. More data are needed to confirm this.

4 CONCLUSIONS

We have obtained a number of polarization spectra of asteroids and the Moon. From their analysis, we *tentatively* suggest that P_Q spectra of low-albedo asteroids always have a positive gradient, and intermediate-albedo asteroids always have a negative gradient: this would be a confirmation of preliminary results obtained in the pioneering works by Lupishko & Kiselev (1995) and Belskaya et al. (2009), based on multicolour BBLP data. Polarization spectra of high-albedo asteroid (44) Nysa have a positive gradient in the positive branch, and a negative gradient in the negative branch, but more observations are needed to check if this result can be generalized to a wide class of asteroids.

We have found strong evidence that the Umov law may be violated: observed in the negative branch, both reflectance and polarization of asteroid (236) Honoria strongly increase with wavelength. We have also discovered that (599) Luisa is a member of the Barbarian class of asteroids.

We have shown that two objects belonging to the S-class observed at the same phase-angle have nearly identical polarization spectra, but we have also found that three objects, (7) Iris, (236) Honoria, and (599) Luisa, have relatively similar optical reflectance spectra but totally different polarizations spectra. Particularly puzzling is the difference between the polarization spectra of Honoria and Luisa, which are both members of the Barbarian class of asteroids. We do not know if this diversity is a consequence of the fact that these objects were observed at a different phase-angle, or if it originates from a remarkably different surface structure.

In asteroid spectroscopy, the choice of the solar analogue used for the normalization of the intensity spectra and the quality of the calibration of the atmospheric extinction play a crucial role in the final data quality, and, ultimately, in the spectral classification of asteroids. By contrast, spectropolarimetric measurements are robust, nearly independent of atmospheric conditions, and do not require any calibration with a solar analogue star. Provided that instrumental polarization is low and under control, they may be perfectly reproduced even with different instruments. Spectropolarimetric techniques still allow us to simultaneously obtain reflectance spectra, provided that the usual calibrations are performed.

We suggest that spectropolarimetric analysis of asteroids should complement traditional spectroscopic measurements and classification. In the longer term, any physical model capable of reproducing the observed reflectance spectra should also be tested against its capability to reproduce the observed spectropolarimetric data.

ACKNOWLEDGEMENTS

SB and AC gratefully acknowledge support from COST Action MP1104 ‘Polarimetry as a tool to study the solar system and beyond’. Observations were performed with ING Telescopes under programme W/2014A/5 and with ESO Telescopes at the La Silla-Paranal Observatory under programme IDs 087.C-0040 and 092.C-0639. We thank WHT staff, the ESO Paranal SCIOPS Team and User Support Division for their excellent support to the observations.

REFERENCES

Appenzeller I. et al., 1998, The Messenger, 94, 1

Bagnulo S., Landolfi M., Landstreet J. D., Landi Degl’Innocenti E., Fossati L., Sterzik M., 2009, PASP, 121, 993

Belskaya I., Levasseur-Regourd A.-C., Cellino A., Efimov Y. S., Shakhovskoy N. M., Hadamcik E., Bendjoya P., 2009, Icarus, 199, 97

Bus S. J., Binzel R., 2002, Icarus, 158, 146

Cellino A., Belskaya I. N., Bendjoya Ph., Di Martino M., Gil-Hutton R., Muinonen K., Tedesco E. F., 2006, Icarus, 180, 565

Cellino A., Gil-Hutton R., Dell’Oro A., Bendjoya Ph., Cañada-Assandri M., Di Martino M., 2012, J. Quant. Spectrosc. Radiat. Transfer, 113, 2552

Cellino A., Bagnulo S., Tanga P., Novaković B., Delbo M., 2014, MNRAS, 439, 75

DeMeo F., Binzel R. P., Slivan S. M., Bus S. J., 2009, Icarus, 202, 160

Dollfus A., Bowell E., Titulaer C., 1971, A&A, 10, 450

Izzo C., de Bilbao L., Larsen J., Bagnulo S., Freudling W., Moehler S., Ballester P., 2010, Proc. SPIE, 7737, 773729

Lupishko D. F., Kiselev N. N., 1995, BASS, 27, 1064

Milani A., Cellino A., Knežević Z., Bojan N., Federica S., Paolo P., 2014, Icarus, 239, 46

Muinonen K., Piironen J., Shkuratoc Yu. G., Ovcharenko A., Clark B. E., 2002, in Bottke W. F., Cellino A., Paolicchi P., Binzel R. P., eds, Asteroids III. Univ. Arizona Press, Tucson, AZ, p. 123

Novaković B., Cellino A., Knezević Z., 2011, Icarus, 216, 69

Penttilä A., Lumme K., Hadamcik E., Levasseur-Regourd A.-C., 2005, A&A, 432, 1081

Sterzik M. F., Bagnulo S., Pallé E., 2012, Nature, 483, 64

Sunshine J. M., Connolly H. C., McCoy T. J., Bus S. J., La Croix L. M., 2008, Science, 322, 1005

Tholen D. J., 1984, PhD thesis, Univ. Arizona

Umov E., 1905, Phys. Z., 6, 674

This paper has been typeset from a TEX/LATEX file prepared by the author.

Downloaded from https://academic.oup.com/mnrasl/article-abstract/446/1/L11/1051874 by guest on 17 March 2020