The challenge of turbulent acceleration of relativistic particles in the intra-cluster medium

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Abstract. Acceleration of cosmic-ray electrons (CRe) in the intra-cluster-medium (ICM) is probed by radio observations that detect diffuse, Mpc-scale, synchrotron sources in a fraction of galaxy clusters. *Giant radio halos* are the most spectacular manifestations of non-thermal activity in the ICM and are currently explained assuming that turbulence driven during massive cluster-cluster mergers reaccelerates CRe at several GeV. This scenario implies a hierarchy of complex mechanisms in the ICM that drain energy from large-scales into electromagnetic fluctuations in the plasma and collisionless mechanisms of particle acceleration at much smaller scales. In this paper we focus on the physics of acceleration by compressible turbulence. The spectrum and damping mechanisms of the electromagnetic fluctuations, and the mean-free-path (mfp) of CRe are the most relevant ingredients that determine the efficiency of acceleration. These ingredients in the ICM are however poorly known and we show that calculations of turbulent acceleration are also sensitive to these uncertainties. On the other hand this fact implies that the non-thermal properties of galaxy clusters probe the complex microphysics and the *weakly collisional* nature of the ICM.

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1. Introduction

Clusters of galaxies are the largest virialized structures in the universe [1]. They form via a hierarchical sequence of mergers and accretion of smaller systems driven by dark matter that dominates the gravitational field. The majority of baryonic matter in clusters is in the form of a hot (T $\sim 10^8$ K) and tenuous ($N_{gas} \sim 10^{-1} - 10^{-4}$ cm^{-3}) gas, the intra-cluster-medium (ICM).

The ICM is a unique environment for plasma physics due to the combination of its weakly collisional nature and high plasma beta. It also provides situations for CR acceleration and dynamics that differ significantly from those in other astrophysical environments. First of all, the bulk of CRs are confined and accumulated in the very large volumes of galaxy clusters for Hubble time [2, 3, 4, 5]. Second, the life-time of CRs in the ICM is very long as CRs diffuse in a dilute high-beta plasma [6, 5]. Under these conditions particle acceleration mechanisms that are notoriously not very efficient, such as stochastic Fermi-II mechanisms, become important because they can act for very long times-scales and on very large volumes. Another point is that the combination of CRs confinement and long life-time with the complex dynamics of the ICM implies that CRs can experience phases of cooling and (re)acceleration by different mechanisms. This results in a complex energy and spatial distribution of the CRs in clusters[5, 7].

Clearly galaxy clusters contain several *discrete* sources of CRs, including galaxies and AGNs. However there is much more. Indeed radio observations show cluster-scale diffuse synchrotron sources in a fraction of galaxy clusters, the most prominent ones are *giant radio halos* that cover Mpc^3 volumes [8]. These emissions are not directly associated with discrete sources and require *in situ* mechanisms of particle acceleration in the ICM [5]. *Radio halos* are observed in cluster mergers [8, 9, 10, 11], suggesting that a fraction of the kinetic energy of large-scale motions can be channelled into electromagnetic fluctuations and particle acceleration at smaller scales in the ICM. In fact this point is telling us that a complex hierarchy of processes are active in the ICM at different scales, and offers a possibility to constrain fundamental aspects of the microphysics of the ICM.

The synchrotron spectra of *radio halos* are steep with spectral slopes measured at GHz frequencies $\alpha \sim 1 - 2$ (flux $\propto \nu^{-\alpha}$) suggesting that the underlying acceleration mechanisms are not very efficient [5]. A popular scenario for the origin of *radio halos* is based on the possibility that turbulence generated during cluster-cluster mergers (re)accelerates seeds electrons distributed on Mpc^3-volumes to energies $\geq$ few GeV, that are requested to produce the synchrotron radiation observed in the radio band [12, 13, 14, 15, 16, 17, 18, 19, 20]. This scenario allows to explain the very extended and diffuse nature of the observed emission of *radio halos* and the tight connection between *radio halos* and dynamics of the hosting clusters. On the other hand crucial ingredients in this scenario are poorly known. Primarily the challenge is to understand the chain of mechanisms that transport energy from large scales to collisionless small-scales in the ICM.

In this paper we focus on CRe acceleration by compressive turbulence driven at large scales in the ICM. Specifically we explore the changes in the efficiency of acceleration that are induced by different assumptions on the turbulent spectrum and ICM microphysics.

2. A brief overview of turbulence in the ICM

In this Section we briefly review the current view of the properties of turbulence in galaxy clusters. Galaxy clusters contain many potential sources of turbulence [5, 21], the most important for large-scale turbulence are mergers between clusters. Mergers deeply stir and rearrange the cluster structure generating turbulence through sloshing of cluster cores, shearing instabilities in the ICM and via the complex patterns of interacting shocks that form during mergers and structure formation more generally. Such a complex ensemble of mechanisms should drive both compressive and incompressive turbulence, as also supported by the analysis of numerical (fluid) simulations[22, 18, 23].

Large-scale motions that are driven during cluster-cluster mergers and dark matter sub-halo motions are expected on scales comparable to cluster cores scales, $L_o \sim 100-400$ kpc, and might have typical velocities $\delta V \sim 300-700$ km s^{-1} [21, 15, 24, 25, 26, 23]. These motions are subsonic, typically with $M_o = \delta V/c_s \approx 0.2-0.5$, but super-Alfvénic, with $M_A = \delta V/V_A \approx 2-8$. An open question is whether these motions generate an efficient turbulent cascade at smaller scales or if they are dissipated at larger scales. The Coulomb mfp of particles in the ICM is very large, $l_C \approx 10$ kpc, which in turns implies a moderate value of the *Reynolds* number in the ICM, $\mathcal{R}e \sim 100$, for typical parameters. Such a moderate *Reynolds* number would not guarantee *per se* an efficient turbulent cascade. However the ICM is a magnetised and *weakly collisional* high-beta medium, that is notoriously unstable to several instabilities [27, 28, 29, 30, 31, 32], implying that collisionless effects govern microphysics and make the medium more turbulent. The combination of these facts suggests that the ICM is turbulent. We note that this conclusion is also supported by similarities with the IPM, that is also *weakly collisional*, magnetised and (moderately) high-beta plasma. In fact the *Reynolds* number (considering Coulomb collisions) of the IPM would not be large, but the IPM is *observed* to be turbulent [33].

At large scales, where the bulk of turbulence is generated by cluster dynamics in the ICM, turbulence is hydrodynamic with kinetic energy of motions being in excess of magnetic energy. At scales smaller than the MHD-scale, $l_A = L_o M_A^{-3}$ (using Kolmogorov scaling for hydro-turbulence), turbulence becomes MHD provided that the ICM behaves a fluid and that collisionless effects play a role only at smaller scales. MHD turbulence can be described by Goldreuch-Sridhar model with Alfvén and slow modes developing anisotropic spectrum[34, 35]. On one hand we can thought that solenoidal and compressive turbulence are generated at large scales and cascade at smaller scales, at the same time however we should expect that turbulence can also be generated directly at smaller scales in response of plasma/kinetic instabilities and coherent wave phenomena driven by the cascade of strong MHD turbulence [28, 31, 36, 37]; the energy associated with small-scale turbulence is however subdominant. Both large-scale motions (and their cascading at smaller scales) and the variety of waves excited at small scales should play crucial roles in governing the micro-physics of the ICM through the scattering of particles and the perturbation and amplification of the magnetic field. However, the efficiency of the transport of turbulent energy from large to small scales and the efficiency of generation of waves at smaller scales are still open issues.

The nonlinear interplay between particles and turbulent waves/modes induces a stochastic process that drains energy from plasma turbulence to particles [38, 39, 40]. Turbulent acceleration is invoked for the origin of *giant radio halos.* Acceleration of CRs directly from the thermal pool to relativistic energies by MHD turbulence in the ICM is very inefficient and faces serious problems due to associated energy arguments [41]. Consequently, turbulent acceleration in the ICM is rather a matter of reacceleration of pre-existing (seed) CRs rather than *ab initio* acceleration of CRs. This poses the problem of the origin of the seed particles, a problem that is currently subject of active discussion[17, 7], but that will not be addressed in this paper.

Presumably the many types of waves generated/excited in the ICM, both at large and very small scales, jointly contribute to the scattering process and (re)acceleration of CRs. In the last years much attention has been devoted to CR reacceleration due to compressible turbulence that is driven at large scales in the ICM from cluster mergers and that cascades to smaller scales[16, 20]. This is the simplest scenario that can be thought, nevertheless it naturally predicts a direct connection between cluster mergers and *radio halos*. In fact several studies suggest that turbulent reacceleration by compressive modes provides a plausible explanation for *radio halos*[16, 17, 19, 18, 5, 7], although several ingredients of the physics of this mechanim are still poorly known.
In the following we will discuss Transit-Time-Damping (TTD) resonance (Sect.3) and non-resonant acceleration by turbulent compressions (Sect.4), exploring the effects induced on the efficiency of these mechanisms by different assumptions.

3. Transit Time Acceleration mechanism: turbulent spectrum and mfp

Compressible component of the magnetic field of compressible modes (i.e. the component along B_o in the case of oblique propagation) can interact with particles through TTD resonance [42, 43]. The condition for resonance is :

$$\omega - k_{\|} v_{\|} = 0 \tag{1}$$

where $k_{\|}$ and $v_{\|}$ are the components of the waves wavenumber and particle velocity parallel to the magnetic field; $\omega = c_s k$ for fast modes (magnetosonic waves) in the ICM. This interaction is essentially a coupling between the magnetic moment of particles and the (parallel) magnetic field gradients. This interaction, in combination with *additional/external* sources of pitch-angle scattering/isotropization during acceleration[44, 45, 16]‡, is considered as a fundamental way to accelerate particles in different astrophysical environments, including the ICM.
Stochastic acceleration can be described as a diffusion in the momentum space of particles. For TTD the expression of diffusion coefficient, assuming quasi-isotropic turbulent cascade and high-beta plasma (conditions suitable for the ICM), is given in [16]:

$$D_{pp}(p) = \frac{\pi^2}{2c} \frac{p^2 c_s^2}{B_o^2} \int_0^1 d\mu \frac{1-\mu^2}{\mu} \mathcal{H}\left(1 - \frac{c_s}{c\mu}\right) \left[1 - (\frac{c_s}{c\mu})^2\right] \int_{k_o}^{k_{cut}} dk W_B(k) k \tag{2}$$

where μ= cosine of pitch angle, $\mathcal{H}(x)$ is the Heaviside step function (1 for $x > 0$, and 0 otherwise), W_B is the energy spectrum of magnetic field fluctuations and k_o and k_{cut} are the injection and cut-off scales (wavenumber) of the fluctuations.
The most important ingredients in Eq.2 are the energy density of electromagnetic fluctuations and their spectrum, and the cut-off scale of the turbulent (magnetic

‡ indeed the $n = 0$ resonance changes only the component of particle momentum parallel to the seed magnetic field that would increase the degree of anisotropy of particle distribution and decrease the acceleration efficiency with time

fluctuations) spectrum k_{cut}. In the simple scenario adopted in this paper, where turbulence is generated at large scale and cascade at smaller scales, the minimum scale k_{cut} is the scale where the mechanisms of damping of turbulence become faster than turbulent cascade. During acceleration energy goes into particles increasing the damping of turbulence due to particles themselves (the cut-off scale $1/k_{cut}$ increases) and reducing the acceleration efficiency. Constraining k_{cut} is thus critical to obtain meaningful estimates of the acceleration efficiency.

To do that we need to calculate the damping of turbulence in the ICM and the efficiency of turbulent cascade. Following the motivations given in [16] we assume that collisionless dampings with particles are the dominant ones in the ICM. The damping is obtained assuming quasi-linear-theory [39, 40]:

$$\Gamma = -i \Big(\frac{E_i^* K_{ij}^a E_j}{16\pi W} \Big)_{\omega_i = 0} \omega_r \tag{3}$$

where K_{ij}^a is the anti-Hermitian part of the plasma dielectric tensor [39], W is the total energy in the modes, E_i is the electric field (fluctuations) and ω_r is the real part of the mode frequency.

TTD determines the strongest collisionless interaction between particles and compressive fast modes in the ICM. The collisionless damping with thermal electrons and protons is [16]:

$$\Gamma_{e/p}(k,\theta) = \sqrt{\frac{\pi}{8}} \frac{|B_k|^2}{W(k,\theta)} \mathcal{H}\Big(1 - \frac{c_{\rm s}}{c}\frac{k}{|k_\parallel|}\Big) \frac{c_{\rm s}^2}{B_o^2} \left(\frac{k}{|k_\parallel|}\right) \left(\frac{k_\perp}{k}\right)^2 \times$$
$$\frac{\Big(m_{e/p} k_B T\Big)^{1/2}}{1 - (\frac{c_{\rm s} k}{c k_\parallel})^2} N_{e/p} \exp\Big\{ - \frac{m_{e/p} c_{\rm s}^2}{2 k_B T} \frac{\Big(k/k_\parallel\Big)^2}{1 - (\frac{c_{\rm s} k}{c k_\parallel})^2} \Big\} k \tag{4}$$

where the ratio of magnetic field fluctuations and total energy density is calculated in the collisionless regime in [16], $\beta_{pl} = 2c_s^2/V_A^2$, and $\langle |B_k|^2/W \rangle \approx 16\pi/\beta_{pl}$, $< .. >$ is the average over pitch-angles.

The other source of damping of fast modes in the ICM is due to TTD interaction with CRs [16]:

$$\Gamma_{e/p}(k,\theta) = -\frac{\pi^2}{8} \frac{|B_k|^2}{W(k,\theta)} \left(\frac{k_\perp}{k}\right)^2 \left(\frac{k}{|k_\parallel|}\right) \mathcal{H}\Big(1 - \frac{c_{\rm s}}{c}\frac{k}{|k_\parallel|}\Big) \frac{N_{e^\pm/p}\, c_{\rm s}^2}{B_o^2}\, k \times$$
$$(1 - (\frac{c_{\rm s} k}{c k_\parallel})^2)^2 \int^\infty p^4 dp \left(\frac{\partial \hat{f}(p)}{\partial p}\right)_{e/p} \tag{5}$$

where $N_{e^\pm/p} \hat{f}(p)_{e/p}$ is the distribution function of CRe/p in the momentum space. The second element that is necessary to constrain k_{cut} is the time-scale of turbulent cascade. We shall use MHD as a guide and derive the cascading of isotropic fast modes using the

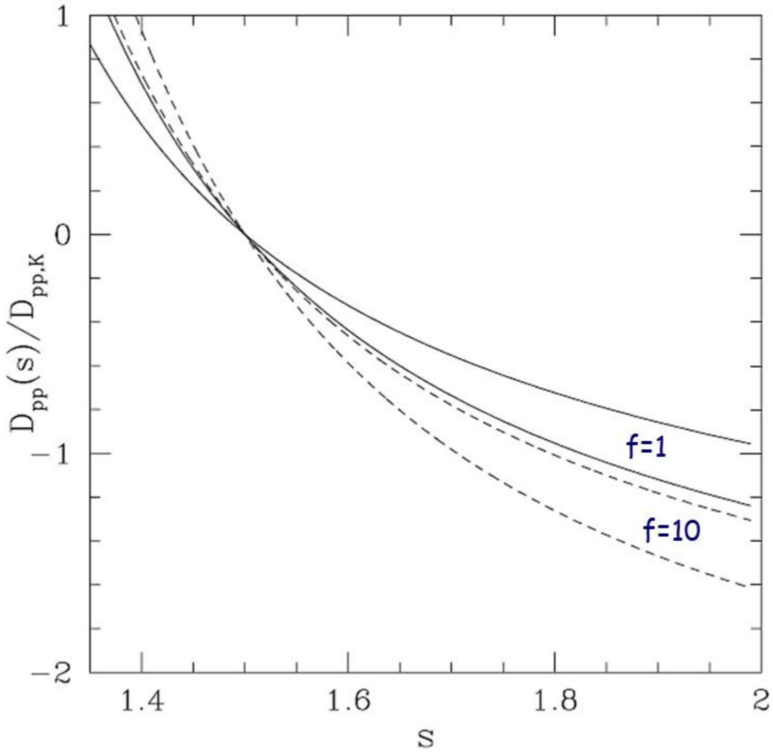

Figure 1. The ratio between acceleration rate assuming $W_B \propto k^{-s}$ and that using a Kraichnan spectrum. Solid lines assume collisionless interaction with both thermal particles and CRs (case (i) in Sect.3), dashed lines assume an increased collisionality, with $k_{cut} = f k_{cut,K}$ (to mimic case (ii) in Sect.3). We adopt $\delta V = 500$ (lower) and 800 km/s (upper lines), $L_o = 300$ kpc, $c_s = 1500$ km/s, and $V_A = 300$ km/s.

Kraichnan treatment. In this case the wave-wave diffusion coefficient in k-space is [46] $D_{kk} \sim k^4 W(k)/(\rho c_s)$ and the resulting cascading time is :

$$\tau_{kk} \approx \frac{k^3}{(\partial/\partial k)(k^2 D_{kk})} \sim \frac{2}{9} \frac{c_s}{\delta V^2} (k k_o)^{-1/2} \tag{6}$$

In Eq.6 and in the following we assume $W(k_o)k_o \sim \delta V^2$, δV is the velocity of large-scale eddies.

Having derived damping and cascading coefficients, the cut-off scale is obtained requiring $\tau_{kk} \sim \Gamma^{-1}$:

$$k_{cut,K} \simeq \frac{81}{4} \left(\frac{\delta V^2}{c_s} \right)^2 \frac{k_o}{(\sum_\alpha \langle \Gamma_\alpha \rangle k^{-1})^2} \tag{7}$$

where $< .. >$ marks pitch-angle averaging.

Given the uncertainties in the ICM microphysics, we consider two scenarios:

(i) assume that the interaction between turbulent modes and both thermal and CRs is fully collisionless. This happens when particles collision frequency in the ICM is $\omega_{ii} < \omega = k c_s$; for example this is the case where ion-ion collisions in the thermal ICM are due to Coulomb collisions. Under this condition the damping of

compressive modes is dominated by TTD with thermal particles (Eq.4) [16]. For large β_{pl} the dominant damping rate is due to thermal electrons and can be approximated by $\Gamma_e \sim c_s k \sqrt{3\pi(m_e/m_p)/20\mu^2}\exp(-5(m_e/m_p)/3\mu^2)(1-\mu^2)$. Combining pitch-angle averaging of this expression with Eq.7 gives $k_{cut,K} \sim 10^4 k_o M_o^4$.
Assuming a Kraichnan spectrum of magnetic field fluctuations, $W_B \propto k^{-3/2}$, the resulting acceleration time-scale (from Eq. 2) is :

$$\tau_{acc} = \frac{p^2}{4D_{pp}} \simeq 2.5\langle\frac{\beta_{pl}|B_k|^2}{16\pi W}\rangle^{-1} f_x^{-1} (\frac{M_o}{1/2})^{-4} \left(\frac{L_o/300\,\mathrm{kpc}}{c_s/1500\,\mathrm{km\,s^{-1}}}\right) \;(\mathrm{Myr}) \qquad (8)$$

where $< .. > \sim 1$ independent of scale, $x = c_s/c$ and

$$f_x = x\left(\frac{x^4}{4} + x^2 - (1+2x^2)\ln(x) - \frac{5}{4}\right) \simeq 0.02 \qquad (9)$$

For typical conditions this scenario predicts acceleration time-scales $\sim$ 100 Myr that are sufficient to explain *radio halos* and is commonly adopted to calculate turbulent acceleration in the ICM [16, 47, 20, 7].

(ii) The other possibility is that thermal particles do not contribute very much to the collisionless dampings. The ICM is a *weakly collisional* high-beta plasma that is unstable to several instabilities (see Sect.2). Scattering induced by magnetic field perturbations driven by instabilities may increase the collision frequencies in the thermal plasma because charged particles can be randomized if they interact with the perturbed magnetic field. This process can be viewed as the collective interaction of an individual ion with the rest of the plasma. Under this condition one may thought that the interaction between modes and thermal particles behave *collisional*. § In this case the dominant collisionless damping in the ICM is due to the CRs [30], and combining Eqs.5 and 7 :

$$k_{cut,K} \simeq \left(\frac{18}{\pi^2}\right)^2 M_o^4 f_x^{-2} \frac{(\rho c_s^2)^2}{\left[c\int p^4 dp \frac{\partial f}{\partial p}\right]^2} k_o \qquad (10)$$

Under typical conditions in the ICM this is $k_{cut,K} \sim 1000\, k_o M_o^4 (\epsilon_{ICM}/\epsilon_{CR})^2$, where the ratio of thermal and CR energy densities is $\epsilon_{ICM}/\epsilon_{CR} \sim 100$. It implies that the turbulent cascade reaches scales that are much smaller than those in the case (i) and consequently the acceleration rate is faster (Eqs.2 and 10) :

$$\tau_{acc} = \frac{p^2}{4D_{pp}} \simeq 6\langle\frac{\beta_{pl}|B_k|^2}{16\pi W}\rangle^{-1} (\frac{M_o}{1/2})^{-4} \left(\frac{L_o/300\,\mathrm{kpc}}{c_s/1500\,\mathrm{km\,s^{-1}}}\right) (\left[\frac{c\int p^4 dp \frac{\partial f}{\partial p}}{\rho c_s^2}\right]/25) \;(\mathrm{Myr}) (11)$$

The acceleration efficiency is inversely proportional to the energy density of CRs. In this scenario CRs drain efficiently energy from the turbulent cascade, however as the CRs energy density increases the acceleration efficiency gets reduced. From the practical

§ It should also be mentioned that scatterings induced by instabilities decrease the effective mfp of thermal particles decreasing the effective viscosity in the ICM and increasing the effective Reynolds number of the ICM.

point of view this is simply because CRs can get a constant energy flux from turbulence implying that the effect on their spectrum is smaller for increasing values of the CRs energy density. As a consequence of this *back reaction* fast acceleration cannot be maintained for long time. Assuming typical conditions in the ICM, the value of the acceleration time averaged on a time-period of few 100 Myr is generally found $\tau_{acc} \sim 10$ Myrs [30]. This is up to 10 times more efficient than in the case (i) resulting in harder spectra of both CRe and synchrotron radiation[30, 20].

All the results discussed above are based on Kraichnan spectrum of fast modes, $W_B(k) \propto k^{-3/2}$. On the other hand part of the energy of the MHD cascade of fast modes may be dissipated into weak shocks producing a steeper spectrum[48]. In general if the spectrum of compressive turbulence gets steeper the turbulent acceleration rate decreases. Assuming a spectrum $W_B(k) \propto k^{-s}$ in Eq.2, the acceleration rate will be changed with respect to that evaluated using a Kraichnan spectrum by a factor‖ $\sim \frac{1/2}{2-s} k_o^{s-1} k_{cut,s}^{2-s} / \sqrt{k_o k_{cut,K}}$, where $k_{cut,s}$ is the cut-off in the turbulent spectrum assuming a slope s.

In order to evaluate this $k_{cut,s}$ we follow a simple procedure. We still assume the Kraichnan diffusion coefficient $D_{kk} \sim k^4 W(k)/\rho c_s$ to model wave-wave coupling at scale k, but we use a spectrum $W(k) \propto k^{-s}$. The resulting cascading time scale of fast modes is :

$$\tau_{kk} \approx \frac{1}{5-s} \frac{c_s}{\delta V^2} \frac{k^{s-2}}{k_o^{s-1}} = \frac{7/2}{5-s} \tau_{kk}(s = \frac{3}{2}) \left(\frac{k}{k_o} \right)^{s-\frac{3}{2}} \tag{12}$$

and the cut-off, obtained from the condition $\tau_{kk} \sim 1/\sum \langle \Gamma \rangle$, is :

$$k_{cut,s} = \left[\frac{2}{7} (5-s) k_{cut,K}^{1/2} k_o^{s-3/2} \right]^{\frac{1}{s-1}} \tag{13}$$

where $k_{cut,K}$ refers to the cut-off in both Eqs.7 and 10.

Figure 1 shows the effect on the acceleration rate due to different assumptions for the turbulent spectrum. The acceleration rate decreases with increasing slope. For Burgers spectra (s =2) the acceleration is 10 times less efficient that that in the Kraichnan case. This has important consequences, for example in the collisional case (case i) steep spectra of magnetic field fluctuations produce acceleration rate via TTD that are generally too small to explain *radio halos* [20].

4. Stochastic Acceleration by large-scale compression: turbulent spectrum and mfp of relativistic particles

Relativistic particles diffusing through large-scale compressible turbulence experience a statistical acceleration effect [49, 50, 16]. In the case of subsonic turbulence, $\delta V^2 << c_s^2$,

‖ in the case of Burgers spectum, $s = 2$, the factor is $\sim (1/2) \ln(k_{cut,2}/k_o) \sqrt{k_o/k_{cut,K}}$

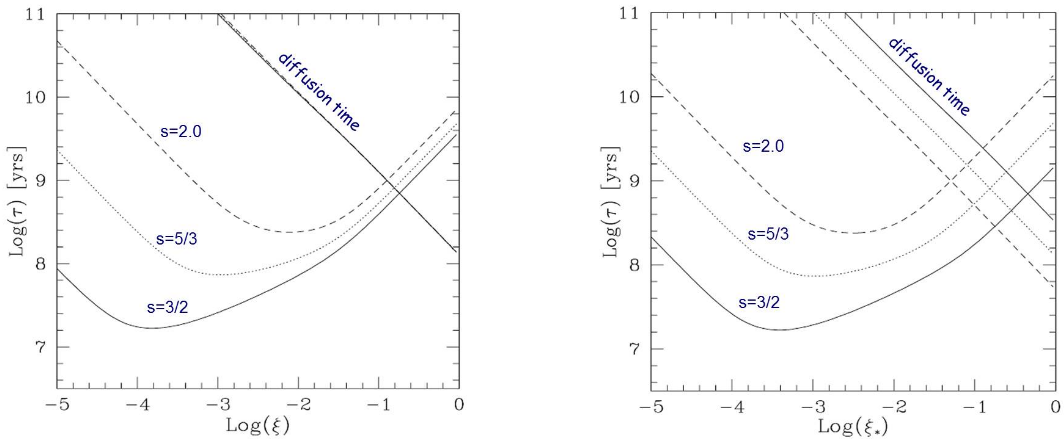

Figure 2. Acceleration time and (Mpc-scale) diffusion time due to non-resonant acceleration as a function of $\xi = \lambda_{mfp}/l_\lambda$. Left panel is for $l_\lambda = M_o^{-3} L_o$ and right panel is for $l_\lambda = \max\{M_o^{-4} L_o, 2\pi/k_{cut}\}$ (see text). The collisionless scenario (case (i), Sect. 3) is adopted to calculate the turbulent cut-off scale. In the calculations we assume δV=750 km/s, and s =3/2, 5/3, 2.0, other parameters being equal to Fig.1.

and provided that turbulence has correlation scales much longer than the particles mfp, the diffusion coefficient in the particle momentum space is :

$$D_{pp} = \frac{2}{9} p^2 D \int_k \frac{dy y^2 \mathcal{K}(y)}{c_s^2 + y^2 D^2} \tag{14}$$

where D is the particle spatial diffusion coefficient and $\mathcal{K}$ is the kinetic spectrum of turbulence, $k_o \mathcal{K}(k_o) \sim \delta V^2/2$.

The spatial diffusion coefficient of CR in the ICM is unknown and we shall consider it as a free parameter. We assume $D = \frac{1}{3} c \lambda_{mfp}$ with a CR mfp $\lambda_{mfp} = \xi l_\lambda$, where l_λ is the minimum scale of magnetic field reversal in the ICM. Under these assumptions Eq.14 reads :

$$D_{pp} = \frac{p^3}{3} \left(\frac{\delta V^2}{c l_\lambda} \right) \xi \int_1^{x_c} \frac{dx x^{2-s}}{(\frac{3 c_s L_o}{2\pi c l_\lambda})^2 + x^2 \xi^2} \tag{15}$$

where $x_c = k_{cut}/k_o$. The acceleration rate depends on the turbulent energy and spectrum, but also on the CRs mfp. This mechanims is characterised by two regimes : *fast diffusion*, for $\xi^2 >> (3 c_s L_o)^2/(2\pi c l_\lambda)^2$, in which case particles leave the eddies before they turnover and the acceleration is dominated by the largest eddies, and *slow diffusion*, for $x_c^2 \xi^2 << (3 c_s L_o)^2/(2\pi c l_\lambda)^2$, in which case the acceleration is mainly dominated by the smallest eddies. In the two regimes the dependencies of the acceleration rate on the physical parameters are different: $D_{pp} \propto p^2 M_o^2 D k_o^{s-1} k_{cut}^{3-s}$ in the *slow* regime and $D_{pp} \propto p^2 M_o^2 c_s^2/D$ in the *fast* regime (see also [50]).

The total (turbulent advection and diffusion) spatial diffusion coefficient of particles diffusing and interacting with compressible/acoustic turbulence is [49, 50, 16] :

$$D_* = \frac{1}{3} l_\lambda c \xi \left[1 + \frac{3}{2\pi^2} \left(\frac{L_o \delta V}{l_\lambda c} \right)^2 \int_1^{x_c} \frac{dx x^{-s}}{(\frac{3 c_s L_o}{2\pi c l_\lambda})^2 + x^2 \xi^2} \right] \tag{16}$$

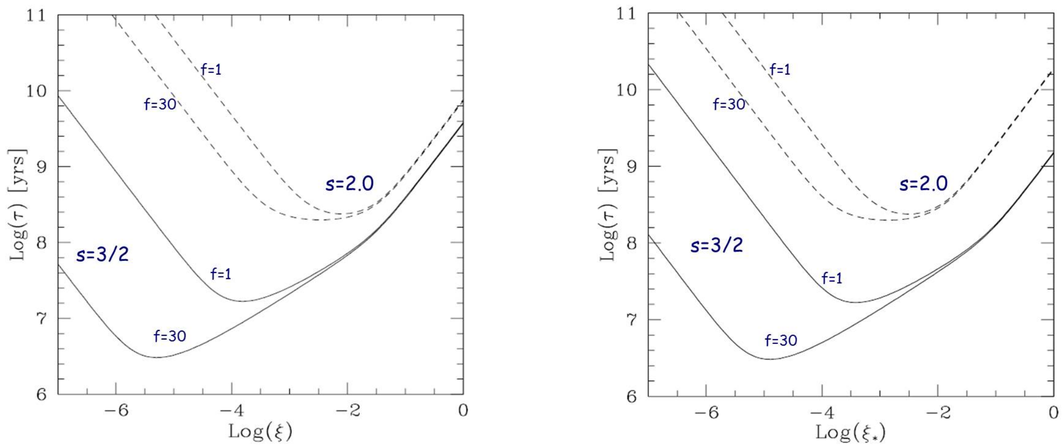

Figure 3. Acceleration time due to non-resonant acceleration as a function of $\xi = \lambda_{mfp}/l_\lambda$. Left panel is for $l_\lambda = M_o^{-3} L_o$ and right panel is for $l_\lambda = \max\{M_o^{-4} L_o,\, 2\pi/k_{cut}\}$ (see text). Turbulent cut-off scale is calculated both in the collisionless scenario ($k_{cut,s}$ Eqs.7 and 10, case (i), Sect. 3) and assuming $k_{cut} = f k_{cut,s}$ (to mimic case (ii) in Sect.3). We show the case s =3/2 (solid) and 2.0 (dashed), other parameters being equal to Fig. 2.

the time-scale necessary to diffuse on scale L_* is $\tau_{diff} \simeq L_*^2/4D_*$.
In order to quantify the acceleration rate we should estimate l_λ. This reversal depends on turbulent properties. We estimate l_λ following two hypothesis : (a) we assume that both solenoidal and compressible turbulence are generated in the ICM at scale L_o with similar energies, $\delta V_s^2 \approx \delta V^2$, and use solenoidal turbulence to estimate $l_\lambda = l_{As}$, where $l_{As} = L_o M_A^{-3}$ is the MHD scale assuming Kolmogorov spectrum of the (super-Alfvénic) solenoidal turbulence; (b) we assume that turbulence in the ICM is only compressible, in this case $l_\lambda = \max\{l_A, 2\pi/k_{cut}\}$ where $l_A = L_o M_A^{-4}$ is the MHD scale (for Kraichnan turbulence) and k_{cut} is given in Sect.3.
Fig.2 shows the acceleration rate vs ξ assuming different slopes of the kinetic turbulent spectrum. The cut-off scale is derived according to Sect.3, case (i). Although ξ is a free parameter, we note that mfp>> r_L implies $\xi > 10^{-6}$ for multi-GeV – TeV particles in the ICM. At the same time the upper bound of ξ is set by the condition that CRe must be confined in Mpc-volumes for ≥Gyrs in order to generate *giant radio halos*, this typically requires $\xi < 0.1$. For $\delta V \sim 700 - 800$ km/s, we conclude that the acceleration time ranges between $10^7 - \text{few} \times 10^9$ yrs depending on the slope of the turbulent spectrum and on the mfp of CRs.
In Fig.3 we show the effect of increasing plasma collisionality in the ICM (i.e. case (ii) in Sect.3) by assuming a cut-off $k_{cut} = f k_{cut,s}$. This increases the acceleration rate in the branch of *slow* diffusion regime. The effect is stronger for flatter spectra of the turbulence, for instance $f = 30$ implies a boost of the acceleration efficiency by more than 2 orders of magnitude in the Kraichnan case.

5. Discussion and conclusions

A popular scenario that is adopted to explain *giant radio halos* is based on turbulent reacceleration, with turbulence generated during cluster-cluster mergers. If true this scenario implies that a hierarchy of complex mechanisms drain a fraction of the energy of the large-scale motions that are generated by the process of cluster formation into electromagnetic fluctuations and collisionless mechanisms of particle acceleration at much smaller scales. It has been realised that the existence of these complex collisionless mechanisms, opens new prospects to understand the micro-physics of the ICM [18, 5, 20].

In this paper we focused on the problem of stochastic acceleration of CRs by compressible turbulence in the ICM. The efficiency of acceleration depends on the particles mfp and on the spectrum of compressible turbulent motions, in particular on that of the electromagnetic fluctuations. The extent and the shape of this spectrum, in turn depend on the processes of plasma damping and on the way turbulence is generated and transported at smaller scales. We have explored this subject by considering two mechanisms that are commonly adopted to explain *giant radio halos*, TTD due to fast modes (Sect. 3) and non-resonant acceleration due to turbulent compressions (Sect. 4). In the case of TTD we analyzed two extreme situations that differs in the efficiency of collision frequencies between thermal particles in the ICM, in particular whether collisions occur via Coulomb scattering or mainly via collective processes induced by plasma instabilities. In the latter case collisionless damping of compressive turbulence is dominated by CRs and the acceleration is more efficient than in the other case. We also discussed the changes in the acceleration rate that are induced by different slopes of the turbulent spectrum.

In the case of non-resonant acceleration we explored the combined effect induced on the acceleration rate by different assumptions for the turbulent spectra and mfp of CRe. The latter parameter is very uncertain but plays a crucial role as it determine the regime of diffusion of CRe in the turbulent field and consequently has the potential to strongly change the acceleration efficiency.

In conclusion we have shown that the uncertainties about the ICM microphysics induce substantial variations in the acceleration efficiency of both mechanisms. On the other hand however this also implies that radio halos and the non-thermal properties of galaxy clusters are effective probes of the complex microphysics of the ICM.

In this paper we do not investigate reacceleration by solenoidal/incompressive turbulence. Several works attempted to model this situation to explain radio halos[51, 14, 52]. We note however that these calculations did not take into account the scale-dependent anisotropies in Alfvénic turbulence, and consequently addressing the role of Alfvénic turbulence in the acceleration process in the ICM requires further investigations.

At this point we believe that future advances in the field will derive from studies that aim at addressing the generation of plasma instabilities in the ICM and from attempts to model self-consistently the way these instabilities generate small-scale fluctuations, and affect particles mfp and acceleration rates. The importance of this step has been clearly highlighted in Sect.3 and 4 where we have compared acceleration rates obtained in the collisional and collisionless cases. Situation may be even more complicated because, similarly to the IPM, it can be thought that the collisional properties of the ICM may evolve with time and space. Finally we believe that magnetic reconnection in the ICM and its interplay with turbulence is another piece of this complex puzzle and it may play a role in the acceleration and reacceleration of CRe.

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