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$$\begin{aligned} \mathbf{X}' &= \begin{pmatrix} x'(x, y) \\ y'(x, y) \end{pmatrix} = \mathbf{R}^T (\mathbf{X} - \mathbf{X}_0) \\ &= \begin{pmatrix} (x - x_0) \cos \theta + (y - y_0) \sin \theta \\ -(x - x_0) \sin \theta + (y - y_0) \cos \theta \end{pmatrix}, \end{aligned}$$

the BSN PDF can be expressed as:

$$p(x, y) = \mathcal{N} \times \begin{cases} \exp \left[-\frac{1}{2} \left(\frac{x'(x, y)}{\lambda_x} \right)^2 - \frac{1}{2} \left(\frac{y'(x, y)}{\lambda_y} \right)^2 \right] \\ \text{if } x'(x, y) < 0 \text{ and } y'(x, y) < 0 \\ \exp \left[-\frac{1}{2} \left(\frac{x'(x, y)}{\lambda_x \tau_x} \right)^2 - \frac{1}{2} \left(\frac{y'(x, y)}{\lambda_y} \right)^2 \right] \\ \text{if } x'(x, y) \geq 0 \text{ and } y'(x, y) < 0 \\ \exp \left[-\frac{1}{2} \left(\frac{x'(x, y)}{\lambda_x} \right)^2 - \frac{1}{2} \left(\frac{y'(x, y)}{\lambda_y \tau_y} \right)^2 \right] \\ \text{if } x'(x, y) < 0 \text{ and } y'(x, y) \geq 0 \\ \exp \left[-\frac{1}{2} \left(\frac{x'(x, y)}{\lambda_x \tau_x} \right)^2 - \frac{1}{2} \left(\frac{y'(x, y)}{\lambda_y \tau_y} \right)^2 \right] \\ \text{if } x'(x, y) \geq 0 \text{ and } y'(x, y) \geq 0, \end{cases}$$

where the normalization constant is:

$$\mathcal{N} = \frac{2}{\pi \lambda_x \lambda_y (1 + \tau_x)(1 + \tau_y)}.$$

We start by estimating the means ($\langle x \rangle, \langle y \rangle$), standard-deviations (σ_x, σ_y), skewnesses (γ_x, γ_y) and the correlation coefficient (ρ) of the posterior. We detail how we estimate these moments in Appendix F.2. These moments can be expressed as a function of the BSN's parameters ($x_0, y_0, \lambda_x, \lambda_y, \tau_x, \tau_y, \theta$):

$$\langle x \rangle = \sqrt{\frac{2}{\pi}} \left[\lambda_x (\tau_x - 1) \cos \theta - \lambda_y (\tau_y - 1) \sin \theta \right] + x_0 \quad (\text{F.4})$$

$$\langle y \rangle = \sqrt{\frac{2}{\pi}} \left[\lambda_x (\tau_x - 1) \sin \theta + \lambda_y (\tau_y - 1) \cos \theta \right] + y_0 \quad (\text{F.5})$$

$$\sigma_x^2 = \lambda_x^2 B(\tau_x) \cos^2 \theta + \lambda_y^2 B(\tau_y) \sin^2 \theta \quad (\text{F.6})$$

$$\sigma_y^2 = \lambda_x^2 B(\tau_x) \sin^2 \theta + \lambda_y^2 B(\tau_y) \cos^2 \theta \quad (\text{F.7})$$

$$\rho \sigma_x \sigma_y = \left(\lambda_x^2 B(\tau_x) - \lambda_y^2 B(\tau_y) \right) \cos \theta \sin \theta \quad (\text{F.8})$$

$$\gamma_x \sigma_x^3 = \sqrt{\frac{2}{\pi}} \left(\lambda_x^3 C(\tau_x) \cos^3 \theta - \lambda_y^3 C(\tau_y) \sin^3 \theta \right) \quad (\text{F.9})$$

$$\gamma_y \sigma_y^3 = \sqrt{\frac{2}{\pi}} \left(\lambda_x^3 C(\tau_x) \sin^3 \theta + \lambda_y^3 C(\tau_y) \cos^3 \theta \right), \quad (\text{F.10})$$

with:

$$B(\tau) = \left(1 - \frac{2}{\pi} \right) (\tau - 1)^2 + \tau \quad (\text{F.11})$$

$$C(\tau) = \left[\left(\frac{4}{\pi} - 1 \right) \tau^2 + \left(3 - \frac{8}{\pi} \right) \tau + \frac{4}{\pi} - 1 \right] (\tau - 1). \quad (\text{F.12})$$

We then simply need to solve the system of Eqs. (F.4)–(F.10). To do that, we first solve θ :

$$\theta = \frac{1}{2} \arctan \left(\frac{2\rho\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2} \right). \quad (\text{F.13})$$

We then solve numerically for τ_x and τ_y , independently, from the two equations:

$$\begin{aligned} (\text{F.1}) \quad \frac{C(\tau_x)}{B(\tau_x)^{3/2}} &= \sqrt{\frac{\pi}{2}} \frac{\gamma_x \sigma_x^3 \cos^3 \theta + \gamma_x \sigma_x^3 \sin^3 \theta}{\cos^6 \theta + \sin^6 \theta} \\ &\times \left(\frac{\cos^2 \theta - \sin^2 \theta}{\sigma_x^2 \cos^2 \theta - \sigma_y^2 \sin^2 \theta} \right)^{3/2} \end{aligned} \quad (\text{F.14})$$

$$\begin{aligned} \frac{C(\tau_y)}{B(\tau_y)^{3/2}} &= \sqrt{\frac{\pi}{2}} \frac{\gamma_y \sigma_y^3 \cos^3 \theta - \gamma_x \sigma_x^3 \sin^3 \theta}{\cos^6 \theta + \sin^6 \theta} \\ &\times \left(\frac{\cos^2 \theta - \sin^2 \theta}{\sigma_y^2 \cos^2 \theta - \sigma_x^2 \sin^2 \theta} \right)^{3/2}. \end{aligned} \quad (\text{F.15})$$

We derive the remaining parameters, using the following equations:

$$\lambda_x = \sqrt{\frac{1}{B(\tau_x)} \frac{\sigma_x^2 \cos^2 \theta - \sigma_y^2 \sin^2 \theta}{\cos^2 \theta - \sin^2 \theta}} \quad (\text{F.16})$$

$$\lambda_y = \sqrt{\frac{1}{B(\tau_y)} \frac{\sigma_y^2 \cos^2 \theta - \sigma_x^2 \sin^2 \theta}{\cos^2 \theta - \sin^2 \theta}} \quad (\text{F.17})$$

$$x_0 = \langle x \rangle - \sqrt{\frac{2}{\pi}} \left[\lambda_x (\tau_x - 1) \cos \theta - \lambda_y (\tau_y - 1) \sin \theta \right] \quad (\text{F.18})$$

$$y_0 = \langle y \rangle - \sqrt{\frac{2}{\pi}} \left[\lambda_x (\tau_x - 1) \sin \theta + \lambda_y (\tau_y - 1) \cos \theta \right]. \quad (\text{F.19})$$

F.2. Robust estimate of the posterior distribution moments

We estimate the moments of Eqs. (F.4)–(F.10) numerically, from the marginalized posterior. Some outliers can be present due to incomplete burn-in removal, requiring robust estimators. We choose M-estimators (Mosteller & Tukey 1977), which are a generalization of maximum-likelihood estimators. They have been popularized in astrophysics by Beers et al. (1990). Location and scale M-estimators, using Tukey's biweight loss function were presented by Lax (1975). Posing $u_i = (x_i - l_x)/(c \times s_x)$, where $l_x = \text{med}(X)$ is the median of the sample $X = \{x_i\}_{i=1\dots N}$, $s_x = 1.4826 \times \text{med}(|X - \text{med}(X)|)$ is the median absolute deviation (MAD) of X , and c is a tuning parameter, the M-estimator of the mean is:

$$\hat{\mu}(X) = l_x + \frac{\sum_{|u_i| < 1} (x_i - l_x)(1 - u_i^2)^2}{\sum_{|u_i| < 1} (1 - u_i^2)^2}, \quad (\text{F.20})$$

and for the variance:

$$\hat{V}(X) = \frac{N \times \sum_{|u_i| < 1} (x_i - l_x)^2 (1 - u_i^2)^4}{\left[\sum_{|u_i| < 1} (1 - u_i^2)(1 - 5u_i^2) \right] \left[\sum_{|u_i| < 1} (1 - u_i^2)(1 - 5u_i^2) - 1 \right]}. \quad (\text{F.21})$$

Considering a second sample $Y = \{y_i\}_{i=1\dots N}$, an M-estimator of the covariance is presented by Mosteller & Tukey (1977). Posing $v_i = (y_i - l_y)/(c \times s_y)$, the covariance can be estimated as:

$$\hat{V}(X, Y) = \frac{N \times \sum_{|u_i| < 1, |v_i| < 1} (x_i - l_x)(1 - u_i^2)^2 (y_i - l_y)(1 - v_i^2)^2}{\left[\sum_{|u_i| < 1} (1 - u_i^2)(1 - 5u_i^2) \right] \left[\sum_{|v_i| < 1} (1 - v_i^2)(1 - 5v_i^2) \right]}. \quad (\text{F.22})$$