$$\boldsymbol{X}' = \begin{pmatrix} x'(x,y) \\ y'(x,y) \end{pmatrix} = \mathbf{R}^T(\boldsymbol{X}-\boldsymbol{X}_0) = \begin{pmatrix} (x-x_0)\cos\theta + (y-y_0)\sin\theta \\ -(x-x_0)\sin\theta + (y-y_0)\cos\theta \end{pmatrix}, \quad \text{(F.1)}$$

the BSN PDF can be expressed as:

$$p(x,y) = \mathcal{N} \times \begin{cases} \exp\left[-\frac{1}{2}\left(\frac{x'(x,y)}{\lambda_x}\right)^2 - \frac{1}{2}\left(\frac{y'(x,y)}{\lambda_y}\right)^2\right] \\ \text{if } x'(x,y) < 0 \text{ and } y'(x,y) < 0 \\ \exp\left[-\frac{1}{2}\left(\frac{x'(x,y)}{\lambda_x\tau_x}\right)^2 - \frac{1}{2}\left(\frac{y'(x,y)}{\lambda_y}\right)^2\right] \\ \text{if } x'(x,y) \geq 0 \text{ and } y'(x,y) < 0 \\ \exp\left[-\frac{1}{2}\left(\frac{x'(x,y)}{\lambda_x}\right)^2 - \frac{1}{2}\left(\frac{y'(x,y)}{\lambda_y\tau_y}\right)^2\right] \\ \text{if } x'(x,y) < 0 \text{ and } y'(x,y) \geq 0 \\ \exp\left[-\frac{1}{2}\left(\frac{x'(x,y)}{\lambda_x\tau_x}\right)^2 - \frac{1}{2}\left(\frac{y'(x,y)}{\lambda_y\tau_y}\right)^2\right] \\ \text{if } x'(x,y) \geq 0 \text{ and } y'(x,y) \geq 0, \end{cases} \quad \text{(F.2)}$$

where the normalization constant is:

$$\mathcal{N} = \frac{2}{\pi\lambda_x\lambda_y(1+\tau_x)(1+\tau_y)}. \quad \text{(F.3)}$$

We start by estimating the means $(\langle x\rangle, \langle y\rangle)$, standard-deviations (σ_x, σ_y), skewnesses (γ_x, γ_y) and the correlation coefficient (ρ) of the posterior. We detail how we estimate these moments in Appendix F.2. These moments can be expressed as a function of the BSN's parameters $(x_0, y_0, \lambda_x, \lambda_y, \tau_x, \tau_y, \theta)$:

$$\langle x\rangle = \sqrt{\frac{2}{\pi}}\left[\lambda_x(\tau_x-1)\cos\theta - \lambda_y(\tau_y-1)\sin\theta\right] + x_0 \quad \text{(F.4)}$$

$$\langle y\rangle = \sqrt{\frac{2}{\pi}}\left[\lambda_x(\tau_x-1)\sin\theta + \lambda_y(\tau_y-1)\cos\theta\right] + y_0 \quad \text{(F.5)}$$

$$\sigma_x^2 = \lambda_x^2 B(\tau_x)\cos^2\theta + \lambda_y^2 B(\tau_y)\sin^2\theta \quad \text{(F.6)}$$

$$\sigma_y^2 = \lambda_x^2 B(\tau_x)\sin^2\theta + \lambda_y^2 B(\tau_y)\cos^2\theta \quad \text{(F.7)}$$

$$\rho\sigma_x\sigma_y = \left(\lambda_x^2 B(\tau_x) - \lambda_y^2 B(\tau_y)\right)\cos\theta\sin\theta \quad \text{(F.8)}$$

$$\gamma_x\sigma_x^3 = \sqrt{\frac{2}{\pi}}\left(\lambda_x^3 C(\tau_x)\cos^3\theta - \lambda_y^3 C(\tau_y)\sin^3\theta\right) \quad \text{(F.9)}$$

$$\gamma_y\sigma_y^3 = \sqrt{\frac{2}{\pi}}\left(\lambda_x^3 C(\tau_x)\sin^3\theta + \lambda_y^3 C(\tau_y)\cos^3\theta\right), \quad \text{(F.10)}$$

with:

$$B(\tau) = \left(1-\frac{2}{\pi}\right)(\tau-1)^2 + \tau \quad \text{(F.11)}$$

$$C(\tau) = \left[\left(\frac{4}{\pi}-1\right)\tau^2 + \left(3-\frac{8}{\pi}\right)\tau + \frac{4}{\pi} - 1\right](\tau-1). \quad \text{(F.12)}$$

We then simply need to solve the system of Eqs. (F.4)–(F.10). To do that, we first solve θ:

$$\theta = \frac{1}{2}\arctan\left(\frac{2\rho\sigma_x\sigma_y}{\sigma_x^2-\sigma_y^2}\right). \quad \text{(F.13)}$$

We then solve numerically for τ_x and τ_y, independently, from the two equations:

$$\frac{C(\tau_x)}{B(\tau_x)^{3/2}} = \sqrt{\frac{\pi}{2}}\frac{\gamma_x\sigma_x^3\cos^3\theta + \gamma_x\sigma_x^3\sin^3\theta}{\cos^6\theta+\sin^6\theta} \times \left(\frac{\cos^2\theta-\sin^2\theta}{\sigma_x^2\cos^2\theta-\sigma_y^2\sin^2\theta}\right)^{3/2} \quad \text{(F.14)}$$

$$\frac{C(\tau_y)}{B(\tau_y)^{3/2}} = \sqrt{\frac{\pi}{2}}\frac{\gamma_y\sigma_y^3\cos^3\theta - \gamma_x\sigma_x^3\sin^3\theta}{\cos^6\theta+\sin^6\theta} \times \left(\frac{\cos^2\theta-\sin^2\theta}{\sigma_y^2\cos^2\theta-\sigma_x^2\sin^2\theta}\right)^{3/2}. \quad \text{(F.15)}$$

We derive the remaining parameters, using the following equations:

$$\lambda_x = \sqrt{\frac{1}{B(\tau_x)}\frac{\sigma_x^2\cos^2\theta-\sigma_y^2\sin^2\theta}{\cos^2\theta-\sin^2\theta}} \quad \text{(F.16)}$$

$$\lambda_y = \sqrt{\frac{1}{B(\tau_y)}\frac{\sigma_y^2\cos^2\theta-\sigma_x^2\sin^2\theta}{\cos^2\theta-\sin^2\theta}} \quad \text{(F.17)}$$

$$x_0 = \langle x\rangle - \sqrt{\frac{2}{\pi}}\left[\lambda_x(\tau_x-1)\cos\theta - \lambda_y(\tau_y-1)\sin\theta\right] \quad \text{(F.18)}$$

$$y_0 = \langle y\rangle - \sqrt{\frac{2}{\pi}}\left[\lambda_x(\tau_x-1)\sin\theta + \lambda_y(\tau_y-1)\cos\theta\right]. \quad \text{(F.19)}$$

F.2. Robust estimate of the posterior distribution moments

We estimate the moments of Eqs. (F.4)–(F.10) numerically, from the marginalized posterior. Some outliers can be present due to incomplete burn-in removal, requiring robust estimators. We choose M-estimators (Mosteller & Tukey 1977), which are a generalization of maximum-likelihood estimators. They have been popularized in astrophysics by Beers et al. (1990). Location and scale M-estimators, using Tukey's biweight loss function were presented by Lax (1975). Posing $u_i = (x_i - l_x)/(c \times s_x)$, where $l_x = \text{med}(X)$ is the median of the sample $X = \{x_i\}_{i=1...N}$, $s_x = 1.4826 \times \text{med}(|X - \text{med}(X)|)$ is the median absolute deviation (MAD) of X, and c is a tuning parameter, the M-estimator of the mean is:

$$\hat{\mu}(X) = l_x + \frac{\sum_{|u_i|<1}(x_i-l_x)(1-u_i^2)^2}{\sum_{|u_i|<1}(1-u_i^2)^2}, \quad \text{(F.20)}$$

and for the variance:

$$\hat{V}(X) = \frac{N \times \sum_{|u_i|<1}(x_i-l_x)^2(1-u_i^2)^4}{\left[\sum_{|u_i|<1}(1-u_i^2)(1-5u_i^2)\right]\left[\sum_{|u_i|<1}(1-u_i^2)(1-5u_i^2)-1\right]}. \quad \text{(F.21)}$$

Considering a second sample $Y = \{y_i\}_{i=1...N}$, an M-estimator of the covariance is presented by Mosteller & Tukey (1977). Posing $v_i = (y_i - l_y)/(c \times s_y)$, the covariance can be estimated as:

$$\hat{V}(X,Y) = \frac{N \times \sum_{|u_i|<1,|v_i|<1}(x_i-l_x)(1-u_i^2)^2(y_i-l_y)(1-v_i^2)^2}{\left[\sum_{|u_i|<1}(1-u_i^2)(1-5u_i^2)\right]\left[\sum_{|v_i|<1}(1-v_i^2)(1-5v_i^2)\right]}. \quad \text{(F.22)}$$