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Aplanatic telescopes based on Schwarzschild optical configuration, from grazing incidence Wolter-like X-ray optics to Cherenkov two-mirrors normal incidence telescopes: a review

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ABSTRACT

At the beginning of XX century Karl Schwarzschild defined a method to design large-field aplanatic telescopes based on the use of two aspheric mirrors. The approach was then refined by Couder (1926) who, in order to correct for the astigmatic aberration, introduced a curvature of the focal plane. By the way, the realization of normal-incidence telescopes implementing the Schwarzschild aplanatic configuration has been historically limited by the lack of technological solutions to manufacture and test aspheric mirrors. On the other hand, the Schwarzschild solution was recovered for the realization of coma-free X-ray grazing incidence optics. Wolter-like grazing incidence systems are indeed free of spherical aberration, but still suffer from coma and higher order aberrations degrading the imaging capability for off-axis sources. The application of the Schwarzschild's solution to X-ray optics allowed Wolter to define an optical system that exactly obeys the Abbe sine condition, eliminating coma completely. Therefore these systems are named Wolter-Schwarzschild telescopes and have been used to implement wide-field X-ray telescopes like the ROSAT WFC and the SOHO X-ray telescope. Starting from this approach, a new class of X-ray optical system was proposed by Burrows, Burg and Giacconi assuming polynomials numerically optimized to get a flat field of view response and applied by Conconi to the wide field x-ray telescope (WFXT) design. The Schwarzschild-Couder solution has been recently re-discovered for the application to normal-incidence Cherenkov telescopes, thanks to the suggestion by Vassiliev and collaborators. The Italian Institute for Astrophysics (INAF) realized the first Cherenkov telescope based on the polynomial variation of the Schwarzschild configuration (the so-called ASTRI telescope). Its optical qualification was successfully completed in 2016, demonstrating the suitability of the Schwarzschild-like configuration for the Cherenkov astronomy requirements. Moreover, other Cherenkov telescopes based on Schwarzschild-Couder solutions are currently being completed at Fred Lawrence Whipple Observatory in southern Arizona, USA and at the Observatoire de Paris-Meudon. In this paper we will review the Karl Schwarzschild solution and its application to grazing incidence and Cherenkov telescopes, discussing on future applications in the field of high-energy astronomy.

KEYWORDS: aplanatic telescopes, optical design, grazing incidence X-ray telescopes, Wolter-Schwarzschild design, Schwarzschild-Couder design, Cherenkov Telescopes

1. INTRODUCTION

The most known achievements of KS came from his studies in astrophysics. He gave the exact solution of Einstein's field equation and formulated the metrics used to describe non-rotating black holes. By the way, the work of Karl Schwarzschild (1873-1916, hereafter KS) on the theoretical design of telescopes based on reflecting mirrors is of fundamental interest; let's think that the 1911 Nobel Prize in medicine A. Gullstrand's work was the analogous of KS optical theory but with specific application to human eye.

To understand the impact of his work we should consider that, at that time, the reflecting telescope optical theory was stacking since almost 3 centuries.

The configurations commonly used for reflecting telescopes were Newton, Gregory and Cassegrain, all based on 1634 Descartes's theory of perfect imagery on-axis. When in 1905 KS published three papers giving the complete theory of imagery in the whole field of view (FoV) for any axial symmetric telescope.

We should point out that such a prolonged lack of improvements in optics development was mainly due to the absence of technological solution for eyepiece realization. The available eyepieces were small in size and the telescope focal lengths typically long, this means that the covered FoV were limited and the interest about off-axis aberration theory was consequently poor.

At the end of 19th century, the development of photography was pushing to have optics with good performances across wider FoV, and the study of aberrations as field angle functions become systematic. Here, we would like to cite just two of the main discoveries in aberration theory of that time (more detailed chronology can be found in [1]).

In 1856 Seidel obtained the equations for the optical aberrations (S_I: spherical aberration, S_{II}: coma, S_{III}: astigmatism, S_{IV}: field curvature, S_V: distortion) allowing treating the off-axis imagery degradation mathematically.

In 1873 Abbe discovered the *sine condition* for freedom from coma optics and defined *aplanatic* the systems whereby both spherical and coma are removed. The Abbe *sine condition* can be expressed as: *the surface (called principal surface) generated by the points of intersection between the directions of incoming and last reflection rays should be a sphere.*

In 1905 KS published three optical papers, in the second one he made the statement: *For any geometry (with sufficient spacing between the mirrors), 2 aspheric mirrors allow the correction of 2 Seidel conditions, i.e.* S_I, S_{II} *to give an aplanatic telescope* [2]. This theorem defines a general method to design aplanatic telescopes.

Since its formulation, the KS's condition influenced astronomical optics design. It became a standard for optical telescopes with the Ritchey-Chretien design. In 1952 Wolter adopted it for x-ray optics design [3] and in 2007 it was proposed by Vassiliev [4] to improve the performance of ground based Cherenkov telescopes.

In this paper we will review the impact that KS's work had on the optical design of reflecting telescopes. In particular, in section 2 we will outline the principles of the KS's solution while in the following sections we will report on the applications of KS's theory to: classical normal incidence telescopes (section 3); X-ray grazing incidence optics (section 4) and ground based Cherenkov telescope (section 5).

2. THE SCHWARZSCHILD SOLUTION

The KS's method for designing aplanatic systems derives directly form the analytical solution of the Seidel's equations expressing spherical and coma optical aberrations. To get a solution for these equations KS develop an ad-hoc formalism (this is the same procedure he later applied to solve Einstein's field equations).

The first step of KS's method is the definition of the radial profile of the optical surface as a polynomial series:

$$z=\frac{1}{2r}y^2+\frac{1}{8r^3}(1+b_s)y^4+\frac{1}{16r^5}(1+b_s)^2y^6+...$$

where r is the radius of curvature, y the distance from the optical axis and b_s the conic constant. Trimming this series at the second term, the surface results to be aspheric with profile depending on the b_s value. The novelty in this formalism is that it permits to describe all the conic sections with the same equation, just changing the value of a parameter b_s, called conic constant. This formalism, introduced by KS, is today a standard for optics description. The relation between the conic constant value and the generated surface profile is explicitly reported in Table 1.

Radial Profile	**Hyperbola**	**Parabola**	**Ellipse**	**Sphere**	**Spheroid**
b_s **Value**	< -1	-1	$0> b_s > -1$	0	$b_s > 0$

Table 1: Relation between the values of the conic constant and the generated conic sections.

The second step is to express the Seidel's equations for spherical and coma aberration as functions of the b_s parameter. Hereafter we will refer to the conical constant of the primary mirror as bs_1 and to the conical constant of the secondary mirror as bs_2. Explicit formulas for the aberrations equations can be found in [5].

The conclusive one is to find a relation between the design parameters zeroing S_I and S_{II}*. This result in a set of conditions completely describing the system as function of three independent variables: the system focal length f, the secondary mirror magnification m_2, and the mirrors diameter ratio ε (D_2/D_1). Using the formalism expressed by Wyman in [6], an aplanatic system would be obtained setting the following configuration relations: radii of curvature of the two mirrors $r_1=2f/m_2$ and $r_2=2\varepsilon f/(m_2+1)$; distance between the vertex of the mirrors of $(f/m_2)(1+\varepsilon)$; back focal length εf and system focal ratio $F= m_2F_1$. In addition to this specific configuration, to give an aplanatic solution, the conic constants of the two mirrors would have the following values:

$$b_{S1}=-1+\frac{2\varepsilon}{m_2^2(1+\varepsilon)} \qquad b_{S2}=-\left(\frac{m_2-1}{m_2+1}\right)^2-\frac{2m_2}{(1+\varepsilon)(m_2+1)^3}$$

The chain of relations just reported can be used to describe all aplanatic systems. The same solution can be expressed with different formalism as in [7].

* The first aim of KS was to remove 4 Seidel aberrations, but the resulting solution was impractical since the obtained telescope was completely obscured.

3. CASE 1: OPTICAL TELESCOPES

As previously cited, the configurations available for optical telescopes at the time of KS's theory formulation were based on the Descartes theory to give optimal imagery for on-axis sources. They were the Newtonian telescope, consisting in a spherical mirror followed by a flat folding mirror, the Gregorian and the Cassegrain. Both the last two solutions use two aspheric mirrors, a parabolic primary and a magnifying secondary (respectively elliptical and hyperbolical) with the focal plane positioned behind the primary mirror vertex. For these configurations the resulting corrected FoV is limited and the image quality degrades fast with off-axis angle.

With the KS's theorem a new class of telescopes can be defined. The Schwarzschild configuration (S) is defined a sub-set of the aplanatic solutions with all ε, f and $m_2 <0$ and with $-1< m_2 <0$. This means that the focal plane is placed between the two mirrors and the secondary mirror de-magnifies the image. We can consider the obtained system as a special form of Cassegrain telescope with a concave secondary mirror necessary to cover a wide FoV.

KS himself proposed a design for a telescope aimed to be compact but with large FoV to fit the astronomical photography's demands. The proposed system had *f-ratio F/3,* with the primary mirror's *f-ratio* being *F/7.5* and m_2=-0.4 (S, Figure 1 – Left). This specific set of parameters was chosen by KS in a sub-sub-set of parameters giving finite value of S_{III} and S_{IV} that, in combination, flatten the field. The result was an optical design with large aperture, capable to maintain a proper optical quality across a wide FoV (~ 12'' on a field of ±1.4°). Unfortunately the conical constants of the correlated mirrors had serious manufacturing difficulties. The primary mirror was a very steep hyperbola and the secondary one was an oblate spheroid. This telescope configuration never found practical implementations.

In 1926 Couder proposed two modifications to the KS's configuration: he removed the condition of flat field and set S_{III}=0 to correct the astigmatism. This variation is known as Schwarzschild-Couder (SC) configuration. The SC solution improved the imagery capability on-axis thanks to astigmatism removal. The drawback is that the distance between the two mirrors should be equal to *2f.* Hence a SC system results to be less compact than a S one. Couder proposed two telescopes based on SC solution. A first one maintaining the *F/3* ratio originally proposed by KS (SC telescope) and, later on, a second longer one with *F/6.5* (C telescope). By the way, the corresponding conic constants were once again out of the manufacturing capability at that time. We should point out that both the S and SC solutions present higher order aberrations, but the SC starts from better results on-axis (thanks to the adoption of the curved focal plane).

In 1922 Chretien and Ritchey developed a solution implementing the KS's aplanats theory to the Cassegrain configuration. The result was the Ritchey-Chretien configuration (RC, Figure 1 – Right) that soon became the standard for modern aplanatic telescopes.

The typical parameters used in the modern RC are reported in Table 2 with the S, C and the SC ones. As can be seen the progression of the conical constants tends to more easily manufacturing mirrors. The steep hyperbola of the S configuration primary mirror became almost a parabola while the challenging oblate spheroid of the S configuration secondary mirror is, in RC, replaced by a hyperbola. Moreover the RC configuration presents an optimal focus position to allocate focal plane instrumentation and results more compact than the Gregory configuration (about 45% [8]).

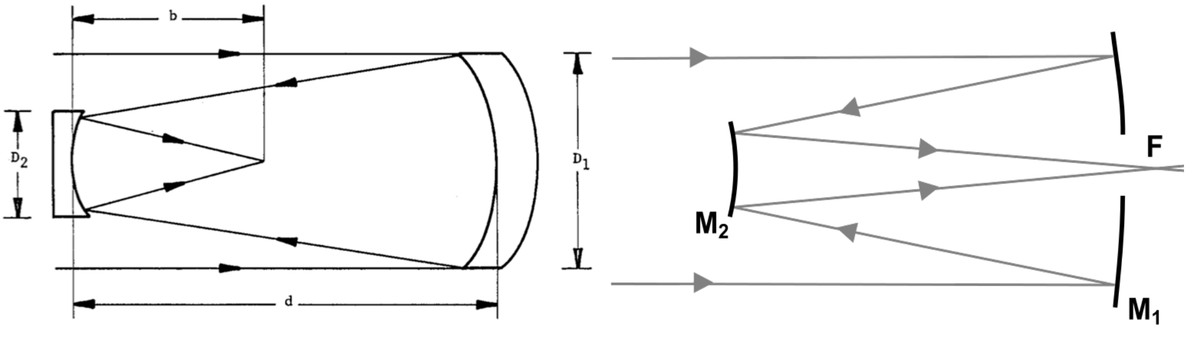

Figure 1: Left - Geometry of Schwarzschild's aplanatic telescope as reported in [6]. Right – Scheme of the Ritchey-Chretien configuration based on KS's aplanatic solution.

	F/#	m_2	b_{s1}	b_{s2}
S	*f*/3.0	-0.4	-13.500	+1.963
SC	*f*/3.0	-0.3	-14.241	-0.554
C	*f*/6.5	-2.5	-1.147	-7.605
RC	*f*/2.0	-4.0	-1.036	-3.160

Table 2: Parameters of aplanats configurations. For S, SC, C and RC configuration we referred to [1], for SC to [6].

4. CASE 2: X-RAY TELESCOPES

The starting point for X-ray optics design is 1923, when Compton demonstrated that it is possible to reflect X-rays. The conditions to have total reflection are: i) grazing incidence reflection; ii) smooth surface. The first of these two conditions directly influences the optical design.

Its physical explanation can be deduced using the refractive index definition in the Snell's law, the resulting expression for the critical angle then become $\alpha_t = k\rho^{1/2}/E$. Where ρ is the density of the reflective material and E the energy of the X-ray. The obtained relation shows that the critical angle decreases with energy. For X-rays (λ of few Å), even when heavy elements (like Gold) are used as reflecting layers, the critical angle for total reflection is in general smaller than ~1 degree. The second point deals with the change from geometrical reflection to scattering regime, which depends on the ratio between the incidence radiation wavelength and the surface irregularities at the corresponding spatial frequency. The scattering effect in X-ray optics is not subject of this work, a detailed discussion can be found in [9].

The first studies on possible optical design to get good imagery in grazing incidence regime were addressed to the realization of X-ray microscopes aiming to work at better resolution than visible light ones.

By the way, these studies were inconclusive because they started from an optical configuration similar to the one used for normal reflection [10]. As explained in detail in [11], using mirror generated as revolution surface around the optical axis to reflect light in grazing incidence, is analogous to use a mirror designed to focus on-axis at ~89° off-axis. In this configuration the focus will move away from the theoretical one depending on the mirror aperture and on the incidence angle. The spherical aberration goes with the square of the off-axis angle, so it can be in principle controlled reducing the FoV. By the way, the reflected image will be dramatically astigmatic, since the ratio between the position of the foci, relative to the directions parallel and perpendicular to the incoming ray trajectory, will be $1/\sin^2\theta$ and even considering a ~ 1 degree incidence angle, the ratio between the foci's position results bigger than > 10^3.

Since working in grazing incidence, intrinsically means to have the incidence angle in the plane formed by the incoming ray's direction and surface's optical axis plane << than the one in the perpendicular plane, there is no way to avoid the shift between the position of the two foci and the resulting huge astigmatism.

In 1948 Kirkpatrick and Baez [12] proposed a clever configuration (KB) to focus X-rays working on the astigmatism removal. They used two cylindrical mirrors to focus X-rays in orthogonal directions with two consecutive reflections. Separating the reflection in the two planes allows treating properly the two different incidence angles compensating for their asymmetry.

The resulting configuration is free from astigmatism but suffers from anamorphotism and coma. The latter can be reduced with a double KB compound arranged to satisfy the Abbe *sine condition*.

4.1 Wolter and Wolter-Schwarzschild optics

The biggest leap foreword in X-ray optics design has to be addressed to Wolter, who in 1952 [13] proposed a proper solution for X-ray total reflection. The biggest innovation in the Wolter design was to imagine a cylindrical close optics, allowing the overlapping of the symmetry axis of the reflecting surface to the rays' incoming direction. This innovation permits to work at small angle off-axis recovering quite well for optical aberrations. The drawback is that the collecting area of a cylindrical optic is small, corresponding just to the thin corona it projects on the plane perpendicular to the optical axes.

The Wolter's solution, which he proposed in 3 possible configurations, is based on coma removal by means of a double reflection on two co-axial, co-focal conics whose principal surface approximates a sphere. The approximation works under the condition $L \ll f$. Where L is the distance of the point of reflection from the intersection plane and f the system focal length.

Soon after Wolter found a way to solve the Abbe condition without any approximation and hence completely removing coma [3].

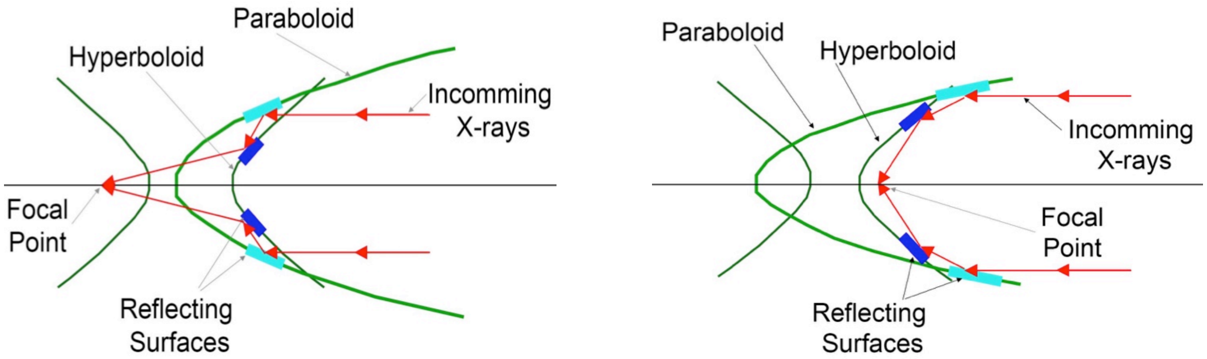

Figure 2: Left - Wolter I configuration. The incoming rays undergo two subsequent reflections. The first reflection is on a parabolic surface. The second one is on a hyperbolic surface co-axial and co-focal to the parabolic one. Right - Wolter II configuration. The incoming rays undergo the first reflection on a parabolic surface and the second one on the outer surface of a hyperbolic one.

Since this solution is the exact application of KS's aplanats theorem to grazing incidence case, the resulting configuration was called Wolter-Schwarzschild (WS). The effect of WS correction on W optics consists in a small modification of the conic constants of the surfaces (with a typical difference in the sagitta of fraction of microns) bringing the principal surface to a perfect sphere.

It was 1960 when Giacconi and Rossi proposed to implement the Wolter configuration in the design of focusing X-ray telescopes, opening the road to X-ray imaging astronomy. Of the three configurations proposed by Wolter, two were implemented in X-ray satellites. Wolter type I optics, the shortest of the Wolter systems, is composed by a parabola and a hyperbola both used for inside reflection (Figure 2 - Left). Thanks to its typical incidence angle smaller than 1 degree, this system became the baseline for hard X-ray (E >1 KeV) telescopes and was implemented in the majority of X-ray observatories (EINSTEIN [14] & [15], EXOSAT [16], ROSAT [17], ASCA [18] & [19], Suzaku [20], Chandra [21], [22], XMM-Newton [23], JET-X telescopes of the SWIFT mission [24] and [25]). Wolter type II (Figure 2 - Right) optics uses the secondary surface (hyperboloid) for external reflection. The resulting incidence angle is larger (> 10 deg) and its typical application is in soft X-ray and EUV telescopes.

Wolter-Schwarzschild's configuration is instead used to take under control aberrations when a large field of view is required and typically for UV and XUV surveys, since the WS correction is more effective when bigger incidence angles are involved. Examples of WS optics implementations are: the Wide Field Camera optics of ROSAT [26] capable to maintain the angular resolution better than 2 arcmin along the whole 5° FoV; the Extreme-Ultraviolet experiment Explorer [27] & [28] with 2 WS type I and 2 WS type II telescope and the WS type II telescope on board of the solar SOHO mission [29].

4.2 X-ray polynomial optics

Once the technological solutions to realize optics with profile accuracy of fractions of microns were demonstrated (and this is the case of both W and WS configurations), other kind of profiles can be taken into account for the optical surfaces. A new leap forward in X-ray telescope design was represented by the introduction of polynomial optics. The KS's solution was based on reflecting surfaces defined as conics, and this is a limit given by the necessity of KS' time to set up a system that can be described and treated analytically. Nowadays, it is possible to take in consideration the introduction of higher order polynomials for the reflecting surfaces definition, and then to let a software simulation choose their coefficient to minimize the effects of higher order aberrations on image degradation along the desired range of field angles. Burrows, Burg and Giacconi firstly proposed this approach in 1992 [30] to widen X-ray telescope fields of view. They firstly defined the two reflecting surfaces as:

$$\frac{\rho_1^2}{\rho_0^2} = \sum_{i=1}^{n1} a_i \left(\frac{z_1}{\rho_0}\right)^i$$

where ρ is the distance from the two reflecting surfaces intersection plane along the optical axis and ρ_0 the focal length ($a_0 = 1$ for both surfaces). Then fixed the polynomial coefficients minimizing a merit function M, given as:

$$M = \sqrt{\frac{\int \theta \sigma^2 \, d\theta}{\int \theta \sigma \, d\theta}}$$

that weights the dimension of the PSF σ along the off-axis angle θ. The imaging capability of such an optical system is degraded on-axis (because higher terms are added), has its best performance at $2^{1/2}\theta_{max}$, and a mean reduction of the PSF dimension on the FoV of a factor ~2.

The effect of the polynomial optimization can be summarized as a flattening of the angular resolution among the FoV obtained allowing a small on-axis degradation of the imaging capability. An interesting point about the on-axis degradation, intrinsic to this method, is that the on-axis perfect imagery, given by the KS's analytics solutions, is never reachable. The best possible result will be always be at least the one permitted by manufacturing errors. Moreover, since the effects of manufacturing errors and optical design aberration should be added quadratically, the opportunity of improving the mean angular resolution along the considered FoV is significant. This kind of optimization is hence, always applicable.

As a possible application for the polynomial solution Burrows, Burg and Giacconi proposed a redesign of the CHANDRA X-ray telescope, aiming at being able to maintain the angular resolution (Half Energy Width - HEW) below 2.5'' on a FoV of 1° (Figure 3). Conconi et al. have later on adopted the same approach for the design of the Wide Field X-ray Telescope [31] capable to maintain the mean angular resolution below 5'' on a F0V of 1 degree.

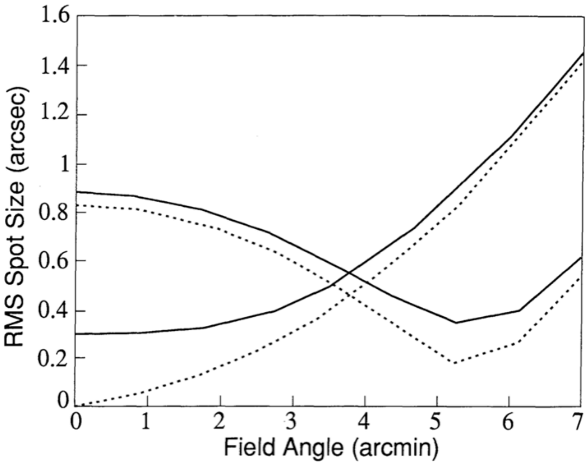

Figure 3: Rms spot size for the outer shell of CHANDRA designed as polynomial and Wolter optics as calculated by Burrows, Burg and Giacconi in [30]. Dotted lines are the given by design, while solid lines take into account a constant 0.5'' degradation due to manufacturing.

5. CASE 3: CHERENKOV TELESCOPES

Cherenkov astronomy is a quite young field of Astronomy. It was 1963 wen Jelley discovered that cosmic ray particles could be detected at ground by the pulses produced by Cherenkov light in liquids [32]. Since that moment efforts were made to realize instrumentation capable to detect such faint and fast pulses.

To understand the optical requirements driving Cherenkov telescopes design, we should start from their scientific target, that is the capability to distinguish the signals from atmospheric cascades induced by high-energy photons emitted by astronomical γ-sources from the night sky background photons induced by hadrons showers. This selection is possible studying the details of the showers' images on the basis of Hillas parameters [33].

The resolution required to discriminate the nature of the shower is an inverse function of the source energy. For sources emitting in the sub GeV band a resolution better than 0.05° is required, for TeV energy range a resolution of tenth of degrees is appropriate. These angular resolution values are quite loose with respect to the ones typical for visible light telescopes and in principle do not represent a challenge in Cherenkov telescopes optics design.

A second fundamental requirement is that, to avoid contamination from the background, the exposure time should be of the order of time of a Cherenkov pulse, i. e. of nanoseconds. This second requirement has strong consequences: limiting the exposure time means that, in Cherenkov astronomy, the desired signal to noise ratio cannot be reached integrating photons in time but just thanks to intrinsic effective area.

By the point of view of optical design, these requirements can be translated as:

1. Ground based Cherenkov telescopes should have large size optics and low absorption factor. The last requirement limits the number of mirrors because the absorption factor scales like $(1-R^n)$, where R is the reflectivity of the mirrors (typically 0.8 for Aluminium coating) and n is the number of reflecting surfaces.
2. To have good estimation of the background and to survey transient phenomena is desirable to have a wide FoV (~ 10 deg is the goal). The dimension of the FoV directly impact on the detector dimension. To avoid an increase of the detector dimension, unbearable both for cost and for the number of individual sensors to operate, a small plate scale is necessary
3. Angular resolution is not a main issue in Cherenkov astronomy, by the way the capability of reconstruction of the photons arrival detection decrease for high aberrated images. To take under control the imagery degradation the *f-ratio* should be increased, and the telescope will result *slow.* Since the plate scale is defined as the inverse of the focal length, this means that a large plate scale would be desirable to take under control the image quality off-axis.

The challenge in Cherenkov telescope design can be addressed to the contradiction contained in items 2, requiring a small plate scale to avoid the spreading of the focal plane instrumentation's dimension, and 3 indicating that a large plate scale should be used to avoid the degradation of the imagery capability off-axis. The only way out is a fine-tuning between plate scale, angular resolution, large field of view and large effective area.

Historically, all IACTs were designed as large apertures prime-focus optical systems, letting the wide filed requirement prevailing on the angular resolution performance. To cover the large area of the reflector they are implemented as segmented mirrors. To be cost-effective, segments are replicated as identical spherical mirrors. Mirrors are then arranged on a paraboloid (like in MAGIC experiment [34]) or in Davies-Cotton (DC) configuration [35] (like in VERITAS [36] and H.E.S.S. I [37]).

The DC is an interesting empirical solution, coming from solar concentrators, based on spherical segments mounted on a parabolic dish and tilted to point the focus of the telescope. This solution takes advantage of the degree of freedom of tessellation to recover for off-axis aberrations. Its performance is similar to a parabolic mirror on-axis and outperforming it off-axis. By the way, this smart empirical solution does not maintain the events isochronicity.

The drawback of this systems, miming a parabolic/aspheric mirror by means of spherical titled segments, is that they should be implemented with large *f-ratio* and resulting large plate scale, to reduce the image degradation due the spherical approximation. Given the large plate scale and requiring a wide FoV implies that the detector should have large area, in the scale of 1 meter. To avoid the excess of sensors multiplicity, only large sensors (Photo Multipliers Tubes - PMTs, with typical diameters of few centimetres) can be used to populate the DC IACTs focal planes, with resolution loss.

For IACTs implemented as single reflectors, the dimension of the focal plane is a criticism. There are almost two key-factors scaling with it: i) the camera weight; ii) the number of sensors to populate its area. Both impact on costs, the first requiring larger and stiffer structures, the second requiring the procurement and management of a great number of sensors and readout channels.

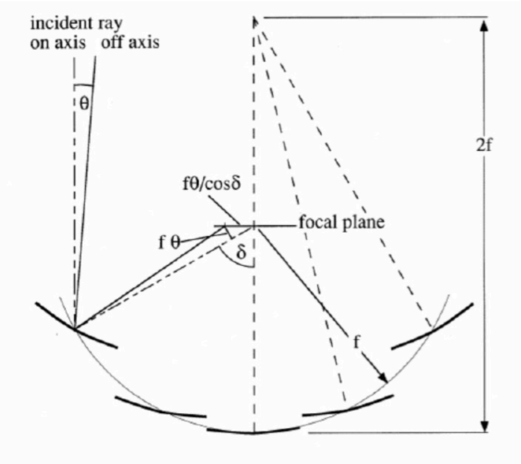

Figure 4: Davies-Cotton configuration: spherical segments with radius of curvature *2f* are arranged on a spheroid with focal length *f* and tilted to concentrate light at the spheroid focus.

5.1 Schwarzschild-Couder optics

In the last decade, the interest about IACTs design arose from the studies for the Cherenkov Telescope Array observatory, foreseeing the installation of tens of telescopes. The CTA observatory will be implemented in two arrays, one in the North and one in the South hemispheres, each composed by three classes Small Size (SST), Medium Size (MST) and Large Size (LST) Telescopes. Different classes of telescopes will be devoted to the observation of different energy bands sources and their designs are hence driven by different requirements.

In the same time, smaller sensors, suitable to populate the IACTs focal plane (like Silicon Photo Multipliers (SiPMs)) became available. SiPMs are more cost-effective and light than PMTs, but cannot be implemented in DC without the addition of concentrators.

These conditions called for having an optical design capable to give good imagery a large FoV, with a small plate scale.

In 2007 Vassiliev et al. [4], proposed a double mirror design for Cherenkov telescopes as demonstrator for the MST class of CTA (MST SC). Considering that the single reflector telescopes are limited by coma (proportional to ϑ/f^2, with ϑ being the field angle) and hence a larger FoV would make the mean optical quality worst, Vassiliev proposed an aplanatic solution. He recovered the KS's method and applied it to Cherenkov telescope design in the SC configuration.

The obtained telescope is the first proposal of a Cherenkov double mirror telescope [38], and represents a big leap forward in Cherenkov telescope design (Figure 5 - Left). The MST SC proposed telescope is capable to maintain the PSF within 3.8 arcmin along a whole filed of view of 8° (Figure 5 - Right), that is about 3 times better than the angular resolution offered by the corresponding MST DC telescope, with primary mirror of the same dimension (~10 m). Moreover, the double mirror solution allows the realization of a more compact system but free from the typical plate scale increase: the extension of the telescope is about 0.5 of the corresponding DC solution and the plate scale results reduced of about ~1/3. This means that the SC solution for the CTA MST foresees a focal plane with linear dimension ~1/3 of the corresponding one for DC configuration. Hence both the number of sensors necessary to populate the same FoV and camera weight would be reduced to ~1/10 (scaling with L^2). SC configuration is then an effective solution to overcome the IACTs focal plane criticism.

The drawback of SC design is the shape of the mirrors. The two mirrors of the MST SC require to be tasselled because of their large size, and being their radial profile aspheric, each segment result in a free-form optics. The availability of technological solutions for manufacturing and characterizing free-form mirrors is a key-point in the implementation of SC configuration. Going back to the starting point, this is what prevent normal incidence SC telescope to be realized since now. Quite obviously, two centuries of technological development advantages Cherenkov new-born applications. For the mirrors realization, the thin glass foils hot-slumping technique is chosen at the more appropriate. It allows realizing lightweight optics with big multiplicity thanks to replica process [39].

The telescope has been placed at Fred Lawrence Whipple Observatory in southern Arizona, USA as part of the prototypal telescopes for CTA medium size class telescopes (MST) and is currently in technological validation phase.

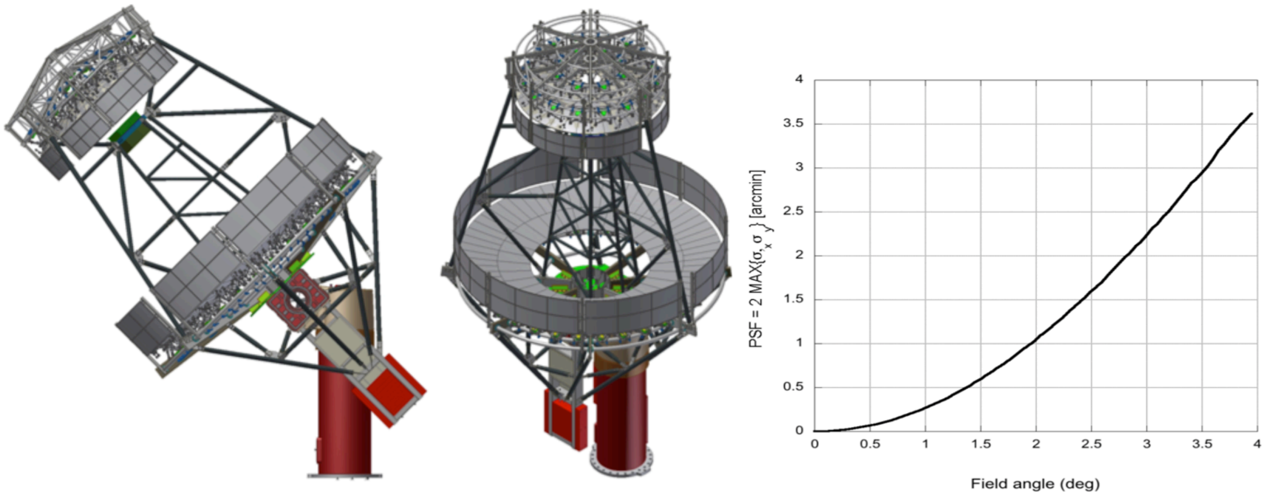

Figure 5: Left - Design proposed for the SC prototypal MST of the CTA observatory (https://www.cta-observatory.org/project/technology/sct/). Right – Angular resolution of the SC MST as function of field angle as reported in [38].

Beyond the proposal of Vassiliev for a SC MST telescope, the SC's configuration resulted to be particularly suitable for the design of CTA SSTs since the small class telescopes require having a larger FoV. In addition to the demonstrator for the MSTs, a SC telescope for the SST class has been realized by a French-English collaboration [40]. The SST SC prototype (called SST GCT) is placed at the Observatoire de Paris – Meudon.

5.2 Cherenkov polynomial optics

As in X-ray telescopes' evolution, also in Cherenkov telescopes optical design, once the technological problem of free-form mirrors realization had been overcome, new possibilities opened. For IACTs the case is represented by ASTRI telescope [41]. ASTRI (Figure 6 – left) is a Cherenkov telescope placed at Serra La Nave (Italy) realized by the Italian National Institute for Astrophysics as a demonstrator for the SST class of CTA project.

ASTRI was designed, starting from the idea of a Cherenkov double mirror solution proposed by Vassiliev et al. [4], applying the polynomial optimization described in section 4.2, we will call this case S-like design. In particular, the optical design was optimized to have an angular resolution better than a SiPM sensor dimension (6.2 mm) on a field of view of 9.6 degrees, to fulfil the requirements of the SST of the CTA observatory.

The polynomial solution yields an optical system presenting great advantages with respect to the SST DC proposed solution. Also in this case the ratio between the ASTRI's plate scale and the SST DC's one is ~1/3, considering telescopes with the same primary mirror dimension (4 m). About the compactness of the telescope, the ASTRI polynomial solution benefit is analogue to the SC solution (~0.5 of the corresponding DC design).

The expected dependence of angular resolution with the field angle for the S-like configuration is shown in the right side of Figure 6 as thick solid line (as reported in [42]). As can be observed its is similar to the one obtained for the polynomial design for X-ray telescopes (Figure 3) but rescaled to cover the required FoV.

The ASTRI telescope optics calibration was performed in 2016 [42]. ASTRI angular resolution was tested across the FoV observing Polaris up to 4.5 degrees off-axis, confirming the capability of the system in maintaining the optical quality constant on a FoV of 9 degrees (left side of Figure 6). This result is of great relevance since represents the first accomplished implementation of SC design in normal incidence telescope. Moreover, ASTRI demonstrated the suitability of S-like configuration for wide FoV Cherenkov telescopes design.

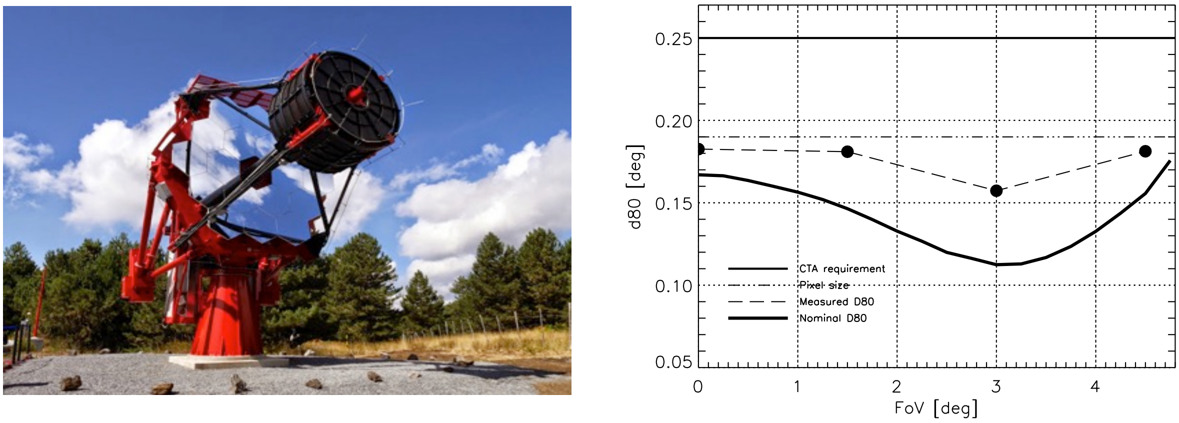

Figure 6: Left - The ASTRI telescope realized by INAF as prototype for the SST of the CTA observatory. Right - Diameter containing the 80% of photons as a function of field angle. Values are reported for the design (solid curve) and as measured at ASTRI telescope (dashed curve). The dimension of the SiPM pixel size (dashed horizontal line) and CTA requirement (solid horizontal line) are also shown for reference.

6. DISCUSSION

In this paper we analysed the optical configurations based on KS's solution. Studying the details of the X-ray and Cherenkov telescopes cases, some similarities arose in their evolutionary path. For both these cases we can identify the following steps:

- The physics at the basis of the phenomena to be observed set peculiar conditions: grazing incidence reflection (angle smaller than 1 degree) for X rays, very fast pulses (few nanoseconds) for Cherenkov light.

- These conditions made difficult the application of classical optical design. Normal incidence optics fails to produce good imagery when used in grazing incidence and cannot maintain good performances across a wide FoV (8-10 deg).
- Empirical solutions were found to overcome these problems. In both cases the solutions are based on simple concentration of the photons toward the focal plane. In X-ray case, the great asymmetry due to grazing incidence was overcome by Kirkpatrick-Baez with a double reflection system. Working on single perpendicular axis allowed the asymmetry corrections. For the Cherenkov telescope case, the solution was found in the Davies-Cotton design. It takes advantage of a segmented large mirror operating differential opportune tilts of the facets to concentrate the light in a common area. These two solutions have in common the working principle: single areas of the reflecting surface work differently in a discontinuous way. They also have in common the drawbacks of non-regular surfaces, KB system is anamorphic, DC system is not isochronous.
- Once the technological advances allowed the manufacturing of the complicated optics necessary to implement KS's solution, a S configuration is recovered. Wolter applied the KS's solution to grazing incidence optics obtaining perfect imagery on-axis and aplanatic behaviour. This solution foresees the realization of quasi-cylindrical optics with requirement in the profile accuracy in the range of fraction of microns. Vassiliev applied the KS's solution for Cherenkov telescopes with the same results. In this case the solution foresees the realization of free-form segments on the meter scale with asphericity of millimetres peak-to-valley.
- Once the manufacturing process has been validated, the design can be optimized allowing the implementation of free-form optics. We reported here examples of polynomial optimized optics for both X-rays and Cherenkov cases. The polynomial optimization represents an empirical solution to customize the optical parameters to achieve better results just on the required FoV.
- We expect that new optimization could be found taking advantages of the mirrors discontinuity. This would be a recovering of the empirical solutions optimized numerically. In the case of X-rays, the optical modules are composed by a number of quasi-cylindrical optics and in the case of Cherenkov telescopes the mirrors are segmented. By the way the polynomial optimization works on continuous surfaces, removing the tessellation degree of freedom.

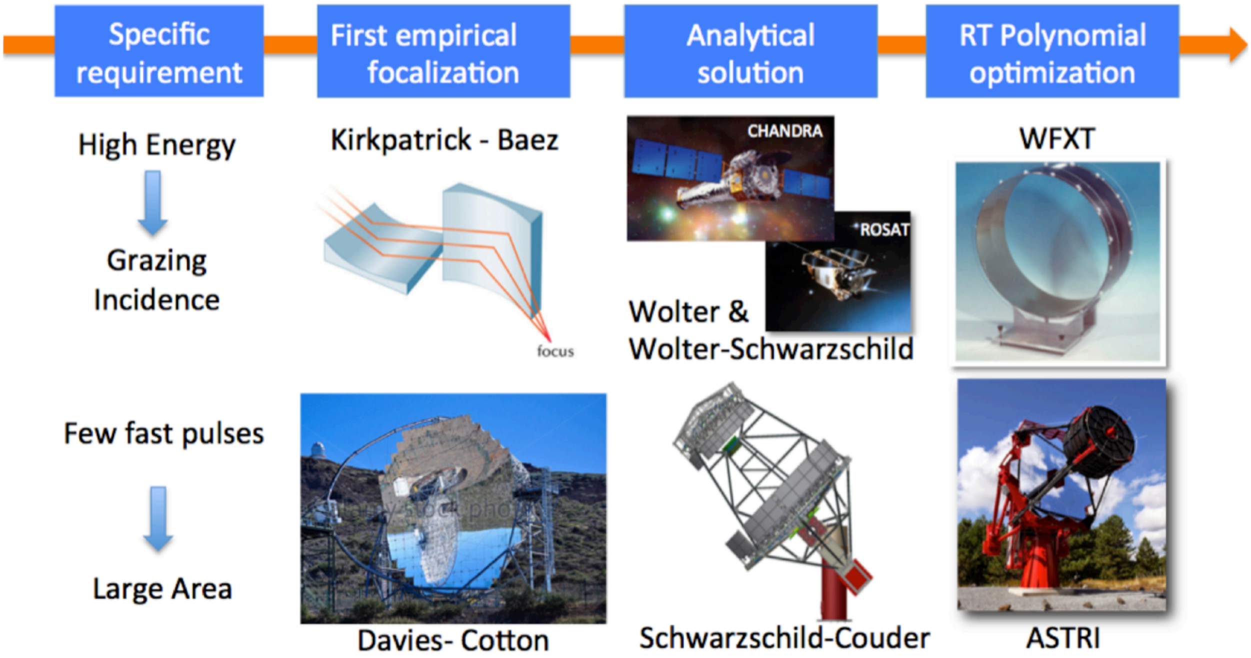

Figure 7: Sketch about the optical design evolution of X-ray and Cherenkov optics traced in this paper.

7. CONCLUSIONS

In this contribution we traced the milestones of the improvements in optical design achieved thanks to KS's theorem. The list is impressive. KS defined a method capable to design aplanatic telescope in every conditions, from normal to grazing incidence. Although the relevance of Schwarzschild's discovery, optical system applying his theory historically struggled to be implemented, mainly because of the complexity in realizing aspheric optics.

The first application of Schwarzschild's theory came from visible light telescopes thanks to the Ritchey-Chretien design. After that, Wolter implemented the Schwarzschild's solution for a variation of his double mirrors solution for X-ray focusing in grazing incidence regime.

In the lasts decade, a novel application for Schwarzschild's optical configuration, was presented: ground based Cherenkov telescope. This case is particularly interesting because it foresees the application of the KS's theorem to realize a large aperture, compact, normal incidence telescope. This is the same aim that originally pushed KS's to study the optical aberrations, attaining to the theorem formulation.

The first proposal for a Cherenkov telescope in Schwarzschild configuration was made by Vassiliev and led to the realization of the MST class telescope construction at Fred Lawrence Whipple Observatory in southern Arizona, USA as a demonstrator for CTA observatory. The same design was adopted by a French-English collaboration to realize a demonstrator for the CTA SST class telescope.

Always in the CTA SST context, the ASTRI telescope represents another application of Schwarzschild's optical configuration. ASTRI is implemented as s Schwarzschild-like configuration with optics customized by polynomials optimization. The positive optical qualification of ASTRI represents the first implementation of a normal incidence Schwarzschild's optical design, definitively proving that KS's configuration can be successfully implemented.

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