Hyper hemispheric lens

Claudio Pernechele*

Italian National Institute for Astrophysics, Astronomical Observatory of Padova, Vicolo dell'Osservatorio, 5, 35122 Padova, Italy
*claudio.pernechele@inaf.it

Abstract: Very wide angle lenses have a field of view greater than one hundred degrees. The paradigm of ultra wide angle lens is widely recognized as the fisheye, which is characterized by a field of view covering a hemispherical field. Recently, due to the availability of low-cost digital sensors, panoramic omnidirectional lenses are also becoming popular. These lenses permit us to capture a panoramic field, i.e. with many tens of degrees above and below the horizon, at the cost of obscuring a region close to the optical axis. We describe here a very wide angle lens which merges the fisheye and the omnidirectional lenses capabilities. The total field of view of the hyper hemispheric lens results in 360 degrees in azimuth angle and up to 270 degrees in zenithal angle, then much more than a hemisphere: we call it "hyper hemispheric lens".

OCIS codes: (220.3620) Lens system design; (220.3630) Lenses.

References and links

1. K. Miyamoto, "Fish-eye lens," J. Opt. Soc. Am. **54**(8), 1060–1061 (1964).
2. R. W. Wood, *Physical Optics* (Mcmillan & Co Ltd, 1919).
3. M. Laikin, "Wide angle lens system," Proc. SPIE **237**, 530–533 (1980).
4. C. B. Martin, "Design issue of hyper-field fisheye lens," Proc. SPIE **5524**, 84–92 (2004).
5. E. P. Wallerstein, E. C. Driscoll, and W. C. Lomax, "Multiple resolution image capture arrangement," EU Patent 1099969 A2 (2000).
6. T. H. Jamieson, "Wide-aperture infrared lenses with hyper-hemispherical fields of view," US Patent 5502592A (1993).
7. J. S. Chahl and M. V. Srinivasan, "Reflective surfaces for panoramic imaging," Appl. Opt. **36**(31), 8275–8285 (1997).
8. I. Powell, "Panoramic lens," Appl. Opt. **33**(31), 7356–7361 (1994).
9. G. I. Kweon, S. Hwang-Bo, G. H. Kim, S. C. Yang, and Y. H. Lee, "Wide-angle catadioptric lens with a rectilinear projection scheme," Appl. Opt. **45**(34), 8659–8673 (2006).

1. Introduction

Lenses with field of view greater than hundred degrees (very wide angle lenses) are able to map, into their focal plane, objects belonging to a space as large as a hemisphere. These lenses work as an inverted telephoto lens producing a large image distortion in the focal plane. The origin of this large distortion resides on the fact that chief rays angles on the object side are not preserved passing through the optics preceding the aperture stop (fore-optics). The distorion is so large that usually it is not even quoted as such by optical designer, and even the focal length looses its meaning. Some authors underline that lenses with extreme field of view, as the fisheye, do not belong anymore to the very-wide-angle-lens class, but rather form an independent class [1]. Despite these drawbacks, very wide angle lenses are becoming popular and this is substantially due to the availability of low-cost large-area digital sensors. In fact, the "native" wide field image, even though extremely distorted, may be manipulated in order to produce, also in real-time, a comprehensible output to the lens user, making it very interesting for many applications. The most popular very wide angle lens is the fisheye [2, 3], able to image a hemispheric space (360° in azimuth angle and about 180° in zenith angle). Fisheye with extreme field of view are also been ideated by some authors both in the visible [4, 5] and thermal infrared [6] range. Another modern lens is the omnidirectional one, which record image at 360° in azimuth and tens of degrees above and below the horizon

#257051 Received 11 Jan 2016; revised 16 Feb 2016; accepted 20 Feb 2016; published 29 Feb 2016

(see Fig. 1). Common omnidirectional lens have a blind spot about the zenith (optical axis) and generate then a “donut shape” image. We will describe here how we merge the fisheye and the omnidirectional lenses capabilities, building a lens able to image a space of 360° in azimuth and 270° in elevation.

2. Omnidirectional lens image forming

A panoramic omnidirectional lens works as depicted in Fig. 1. The lens optical axis point toward the zenith. We define “horizon” a plane orthogonal to the zenith axis and passing through the lens. The Zenith angle Z is measured from zenith, down to the maximum field of view (Zmax). The lens looks to a panoramic field, 360° around the azimuth axis and tens of degrees both above and below the horizon (panel a). A typical omnidirectional lens works between Zmin = 30° and Zmax = 135°. The focal plane image (panel b) has the typical donut-shape, with the inside rim at Zmin and the external one at Zmax. The physical dimension of the “donut” is fixed by the lens (paraxial) focal length and by the lens mapping function, later described on this paper. The blind field around the zenith (frontal field) is projected as a blind spot on the focal plane (the central part of the donut). The central hole of the “donut” depicted in Fig. 1 is a disadvantage also because part of the sensor is not exploited. As a reference we note that a fisheye works from Z = 0° up to about Z = 180° (the horizon).

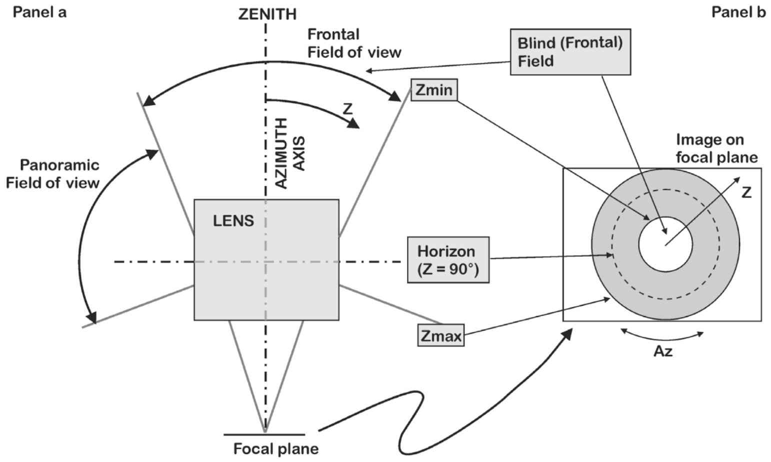

Fig. 1. Omnidirectional lens image forming.

Due to the availability of large area digital sensors, omnidirectional lenses are becoming more and more utilized to catch panoramic fields of view. Common version of such omnidirectional lenses use a curved, usually aspheric, mirror placed in front of a commercial lens to capture a 360° area around the horizon [7]. More recent design use a dedicated catadiopter instead of a mirror [8, 9]. A point to underline is the formal definition of a wide angle lens field of view. When the field angle Z become wide, the entrance pupil is seen slanting and at the limit angle of 90° it will result completely obstructed. In order to permit the visibility of objects at high Z fields, the optical designer has to tilt the entrance pupil. This movement is possible at a cost of shifting (and compress) the pupil.

3. Hyper hemispheric lens image forming

The hyper-hemispheric lens (HH lens) we have designed, built and tested, is able to image both the panoramic and the frontal field, as depicted in the Fig. 2. The left panel shows how the lens works, while the right one shows the focal plane image.

The lens is composed by three logical segments: a fore-optics (the optics preceding the aperture stop AS), a lens for the frontal field (FO) and an objective lens following the aperture stop (OBJ) able to image the field on the focal plane. The fore-optics is composed by a catadiopter C with a reflective concave surface and a lens to speed-down the light beam (SD) making it slow enough when entering the OBJ through the stop AS. The panoramic field is comprised between the chief rays 2 and 3; they enter the catadiopter, refract on the first surface, reflect from the concave one and enter the SD optics. One surface of the SD is made semi-reflective and then half of the light re-enters back to the catadiopter and then it passes across the AS entering the OBJ block to be imaged on the focal plane. The frontal field is a cone with the axis pointing toward the zenith and extending down to the chief ray 1. The frontal field enters the FO, then passes through SD (also in this case only half of the light passes through SD, while the other half is lost back), C, AS, OBJ and is finally imaged on the focal plane, just on the donut hole. In the right panel the primed numbers show the imaged chief rays (1, 2, 3) and the objects reference points 4 and 5.

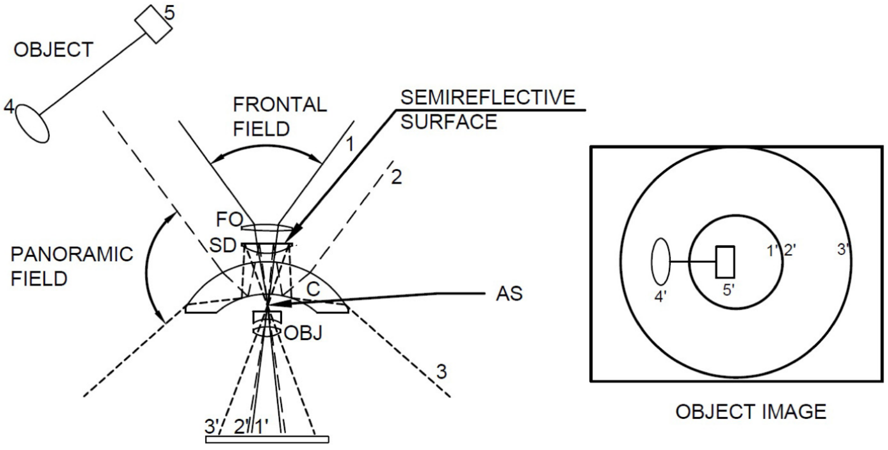

Fig. 2. Hyper hemispheric lens image forming.

The main drawback of this design is the use of a semi-reflecting surface in the optical train, which causes the loss of half the entering light power.

4. Hyper hemispheric lens design

In order to proof the concept we realized an all-spheric hyper hemispheric lens for a 2/3 inch sensor, fixing the dimension of the donut external diameter at 6 mm (paraxial focal length of 2 mm). The relative aperture is F/3.5.

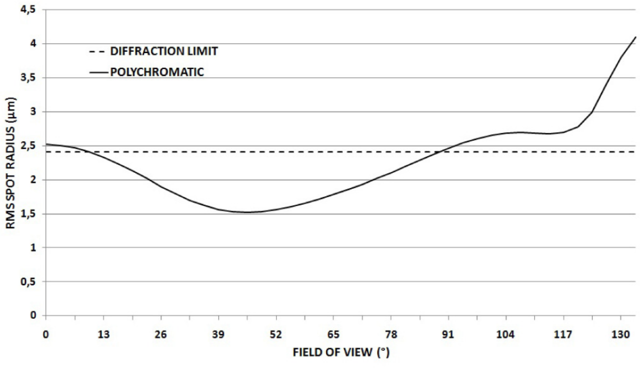

Fig. 3. Optical quality of the designed hyper hemispheric lens.

The panoramic field goes from 30° up to 135° in zenith angle (−45° below the horizon and + 60° above of it). The frontal field is a cone of +/− 30°. The whole FoV is then 360° in azimuth (panorama) and 270° in elevation (hyper-hemisphere). The RMS spot diagrams are shown in the Fig. 3. The continuous line is the performances of the balanced colors (polychromatic) from 0.45 to 0.7 µm. The diffraction limit (at the wavelength of 550 nm and paraxial conditions) is also shown for reference as a dashed line.

The image is almost diffraction limited from the azimuth down to the horizon. For angle below the horizon, the image quality deteriorates due to the anamorphism.

5. Hyper hemispheric lens mapping function

The way in which the 3D object space is mapped onto the 2D focal plane is described by the lens mapping equation. With reference to the Fig. 4, the angular field distribution function at the main plane ψ(Z) is, in general, not linear with respect to Z and a large amount of optical distortion would be present. The focal length f is changing along the field: f = f(Z) (with f(0) the paraxial focal length) and this causes the deformation of the image. In the case of a fisheye Zmax = 90° while in the hyper-hemispheric lens it may go up to 135°. Hereinafter we designate with f the paraxial focal length: f = f(0).

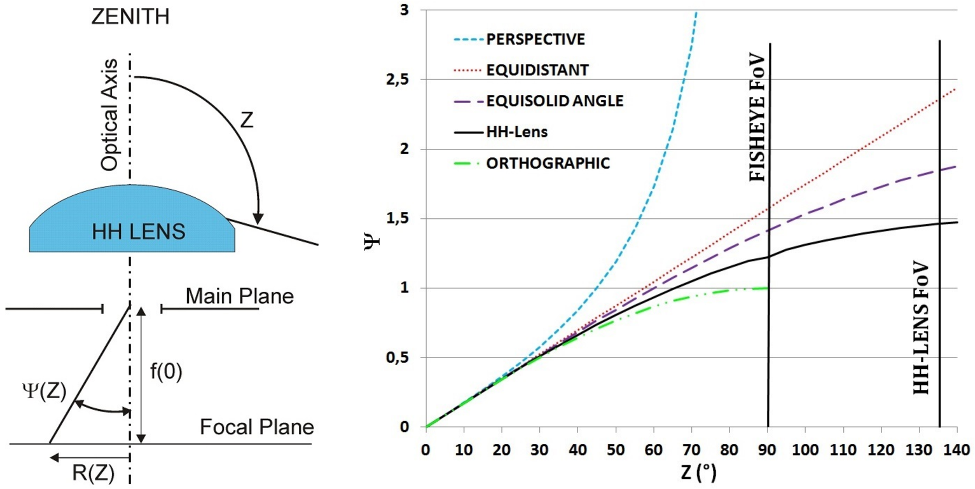

Fig. 4. Very wide angle lenses mapping function.

The space mapping function R(Z) = fψ(Z) may have different forms. The "perfect" undistorted map of the object space is one where R = f tan(Z), where f is the paraxial focal length. In this way every point in the space is mapped maintaining the same angular distribution into the focal plane. The lens works like the pinhole camera and object straight lines remain straight (distortion free) in the focal plane. This function is known as perspective projection. It is well known that it has no practical sense for wide angle lenses, because the focal plane would be infinitely extended and the entrance pupil (the pin-hole) would be completely obscured for Z = 90°. Equidistant (linear scaled) projection is one where R = f Z: it maintains angular distances. The most general projection function has the form:

$$R(Z) = f\psi(Z) = fk_1 \sin(k_2 Z) \qquad (1)$$

Within this class of mapping functions, high compression of the marginal objects is present. Among those there is the equisolid angle (k1 = 2, k2 = 0.5) projection, which maintains surface relations. Each pixel in the detector subtends an equal solid angle, i. e. an equal area on the unit sphere. Finally we cite the orthographic projection (k1 = k2 = 1) where R = f(0) sin(Z). This mapping function maintains planar illuminance. In this projection the marginal fields are extremely compressed at the focal plane and make sense only for Z < 90°. The projection function of our hyper hemispheric lens (described in sec 4) lies in the midway between the equisolid angle and orthographic projections, with f = 2 mm, k1 = 1.48 and k2 =

#257051 Received 11 Jan 2016; revised 16 Feb 2016; accepted 20 Feb 2016; published 29 Feb 2016

0.64. Those parameters are useful for the development of the image restoration software (dewarping).

6. Hyper hemispheric lens plate scale

Due to the high distortion of very wide angle lenses, the angular resolution of the object space is varying at different zenith and azimuth angles. The zenith angle Z, expressed as function of the focal plane radial coordinate R is the inverse of the Eq. (1):

$$Z(R) = \arcsin(\frac{R}{fk_1}) / k_2 \tag{2}$$

The plate scale (°/mm) along the zenith angle is the differential of the Eq. (2):

$$Resz = \frac{\partial Z(R)}{\partial R} = \frac{1}{fk_1k_2\sqrt{1-(\frac{R}{fk_1})^2}} \cdot \frac{180^\circ}{\pi} \quad \circ/mm \tag{3}$$

The angular resolution decreases at higher zentih angle, as expected.

The object space azimuth angle AZ is varying with zenith angle Z following the relation

$$AZ(Z) = \sin(Z) \cdot 360^\circ \tag{4}$$

as depicted in Fig. 5. The AZ angle is 360° at the horizon, where Z = 90°.

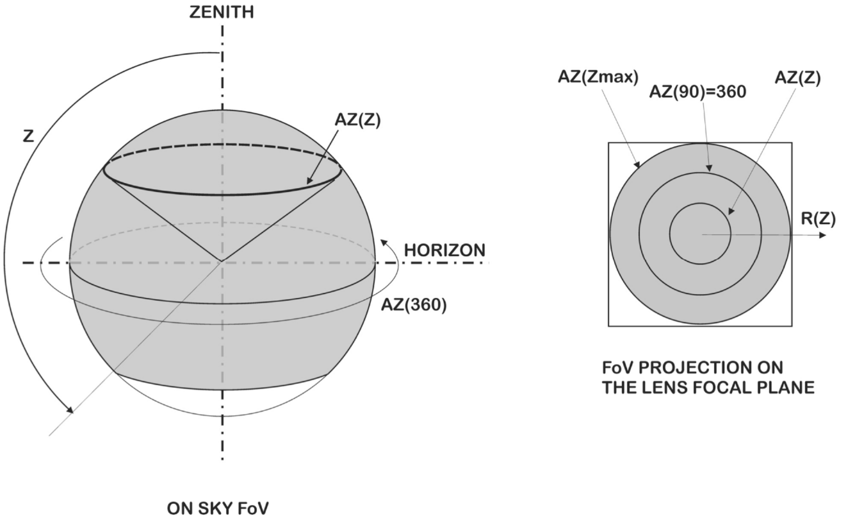

Fig. 5. Azimuth angle as a function of zenith angle.

The plate scale along the azimuth angle, stated in focal plane coordinate R, is:

$$\mathrm{Re}\,s_{az} = \frac{\sin\left[\arcsin(\frac{R}{fk_1}) / k_2\right]}{2\pi R} \cdot 360^\circ \quad \circ/mm \tag{5}$$

The difference in values between the spatial resolution along the zenith [Eq. (3)] and azimuth [Eq. (5)] axes reflect the anamorphism of the lens.

#257051 Received 11 Jan 2016; revised 16 Feb 2016; accepted 20 Feb 2016; published 29 Feb 2016

7. Hyper hemispheric lens realization and test

After the design phase we proceeded to the realization of the lens prototype. A picture of the realized lens is shown in Fig. 6 (top left panel). The dimension of the catadiopter is 60 mm in diameter and the lens has a standard C-Mount, so we can test it with different image sensors. The barrel containing the FO is visible on the top of the catadiopter. In the top right panel of the Fig. 6 a row hyper hemispheric image is shown. Some residual misalignement during the assembly phase has to be fixed. Due to the semireflective coating, which is not precisely neutral in reflection wavelengths, a color difference between the two regions is present. Some reflection ghosts, in presence of bright light (direct sunlight coming from the window), are also present. Despite of this flaws, image analisys shows as the image is almost diffraction limited up to 90° FoV, while its quality is overcame by the anamorphism at higher field angles. In the bottom panel of the Fig. 6 a dewarped image (solely for the panoramic field) is shown. The distorion map decribed in the section 5 above has been used.

Fig. 6. The HH lens prototype (top left) and a "naked" recorded image (top right). Dewarped image (solely for the panoramic field) is shown in the bottom panel.

8. Conclusion

A hyper hemispheric lens able to image a field of view of 360° in azimuth and 270° in zenith angles has been designed. A prototype with paraxial focal length of 2 mm has been realized and tested. The paper gives details on theoretical optical quality, projection map and angular resolution of the lens. An example picture of a hyper hemispheric image taken with the lens is also shown.

Acknowledgments

I wish to thank the Lobre srl (Brescia, Italy) company for the manufacturing of the lens and C. Barbieri (University of Padova) for useful discussion.