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# **MARSIS RADAR: IONOSPHERE PHASE DISPERSION COMPENSATION**

Issue 2, Rev 0

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## 1. INTRODUCTION

One of the main challenges that the Marsis radar (on-board the Mars Express Mission) needs to deal with are the effects introduced in an electromagnetic wave by the Mars ionosphere. According to the recent literature [1], the profile of the electron density and thus of the plasma frequency for increasing altitude raises sharply towards a single well-defined maximum located at an altitude of about 100-150 Km and then drops off smoothly as the altitude increases.

The value of the maximum plasma frequency  $f_{p,max}$  is a function of the solar flux density, as well as of the Solar Zenith Angle (SZA), which is dictated by the illumination condition (day/night). According the behavior of fig.1.

on day time the  $f_{p,max}$  can be as high as 3-4 MHz

- on night time the maximum value of  $f_{p,max}$  should be about 0.8-1 MHz.

We can notice that the phase distortion arising from the Ionosphere produces a delay, an increase of sidelobes level (after matched filter), a distortion of the waveform shape and a loss of signal to noise ratio. Propagation effects on the MARSIS Radar Signals [2], taking into account the very large fractional bandwidth, will be investigated and a compensation technique will be proposed.

## 2. IONOSPHERE MODELS

In order to characterize the distribution of the plasma frequency vs. height and to quantify the amount of distortion due to the ionosphere propagation the “gamma” model can be used.

By comparison of Night [3] time and Day time ionosphere profile [4] with the

### 2.1 Gamma model

$$(1) \quad f_p(z) = f_{p-max} \frac{z - h_0}{b} e^{1 - \frac{z - h_0}{b}} \delta_{-1}(z - h_0)$$

where

$f_{p-max}$  : maximum plasma frequency

$b$  : shape factor

$h_0$  : beginning of the ionosphere layer

we have the behaviors of Fig.2 and the corresponding phase errors (s. Fig.3) are very small, with reference in particular to the requirements of the compressed return echoes; requirements given in terms of pulse widening (<20%) and side lobe level to allow the subsurface interface detection (the level of the return echoes, with reference to the surface echo, can decrease linearly with depth until 60 dB for depth bigger of 3 Km), as will appear more clear in the following.

A suitable range for the parameter  $b$  of eq.(1) appears (s.Fig.4):

$$(2) \quad 20 < b < 50 \text{ Km}$$

We can remember that the phase distortion induced by plasma layer of thickness  $L$  as a function of the frequency  $f$  depends on the profile of the plasma frequency and can be expressed as:



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$$(3) \quad \Delta\phi_{\text{GAMMA}}(f, b, f_{p,\text{MAX}}) = \frac{4\pi}{c} f \int_0^L \left[ \sqrt{1 - \left( \frac{f_p(z)}{f} \right)^2} - 1 \right] dz$$

so that the compensation procedure is related to the estimation of  $b$  and  $f_{p,\text{max}}$

In order to reduce the estimation complexity, we can approximated the eq.(3) with the following:

$$(4) \quad \text{fit}(f) = a_0 + a_1(f - f_0) + a_2(f - f_0)^2 + \dots a_v(f - f_0)^v$$

where  $f_0$  is the carrier frequency. We can notice that  $a_0$  not introduce distortion,  $a_1$  introduce only time displacement and in any case a coarse estimation of  $a_1$  can be obtained from the extra time delay respect to the free-space delay (taking into account the accuracy in the knowledge orbital parameter), moreover the estimation of  $a_2 \dots a_v$  can be obtained by signal optimization procedure. We wish consider the possibility of estimate  $a_2$  by contrast method applied on the received signals and to estimate  $a_3 \dots a_v$  by theoretical extrapolation technique: using closed loop approach, as discussed in the following: in any case we must minimize the  $v$ .

In order to find the coefficients of the polynomial fit( $f$ ) of degree  $v$ , that fits the integral of Gamma model in a least squares sense, we have used of the Levenberg's and Macquardt algorithm, obtaining the values of Tab.1 for  $v=3$  and  $v=4$ , by considering the bandwidth of 1 MHz and night and day side operation with extreme  $b$  values.

					<b>v=3</b>			<b>v=4</b>			
<b>b</b> (km)	<b>h</b> (km)	<b>h<sub>o</sub></b> (Km)	<b>f<sub>o</sub></b> (MHz)	<b>f<sub>p-max</sub></b> (MHz)	<b>a<sub>0-bf</sub></b>	<b>a<sub>1-bf</sub></b> (MHz <sup>-1</sup> )	<b>a<sub>2-bf</sub></b> (MHz <sup>-2</sup> )	<b>a<sub>3-bf</sub></b> (MHz <sup>-3</sup> )	<b>a<sub>2-bf</sub></b> (MHz <sup>-2</sup> )	<b>a<sub>3-bf</sub></b> (MHz <sup>-3</sup> )	<b>a<sub>4-bf</sub></b> (MHz <sup>-4</sup> )
50		120	1.8	0.65	-186	108	-70	45	-64	45	-29
50	800	120	1.8	0.8	-285	170	-118	80	-106	80	-57
50	800	120	1.8	1	-456	285	-224	174	-191	174	-147
20	800	120	1.8	0.65	-464	270	-177	112	-161	112	-73
20	800	120	1.8	0.8	-713	426	-296	201	-264	201	-143
20	800	120	1.8	1	-1139	714	-559	436	-478	436	-368
50	800	120	5	2	-637	135	-30	7	-30	7	-2
50	800	120	5	3	-1495	348	-90	25	-88	25	-8
50	800	120	5	4	-2864	803	-301	139	-283	139	-79
20	800	120	5	2	-1593	338	-75	17	-74	17	-4
20	800	120	5	3	-3739	870	-225	63	-221	63	-19
20	800	120	5	4	-7160	2010	-752	349	-709	349	-197

Tab.1

By considering the range compressed signals, after the compensation (with the best fitting parameters given in Tab.2.1; the Hanning weighting function is used, to allow the required low level of sidelobes) of the ionosphere distortion (gamma model), as shown in Fig.5 and Fig.6, we can conclude that the compensation of the  $v=4$  term is wanted, also if estimation and software constraints must be taken into account.



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## 2.2 Uniform model

In order to reduce the estimation complexity, an alternative simplified ionosphere model characterized by a constant plasma frequency  $f_{p,eq}$  and thickness  $L_{EQ}$  can be introduced, so that the eq. (3) becomes:

$$(5) \quad \Delta\phi_{EQ}(f) = \frac{4\pi L_{EQ}}{c} f \left( \sqrt{1 - \left(\frac{f_{p,eq}}{f}\right)^2} - 1 \right) = 2\pi\tau_0 \left( \sqrt{f^2 - f_{p,eq}^2} - f \right)$$

Moreover a *one-dimensional* version of the uniform equivalent model can be introduced, if the ionosphere equivalent thickness is fixed to an average value  $L_{EQ}=L_m$  (80 km) ( $\tau_0=533 \mu\text{sec}$ ).

In this case the phase errors increase according the behavior of Fig.7, but these values can be accepted, as is shown in Fig.8, where the phase distortion effects on the impulsive response are shown. This estimation procedure entails to obtain  $f_{p,eq}$ ; Fig.9 shows the behavior of  $f_{p,eq}$  vs. the gamma parameters; we can notice that  $f_{p,eq}/f_0$  doesn't depend on  $b$ , it depends on  $f_{p,max}/f_0$  only.

Moreover we can write:

$$(6) \quad \Delta\phi_{EQ}(f) = 2\pi\tau_0 \left( \sqrt{f^2 - f_{p,eq}^2} - f \right) \cong a_0 + a_1(f - f_0) + a_2(f - f_0)^2 + a_3(f - f_0)^3 + \dots$$

where

$$a_0 = 2\pi\tau_0 \left( \sqrt{f_0^2 - f_{p,eq}^2} - f_0 \right) \quad [rad]$$

$$a_1 = 2\pi\tau_0 \left( \frac{f_0}{\sqrt{f_0^2 - f_{p,eq}^2}} - 1 \right) \quad [rad / Hz]$$

$$a_2 = -2\pi\tau_0 \left( \frac{f_{p,eq}^2}{2(f_0^2 - f_{p,eq}^2)^{\frac{3}{2}}} \right) \quad [rad / Hz^2]$$

$$a_3 = 2\pi\tau_0 \left( \frac{f_0 f_{p,eq}^2}{2(f_0^2 - f_{p,eq}^2)^{\frac{5}{2}}} \right) \quad [rad / Hz^3]$$

$$a_4 = -2\pi\tau_0 \left( \frac{4f_0^2 f_{p,eq}^2 + f_{p,eq}^4}{8(f_0^2 - f_{p,eq}^2)^{\frac{7}{2}}} \right) \quad [rad / Hz^4]$$



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To estimate  $a_2$  (that is the most important term of the phase dispersion after the linear one) we use the contrast method, which allows also the estimation of  $f_p$ .

The 3<sup>rd</sup> (and eventually 4<sup>th</sup>) order phase term shall be obtained by  $a_2$  according to the previous equations, as matter of fact we can select among 2 different methods to obtain the cubic term:

according to the single parameter equivalent model from the term  $a_2$  we can obtain the equivalent plasma frequency and then the term  $a_3$

for low value of the ratio  $f_{p,eq}/f_0$ ,  $a_3 \approx -a_2 / f_0$ , this last solution is easier to implement than the preceding one but gives bigger error as more the plasma frequency is close to the transmitted frequency (especially during the daytime when we operate with the higher frequencies)

### 3. CONTRAST METHOD: Design Approach

Considering now only the effect of the quadratic phase term on the chirp signal compression, we can relate easily the quadratic phase distortion in the spectrum with the mismatching in the chirp slope  $\mu$  of matched filter in the receiver.

Taking into account the behavior of Fig.10 [5], we can define the requirement in the mismatching allowed

$$(7) \quad \gamma TB \leq 2$$

where (according to the MARSIS design):

$T$  is the chirp duration (250  $\mu$ sec)

$B$  is the chirp bandwidth (1 MHz)

$\gamma = \frac{\Delta\mu}{\mu}$  is the mismatching factor

$$\mu = \frac{2\pi B}{T} = 2.51 * 10^{10} \text{ sec}^{-2} \rightarrow \gamma \leq 8 * 10^{-3}$$

therefore the required *accuracy* in the compensation of the quadratic term coefficient ( $a_2$ ) is:

$$(8) \quad \Delta a_2 = \frac{\pi\gamma T}{B} = 6 * 10^{-12} \text{ [rad/Hz}^2\text{]}$$

Moreover the *research area* (in terms of  $a_2$ ) during the tracking phase is function of the ionosphere variation in the space covered by the orbiter during each synthetic aperture (integration) time (since the compensation is updated adaptively every frame).

In this case we have supposed that the maximum variation of  $|\Delta f_{p,eq}| \approx |\Delta f_{p,max}| \approx \pm 50\text{KHz}$ , so that we can assume:

$$(9) \quad a_2 = \pm 30 * 10^{-12} \text{ [rad/Hz}^2\text{]}$$

Moreover we can notice that the error, on the cubic term, accepted can be evaluated by considering the distortion introduced from an approximated sinewave term (obtained through the subtraction of the cubic term from the best fitting linear term), the corresponding amplitude must be less than 1 rad:

$$(10) \quad \Delta a_3 \left( \frac{B}{2} \right)^2 (f - f_0) - \Delta a_3 (f - f_0)^3 \Big|_{f-f_0=\frac{B}{4}} \approx \Delta a_3 B^3 \frac{1}{21} \approx 5 * 10^{16} \Delta a_3 < 1 \text{ rad}$$



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$$\rightarrow \Delta a_3 < 20 * 10^{-18} \text{ rad/Hz}^3$$

In order to evaluate the error introduced by

$$(11) \quad a_3 = -a_2 / f_0$$

we can write (s.eq.(5))

$$(12) \quad \frac{a_3}{a_2} = - \left( \frac{f_0}{(f_0^2 - f_{p,eq}^2)} \right) \approx - \frac{1}{f_0} - \frac{f_{p,eq}^2}{f_0 f_0^2} \approx - \frac{1}{f_0} \left( 1 - \frac{a_2 f_0}{\pi \tau_0} \right)$$

therefore from eq (10) we can have:

$$\Delta a_3 \approx (a_2)^2 \frac{1}{\pi \tau_0} < 20 * 10^{-18} \text{ rad/Hz}^3 \rightarrow (a_2)^2 < 20 * 10^{-18} \pi \tau_0 \approx 3.4 * 10^{-20} \rightarrow a_2 < 200 * 10^{-12}$$

The  $a_2$  can be estimated ( $\hat{a}_2$ ) through the Amplitude Contrast Maximization technique [6] (s. Annex 1), that is the estimation of the phase distortion of the received signal based on the principle that the output of the matched filter is maximally sharp, when the reference function matches perfectly the phase distortion spectrum of the received signal. Moreover  $a_3$  can be obtained from eq. (12):  $\tau_0$  can be adapted in order to improve the results.

According to the previous data, some results are shown in Fig.11, to be compared to the data of Fig.5.

The estimation can improved by inserting also the

$$(13) \quad a_4 = -a_3 / f_0$$

and the results are shown in Fig.12, and must be compared with the behavior of Fig.6.

Tab. 2 shows the error of estimation of  $a_2$  and  $a_3$  in the previous cases (s Tab.1)

<b>b</b> (km)	<b>f<sub>0</sub></b> (MHz)	<b>f<sub>p-max</sub></b> (MHz)	<b>v=3</b>		<b>v=4</b>		
			<b>Δa<sub>2</sub></b> (MHz <sup>-2</sup> )	<b>Δa<sub>3</sub></b> (MHz <sup>-3</sup> )	<b>Δa<sub>2</sub></b> (MHz <sup>-2</sup> )	<b>Δa<sub>3</sub></b> (MHz <sup>-3</sup> )	<b>Δa<sub>4</sub></b> (MHz <sup>-4</sup> )
50	1.8	0.65	-2	-2	-4	-4	7
50	1.8	0.8	-4	-5	-2	-15	20
50	1.8	1	-18	-11	-14	-39	73
20	1.8	0.65	-5	5	-2	-9	16
20	1.8	0.8	-16	21	-10	-11	37
20	1.8	1	-39	72	-47	-9	133
50	5	2	-3	0	2	-1	0
50	5	3	-2	-3	1	-4	3
50	5	4	-9	-30	-22	-32	57
20	5	2	-1	1	-3	0	1
20	5	3	-3	6	-4	7	6
20	5	4	-19	97	-30	66	113



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Tab.2

$$\text{Eq.s (12), (13)} \quad \Delta a_2 = \hat{a}_2 - a_{2,\text{bf}} \Leftrightarrow \Delta a_3 = \hat{a}_3 - a_{3,\text{bf}} \Leftrightarrow \Delta a_4 = \hat{a}_4 - a_{4,\text{bf}}$$

In order to improve the estimation technique, taking into account the approximation due to limited number of the parameters  $a_i$  and very large fractional bandwidth, we have analyzed the best fitting data and the optimum relations between  $a_2$ ,  $a_3$  and  $a_4$  can be estimated (s. Annex 2), so that the eq.s (12),(13) become:

$f_0=1.8$  MHz:

$$(14) \quad \begin{aligned} a_3 &\approx -\frac{a_2}{f_{o1}} \left( 1 - \frac{a_2 f_{o1}}{\pi \tau_{o1}} \right) & f_{o1} &= 1.4 \text{ MHz} \\ a_4 &\approx -\frac{a_2}{1.1 \cdot f_{o1}^2} \left( 1 - \frac{a_2 (1.1 \cdot f_{o1})}{0.5 \cdot \pi \tau_0} \right) & \tau_{o1} &= 700 \mu\text{sec} \end{aligned}$$

$f_0=5$  MHz:

$$(15) \quad \begin{aligned} a_3 &\approx -\frac{a_2}{f_{o1}} \left( 1 - \frac{a_2 f_{o1}}{\pi \tau_{o1}} \right) & f_{o1} &= 2.8 \text{ MHz} \\ a_4 &\approx -\frac{a_2}{0.95 \cdot f_{o1}^2} \left( 1 - \frac{a_2 (0.95 \cdot f_{o1})}{0.5 \cdot \pi (0.7 \cdot \tau_0)} \right) & \tau_{o1} &= 1600 \mu\text{sec} \end{aligned}$$

so that the data of Tab.3 and the behaviors of Fig.13 and Fig.14 arrive.

<b>b</b> (km)	<b>f<sub>o</sub></b> (MHz)	<b>f<sub>p-max</sub></b> (MHz)	<b>v=4</b>		
			<b>Δa<sub>2</sub></b> (MHz <sup>-2</sup> )	<b>Δa<sub>3</sub></b> (MHz <sup>-3</sup> )	<b>Δa<sub>4</sub></b> (MHz <sup>-4</sup> )
50	1.8	0.65	2	2	-9
50	1.8	0.8	4	-3	-7
50	1.8	1	-2	-19	10
20	1.8	0.65	11	5	-28
20	1.8	0.8	14	6	-47
20	1.8	1	-1	10	-3
50	5	2	2	3	-2
50	5	3	1	7	-6
50	5	4	-4	-20	23
20	5	2	3	9	-7



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20	5	3	7	22	-19
20	5	4	-3	6	-1

Tab.3

We can notice that in the last figures (15, 16, 17), the increasing of side lobe due to Fresnel ripple to the level [7]:

$$(16) \quad \text{S.L.F.} = 20\log(\tau B) + 3 = 20\log(250 \cdot 1) + 3 = 51 \text{ dB}$$

was software compensate, by ripple extraction (in the 1 MHz bandwidth) by FFT of reference function (Ref-fun-def(f)) and performing Ref-prod (s. Annex 1).

#### 4. CONTRAST: Evaluation of Noise Effects

By analytical point of view, we can assume for simplicity a gaussian pulse  $X_r(t)$  after detection

$$(17) \quad X_r(t) = A e^{-\frac{(t-t_0)^2}{2\sigma_r^2}} + n(t)$$

with

$$(18) \quad \sigma_r^2 = \left(\frac{\tau_0 w}{2}\right)^2$$

where

$\tau_0$  is the minimum pulse duration (after ideal matched filter).

$t_0$  is the delay time due to ionosphere phase dispersion.

$w$  is the pulse duration widening due to mismatching referred to ionosphere quadratic error.

The *Contrast* of the signal  $X_r$  is defined by the ratio of the standard deviation of the signal to its average value and we obtain

$$(19) \quad C = \frac{2\gamma}{w\sqrt{\pi}} \cdot \frac{\text{Erf}\left(\frac{2}{w} \cdot \left(\frac{\gamma}{2} - \alpha\right)\right) + \text{Erf}\left(\frac{2}{w} \cdot \left(\frac{\gamma}{2} + \alpha\right)\right)}{\left\{ \text{Erf}\left(\frac{\sqrt{2}}{w} \cdot \left(\frac{\gamma}{2} - \alpha\right)\right) + \text{Erf}\left(\frac{\sqrt{2}}{w} \cdot \left(\frac{\gamma}{2} + \alpha\right)\right) \right\}^2 - 1}$$



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where  $\gamma = T / \tau_0$  being T the integration time  
 $\alpha = t_0 / \tau_0$

For  $T \gg \tau_0$  and  $\gamma \gg \alpha$  the previous equation becomes

$$(20) \quad C_w = \sqrt{\frac{\gamma}{w\sqrt{\pi}} - 1}$$

where  $w$  is the -3dB widening factor of the pulse due to the quadratic slope mismatching and it can be related to the quadratic phase distortion by the following relation

$$(21) \quad w \cong 1 + 0.025 (\nu TB)^2 = 1 + \delta x$$

where  $\nu = \frac{\Delta\mu}{\mu}$  is the mismatching factor and  $\mu$  is the

chirp slope defined by  $\mu = \frac{2\pi B}{T}$

From figure we notice that in absence of slope mismatching the contrast assume is maximum value.

Moreover being  $a_2 = \frac{2\pi^2}{\mu}$  we can write

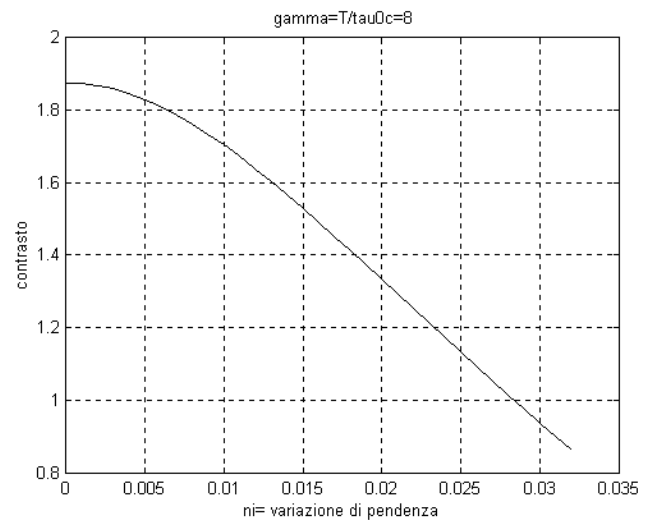
$$w \cong 1 + 0.025 \left( \frac{\delta a_2 B^2}{\pi} \right)^2 \text{ and the contrast given}$$

becomes an explicit function of  $\delta a_2$

(22)

$$C_x = \sqrt{\frac{\gamma}{w\sqrt{\pi}} - 1} = \sqrt{\frac{\gamma}{(1 + \delta x)\sqrt{\pi}} - 1} = \sqrt{\frac{\gamma}{\sqrt{\pi}}(1 - \delta x) - 1} = \sqrt{\frac{\gamma}{\sqrt{\pi}} - 1 - \frac{\gamma}{\sqrt{\pi}} \delta x} \approx \sqrt{\frac{\gamma}{\sqrt{\pi}} - 1} \left( 1 - \frac{\frac{\gamma}{2\sqrt{\pi}} \delta x}{\frac{\gamma}{\sqrt{\pi}} - 1} \right)$$

$$(23) \quad C_x \approx C - \frac{(C^2 + 1)}{2C} \delta x$$





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$$(24) \quad C_n \approx \frac{\sqrt{\sigma_{X_r}^2 + \gamma \sigma_n^2}}{\bar{X}_r + \gamma \bar{x}_n \pm \sqrt{(2 - \frac{\pi}{2})\gamma \sigma_n^2}} \approx \frac{\sigma_{X_r} \left( 1 + \gamma \frac{\sigma_n^2}{2\sigma_{X_r}^2} \right) \left( 1 - \gamma \frac{\bar{x}_n}{\bar{X}_r} \pm \sqrt{(2 - \frac{\pi}{2})\gamma \frac{\sigma_n}{\bar{X}_r}} \right)}{\bar{X}_r} \approx$$

$$\approx C + \gamma C \frac{\sigma_n^2}{2\sigma_{X_r}^2} - \gamma C \frac{\bar{x}_n}{\bar{X}_r} \pm C \sqrt{(2 - \frac{\pi}{2})\gamma \frac{\sigma_n}{\bar{X}_r}}$$

$$(25) \quad C \sqrt{(2 - \frac{\pi}{2})\gamma \frac{\sigma_n}{\bar{X}_r}} = \frac{(C^2 + 1)}{2C} \delta x$$

$$(26) \quad \frac{\bar{X}_r}{\sigma_n} \approx \frac{2C^2}{(C^2 + 1)\delta x} \sqrt{(2 - \frac{\pi}{2})\gamma} = \frac{\gamma - \sqrt{\pi}}{\gamma \delta x} \sqrt{2(4 - \pi)\gamma}$$

$$\bar{X}_r = A \quad \frac{S}{N} = \frac{A}{\sqrt{2}\sigma} = \frac{\gamma - \sqrt{\pi}}{\gamma \delta x} \sqrt{\gamma(4 - \pi)}$$

The maximum value of the contrast can be also obtained by selection of the minimum value of the mean value.

$$(27) \quad E = A^2 \tau = A_0^2 (1 - \delta A)^2 \tau (1 + \delta x) = \cos t \rightarrow 2\delta A \approx \delta x \rightarrow A\tau = A_0 \tau (1 + \frac{\delta x}{2})$$

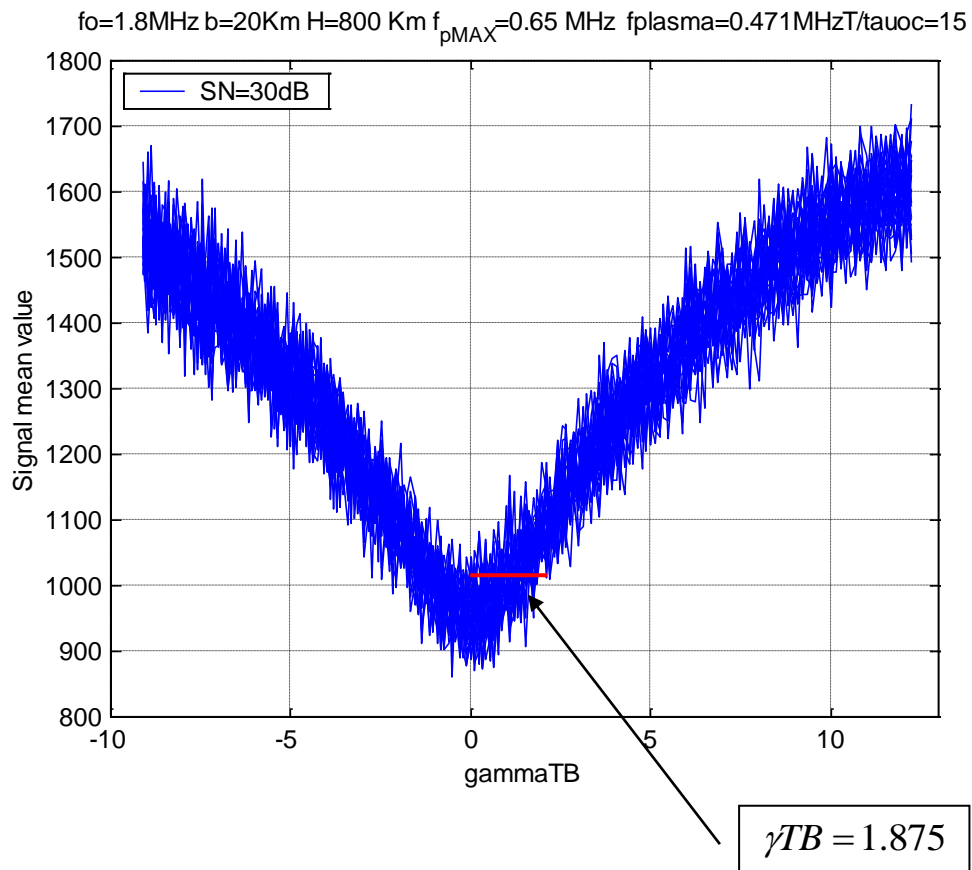
$$(28) \quad C_n|_{\max} \rightarrow \bar{X}|_{\min} \approx \left( \frac{A\tau}{T} + \bar{x}_n \pm \sqrt{(2 - \frac{\pi}{2})\frac{\sigma_n^2}{\gamma}} \right) \Bigg|_{\min} = \left( \frac{A_0}{\gamma} (1 + \frac{\delta x}{2}) + \bar{x}_n \pm \sqrt{\frac{4 - \pi}{2\gamma}} \sigma_n \right) \Bigg|_{\min}$$

$$\rightarrow \frac{A_0}{\gamma} \frac{\delta x}{2} = \sqrt{\frac{4 - \pi}{2\gamma}} \sigma_n \rightarrow \frac{A_0}{\sqrt{2}\sigma_n} = \frac{\sqrt{(4 - \pi)\gamma}}{\delta x}$$



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## EVALUATION OF NOISE EFFECTS

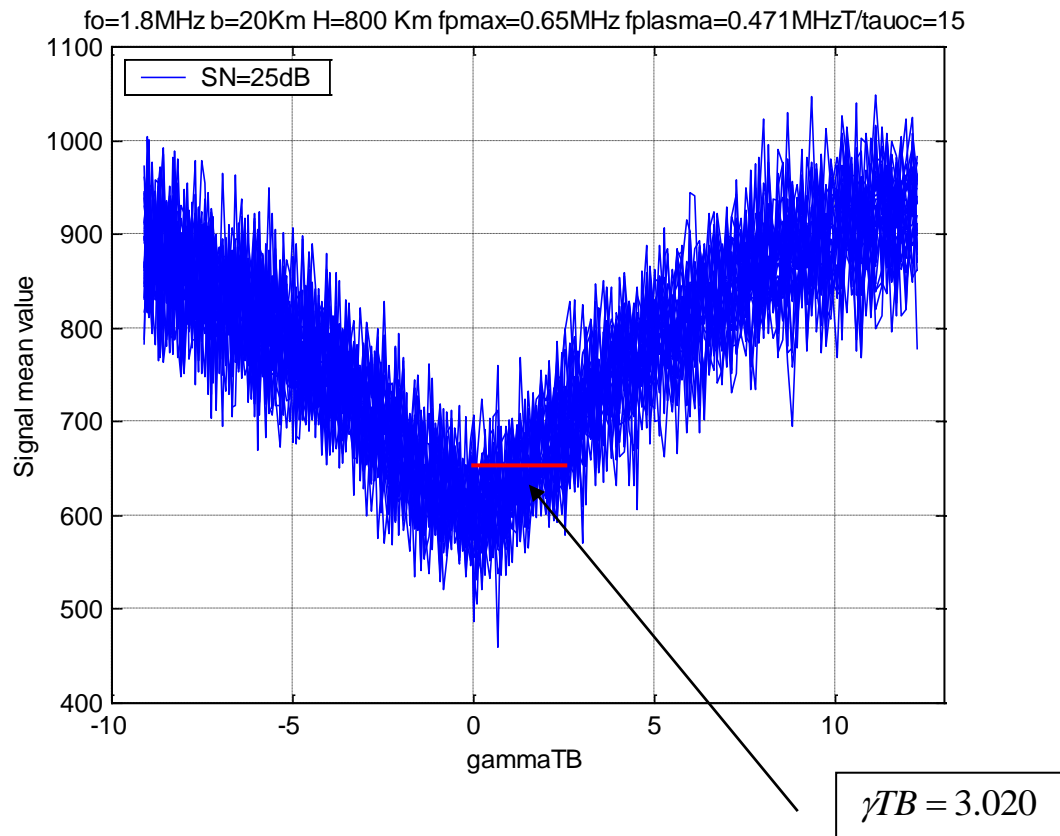


$$\gamma TB = 1.875 \longrightarrow \delta_x = 0.09 \longrightarrow S / N \cong 32dB$$



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## EVALUATION OF NOISE EFFECTS



$$\gamma TB = 3.020 \quad \longrightarrow \quad \delta_x = 0.21 \quad \longrightarrow \quad S/N \cong 24dB$$



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## 5. ACQUISITION TRACKING PHASE

After the acquisition phase and before starting the pulse tracking phase, taking into account that we haven't precise preliminary knowledge of  $a_2$ , the *research area* increases. Hence the estimation of  $a_2$  term shall be initialized, obviously in order to reduce as more as possible the time necessary to get the optimum value. The initialization can be obtained by the measurement of the extra delay time (or  $a_1$ ), due to ionosphere, as available in acquisition phase; since for high value of the ratio  $f_0/f_{p,eq}$  the coefficient  $a_2 \approx -a_1/f_0$  (see section 2) it seems reasonable to choose this value to initialize the coefficient  $a_2 \rightarrow \hat{a}_2$

The extra delay time ( $\tau$ ) corresponding to the frequency  $f_0$  is given (s.eq.(5)) by:

$$(29) \quad \tau = \frac{a_1}{2\pi} = \frac{\tau_0}{\sqrt{1 - \frac{f_{p,eq}^2}{f_0^2}}} - \tau_0 = \tau_0 \varphi\left(\frac{f_{p,eq}}{f_0}\right)$$

The  $\tau$  should be less than 110  $\mu\text{sec}$  (night time  $\rightarrow f_{p,eq} < 1 \text{ MHz}$ ,  $f_0 = 1.8 \text{ MHz}$ ) and 350  $\mu\text{sec}$  (day time  $\rightarrow f_{p,eq} < 4 \text{ MHz}$ ,  $f_0 = 5 \text{ MHz}$ ).

We can notice that the  $\tau$  measurement accuracy is function of:

- orbit knowledge accuracy: uncertainty of orbiter position of 3 sec, radial speed less 1 km/sec entail  $\Delta\tau_1 < 20 \mu\text{sec}$
- surface behavior and altimeter measurement accuracy  $\Delta\tau_2 < 10 \mu\text{sec}$

so that we can obtain:

$$(30) \quad \hat{a}_1 = a_1 \pm 10^{-4} \rightarrow \hat{a}_2 = a_2 \pm \frac{10^{-4}}{f_0} \quad [\text{rad/Hz}^2]$$

in the night time (for high value of the ratio  $f_0/f_{p,eq}$ )

$$(31) \quad \hat{a}_2 = a_2 \pm 50 * 10^{-12} \quad [\text{rad/Hz}^2]$$

In order to improve the estimation of the term  $a_1$ , by mean without the effects of orbit and surface un knowledge, in the *two frequency operative modes* we can use the measurement of time delay difference ( $\Delta\tau_M$ ) between the time delay of two signal with different central frequencies  $f_0$ ,  $f_{02} = f_0 + \Delta f$ .

$$(32) \quad \Delta\tau_M = \tau - \tau_2 = \tau_0 \left[ \varphi\left(\frac{f_{p,eq}}{f_0}\right) - \varphi\left(\frac{f_{p,eq}}{f_{02}}\right) \right] = \tau_0 \Delta\varphi$$

with  $\frac{\tau}{\tau_0} = \varphi\left(\frac{f_{p,eq}}{f_0}\right)$  and  $\frac{\tau_2}{\tau_0} = \varphi\left(\frac{f_{p,eq}}{f_{02}}\right)$

Assuming an exact estimation of  $\Delta\tau_M$  we can write:



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$$(33) \quad \tau = \Delta\tau_M \frac{\varphi\left(\frac{f_{p,eq}}{f_0}\right)}{\Delta\varphi} = \Delta\tau_M \frac{\frac{1}{\sqrt{1-\left(\frac{f_{p,eq}}{f_0}\right)^2}} - 1}{\frac{1}{\sqrt{1-\left(\frac{f_{p,eq}}{f_0}\right)^2}} - \frac{1}{\sqrt{1-\left(\frac{f_{p,eq}}{f_{02}}\right)^2}}} \cong \Delta\tau_M \frac{\frac{1}{f_0^2}}{\left(\frac{1}{f_0^2} - \frac{1}{f_{02}^2}\right)}$$

Concerning the hypothesis of high value of the ratio  $f_0/f_p$  we can write:

$$(34) \quad a_2 = -2\pi\tau_0 \left( \frac{f_{p,eq}^2}{2\sqrt{(f_0^2 - f_{p,eq}^2)}(f_0^2 - f_{p,eq}^2)} \right) = -\frac{2\pi\tau_0}{f_0} \left( \frac{f_{p,eq}^2}{2f_0^2} \right) \left( \frac{\tau}{\tau_0} + 1 \right)^3 = -\frac{2\pi\tau_0}{2f_0} \left( \left( 1 + \frac{\tau}{\tau_0} \right)^2 - 1 \right) \left( 1 + \frac{\tau}{\tau_0} \right) =$$

$$= -\frac{2\pi\tau_0}{2f_0} \left( \frac{2\tau}{\tau_0} + \left( \frac{\tau}{\tau_0} \right)^2 \right) \left( 1 + \frac{\tau}{\tau_0} \right) = -\frac{\pi\tau}{f_0} \left( 2 + \frac{\tau}{\tau_0} \right) \left( 1 + \frac{\tau}{\tau_0} \right) = -\frac{2\pi\tau}{f_0} \left( 1 + \frac{\tau}{2\tau_0} \right) \left( 1 + \frac{\tau}{\tau_0} \right) \approx -\frac{2\pi\tau}{f_0} \left( 1 + \frac{3\tau}{2\tau_0} \right)$$

so that:

$$(35) \quad a_2 = -\frac{a_1}{f_0} \left( 1 + \frac{3\tau}{2\tau_0} \right) \rightarrow \hat{a}_2' = \hat{a}_2(1+y) \rightarrow \begin{array}{l} y < 0.3 \Leftrightarrow \text{night-time} \\ y < 0.98 \Leftrightarrow \text{day-time} \end{array}$$

In conclusion, taking into account the maximum value of  $a_2 < 1000 \text{ rad/MHz}^2$ , we can conclude that it seems reasonable to choose the coefficient  $-a_1/f_0$  to initialize the coefficient  $a_2$ , but some compensation terms should be applied, following the approach of the previous section, taking into account the procedure selected for the start of the tracking phase.

In any case the uncertainty on  $\tau$  entails an error on the estimation on  $a_2$ :

$$(36) \quad \Delta a_2 \approx \frac{2\pi\Delta\tau}{f_0} \left( 1 + \frac{3\tau}{2\tau_0} \right) + \frac{2\pi\tau}{f_0} \frac{3\Delta\tau}{2\tau_0} = \frac{2\pi\Delta\tau}{f_0} \left( 1 + \frac{3\tau}{\tau_0} \right) = \frac{\Delta\tau}{\tau} \hat{a}_2' (1+2y) = \frac{\Delta\tau}{\tau} \hat{a}_2' \frac{1+2y}{1+y} \rightarrow \begin{array}{l} 0.33 * \hat{a}_2' \Leftrightarrow \text{night-time} \\ 0.13 * \hat{a}_2' \Leftrightarrow \text{day-time} \end{array}$$

by considering  $\Delta\tau = \Delta\tau_1 + \Delta\tau_2 \leq 30 \text{ } \mu\text{sec}$ . The values of  $\hat{a}_2'$  are given in fig.3.1.2 and fig.3.2.2 of Annex 3.

## 6. CONCLUSIONS

In this paper the compensation procedure for phase distortion on the pulse chirp compression are envisaged. In particular the estimation procedure can be reduced to the estimation of chirp slope and of cubic term of frequency series development of ionosphere high frequency uniform equivalent model.

The research range necessary in the estimation of the parameters has been taken into account in particular in the transition phase acquisition tracking; in fact we have pointed out that in the transition between acquisition and tracking (due to the few preliminary information we have on



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Mars Ionosphere), the “research area” allowed in a single cycle of the contrast-block is not sufficient.

In addition in annex 3 the quantization step of the frequency variable has been investigated.



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## **ANNEX 1**

### **FLOW CHART CONTRAST METHOD**



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## FLOW CHART CONTRAST

Signal  $s(t)$ , 490 samples:

$$s(t) = A \exp(j\pi\mu(t-t_0)^2) \text{rect}_{250\mu\text{sec}}(t-t_0)$$

A=input

$f_d$ =input → Banda della Ref\_Fun(f) centrata in  $f_0$

$B_c$ =1 MHz

$T$ =250 $\mu$ sec

$\mu$ =  $B_c / T$

$t_0$ =input

$a_{2-in}$  = input

$\Delta a_2$ =6.28\*10<sup>-12</sup> [rad/Hz<sup>2</sup> ]

endloop= input → numero prove del ciclo del contrasto

$t$  → 350 $\mu$ sec (490 samples):

$t_i = -125*10^{-6} + i/fs, \dots, -125*10^{-6} + i*1/fs, \dots, 225*10^{-6}$

$fs$ =1.4 MHz

$S = s(t_i) \quad i = 1, \dots, 490$   
 $S = 0 \quad i = 491, \dots, 512$

$S(f) = \text{FFT}\{s(t_i)\}$ , 512 samples  
 $f = -fs/2 + fs/512 + f_0, \dots, fs/2 + f_0$   
step= $fs/512$



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### Ionospheric Distortion (Gamma Model)

IF  
 GAMMA  
 MODEL

$$f_p(z) = f_{p\text{-max}} \frac{z - h_o}{b} \exp\left(1 - \frac{z - h_o}{b}\right) \delta_{-1}(z - h_o)$$

b=input (km)  
 ho=120 Km  
 H=input (Km)  
 fo= carrier= input  
 fp<sub>max</sub>= input

$$\Delta\phi(f) = \frac{4\pi f}{c} \int_{h_o}^H \left[ \sqrt{1 - \left(\frac{f_p(z)}{f}\right)^2} - 1 \right] dz$$

f= array\_512 elements  
 f=-fs/2+fs/512+f<sub>0</sub>,.....,fs/2+f<sub>0</sub>  
 step=fs/512

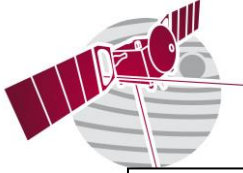
### Ionospheric Distortion (Uniform Model)

IF  
 UNIFORM  
 MODEL

fo= carrier= input  
 fp= input  
 h<sub>i</sub>= input  
 L<sub>eq</sub>= input

$$\Delta\phi(f) = \frac{4\pi f}{c} \int_{h_i}^{h_i + L_{eq}} \left[ \sqrt{1 - \left(\frac{f_p}{f}\right)^2} - 1 \right] dz$$

f= array\_512 elements  
 f=-fs/2+fs/512+f<sub>0</sub>,.....,fs/2+f<sub>0</sub>  
 step=fs/512



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### Ionosferic Distortion (Reduced Model)

$\tau_o = 650 \mu\text{sec}$   
 $f_o = \text{carrier} = \text{input}$   
 $f_p = \text{input}$

$$a_2 = -2\pi\tau_o \left( \frac{f_p^2}{2(f_o^2 - f_p^2)^{\frac{3}{2}}} \right) \left[ \text{rad} / \text{MHz}^2 \right]$$

$$a_3 = 2\pi\tau_o \left( \frac{f_o f_p^2}{2(f_o^2 - f_p^2)^{\frac{5}{2}}} \right) \left[ \text{rad} / \text{MHz}^3 \right]$$

$$a_4 = -2\pi\tau_o \left( \frac{4f_o^2 f_p^2 + f_p^4}{8(f_o^2 - f_p^2)^{\frac{7}{2}}} \right) \left[ \text{rad} / \text{MHz}^4 \right]$$

$$\Delta\phi(f) = a_2(f - f_o)^2 + a_3(f - f_o)^3 + a_4(f - f_o)^4$$

$f = \text{array\_512 elements}$   
 $f = -fs/2 + fs/512 + f_o, \dots, fs/2 + f_o$   
 $\text{step} = fs/512$

IF  
 REDUCED  
 MODEL

### Ionosferic Distortion (TEC Model)

$f_o = \text{input}$

$a_2 = \text{input} \left[ \text{rad} / \text{MHz}^2 \right]$

$a_3 = -\frac{a_2}{f_o} \left[ \text{rad} / \text{MHz}^3 \right]$

$a_4 = -\frac{a_3}{f_o} \left[ \text{rad} / \text{MHz}^4 \right]$

$$\Delta\phi(f) = a_2(f - f_o)^2 + a_3(f - f_o)^3 + a_4(f - f_o)^4$$

$f = \text{array\_512 elements}$   $f = -fs/2 + fs/512 + f_o, \dots, fs/2 + f_o$   
 $\text{step} = fs/512$

IF  
 TEC  
 MODEL



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Definizione Funzione di Riferimento:  
**Ref\_fun**

$$k = \sqrt{\frac{4B_c}{T}}$$

$$h(t_i) = \frac{k}{2} \exp\left[j \frac{\pi B_c}{T} t_i^2\right]$$

$t_i = -125 \cdot 10^{-6} + i/f_s, \dots, -125 \cdot 10^{-6} + i \cdot 1/f_s, \dots, 125 \cdot 10^{-6}$

25

W(t) =  $\cos^2(\pi t_i/T)$   
 $h(t_i) = h(t_i)W(t)$

NO

Fresnel Ripple  
 Extraction?

YES

**Ref\_fun = H(f) = FFT{h(t<sub>i</sub>)}**

**Ref\_fun\_def(f) = Ref\_fun(f)**

$f = f_0 - f_d/2, \dots, f_0 + f_d/2$

**Ref\_fun\_def(f) = Ref\_fun\_def(f<sub>0</sub>)**  
 $f = f_0 - f_s/2, \dots, f_0 - f_d/2 - f_s/512$   
 $f = f_0 + f_d/2 + f_s/512, \dots, f_0 + f_s/2$   
 step =  $f_s/512$

Inizio ciclo Contrasto

YES

Open Loop?

NO

**Best Fitting: Levenberg-Macquardt**  
 B = 1 MHz

$$\Delta\phi'(f) = a'_{2bf}(f - f_o)^2 + a'_{3bf}(f - f_o)^3 + a'_{4bf}(f - f_o)^4$$

A

b=1

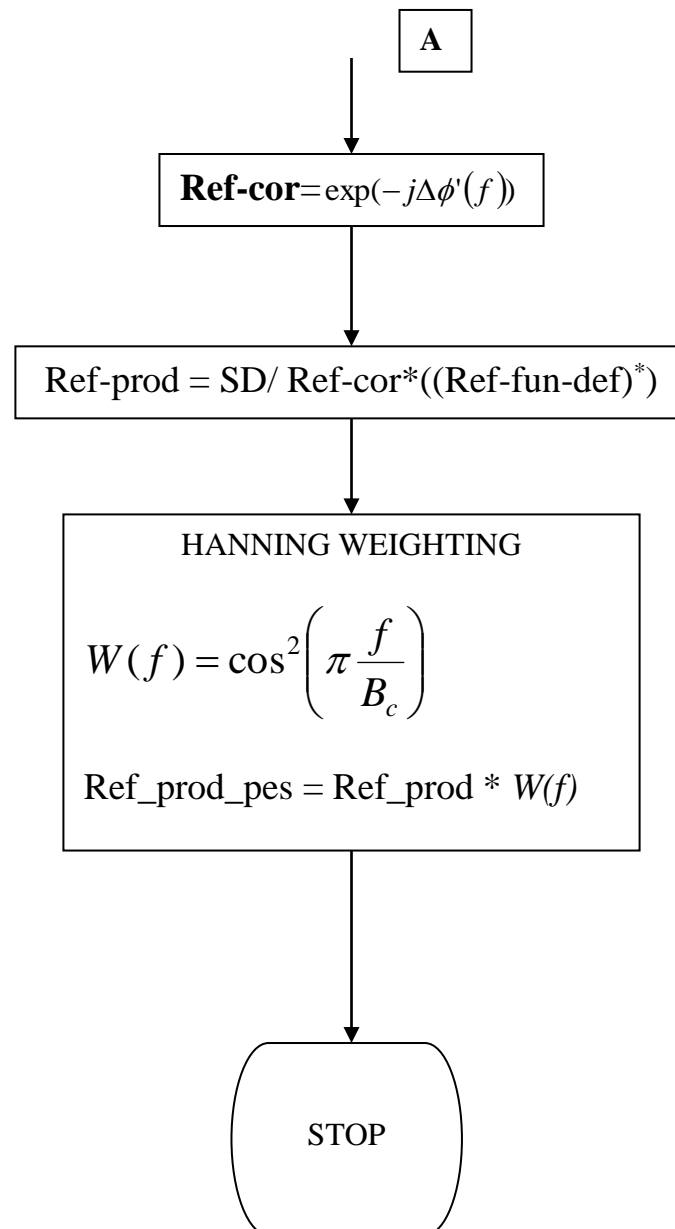
C

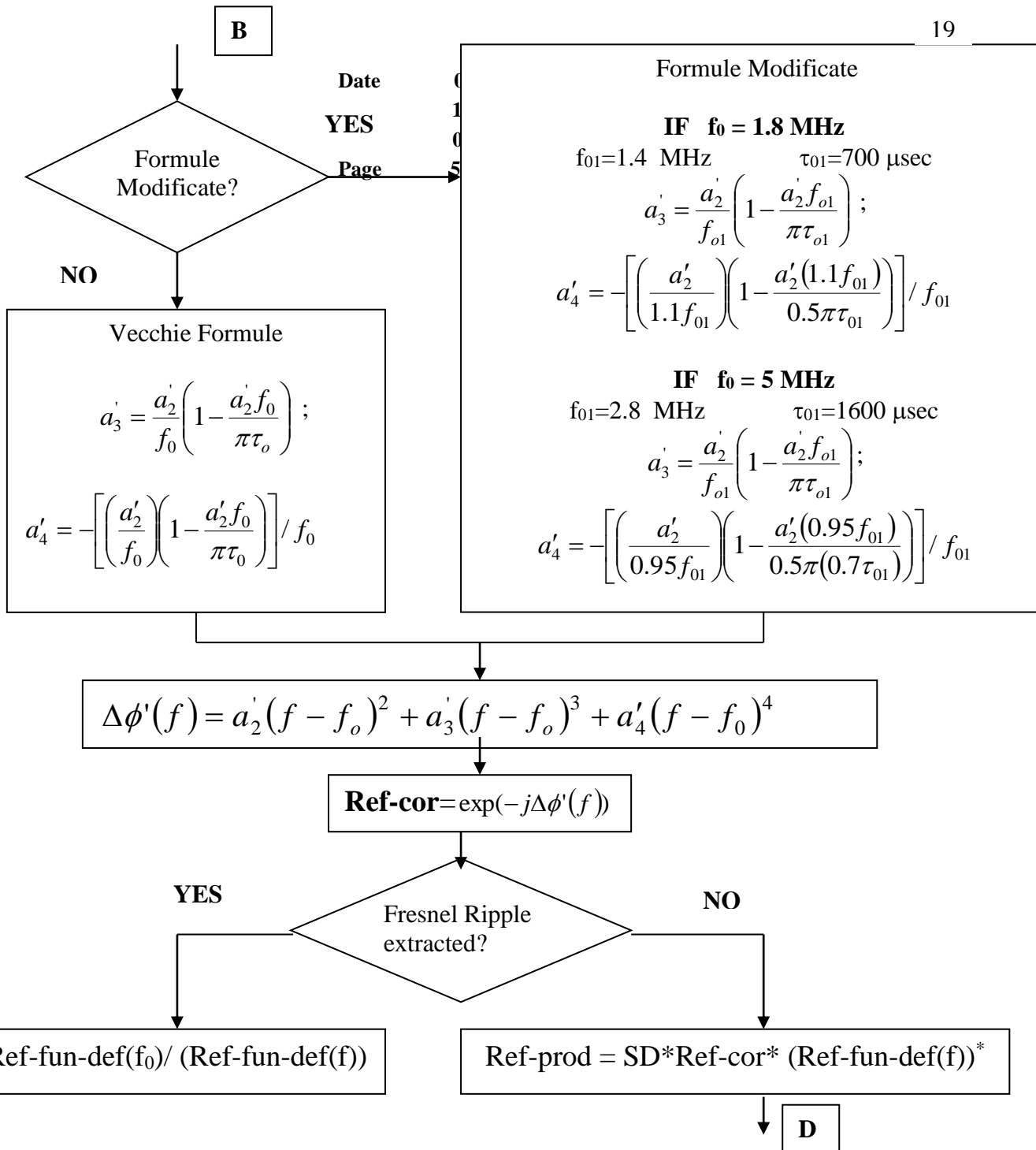
$$a'_2 = a'_{2-in} + (b - \frac{endloop}{2})\Delta a_2$$

B



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**E**

**D**

HANNING WEIGHTING  

$$W(f) = \cos^2\left(\pi \frac{f}{B_c}\right)$$

$$\text{Ref\_prod\_pes} = \text{Ref\_prod} * W(f)$$

Ref\_prod\_pes = Ref-prod

$s(i, a'_2) = \text{IFFT}(\text{Ref-prod\_pes})$

$$C_A(b) = \sum_{n_o - \frac{N}{2}}^{n_o + \frac{N}{2}} |s(i, a'_2)|$$

$$n_o = \text{input}, N = 512$$

$\hat{a}_2 = a'_2$   
 $C_A = C_A(b)$

**YES**

b=1

**NO**

b>1 AND C<sub>A</sub>(b)<C<sub>A</sub>

**NO**

**YES**

$\hat{a}_2 = a'_2$   
 $C_A = C_A(b)$

b < endloop

**YES**

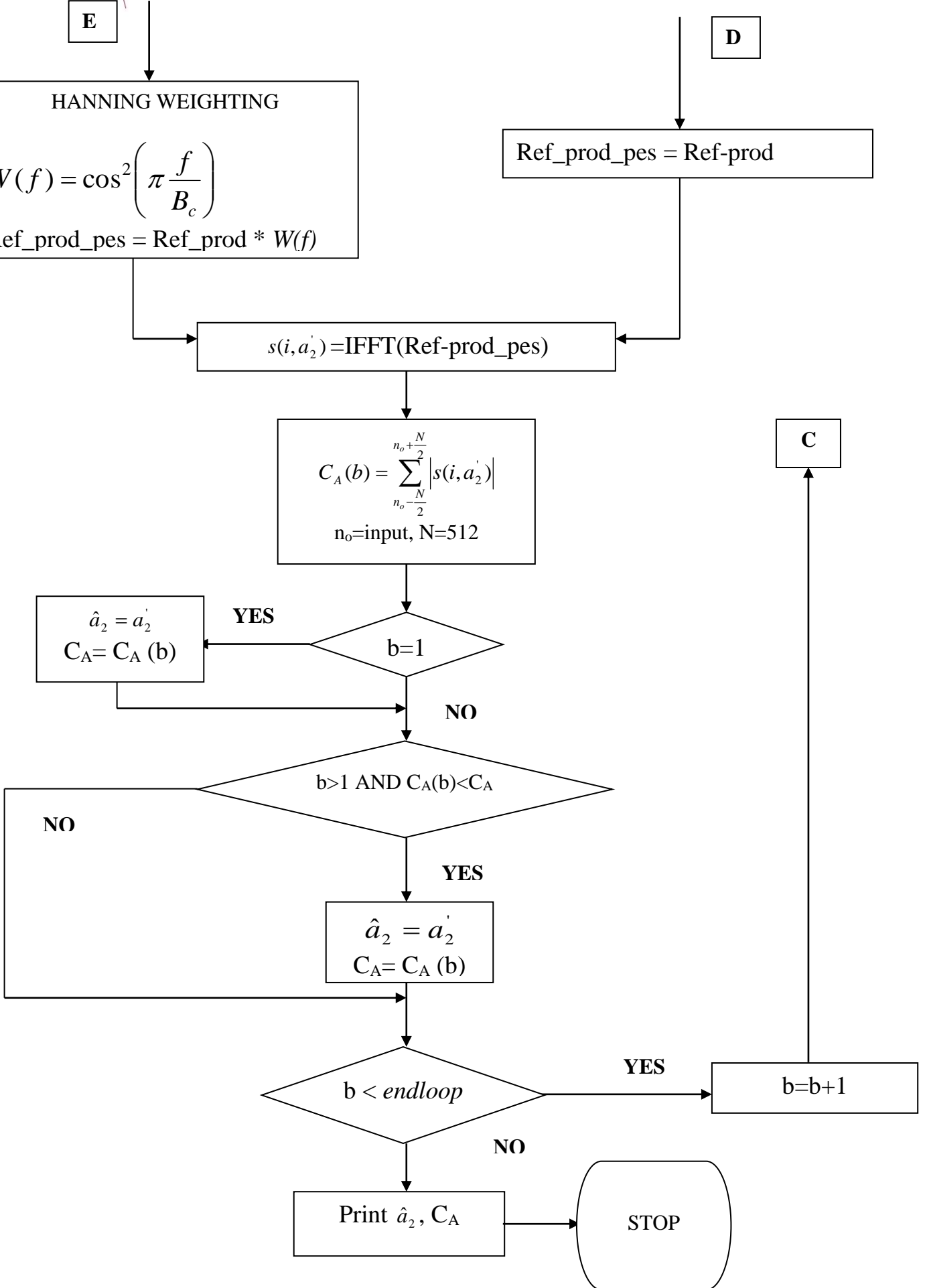
b=b+1

**NO**

Print  $\hat{a}_2, C_A$

STOP

**C**





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- [1] Lillis, R.J., Fillingim, M.O., Brain, D.A., 2011. Three-dimensional structure of the martian night side ionosphere: Predicted rates of impact ionization from Mars Global Surveyor magnetometer and electron reflectometer measurements of precipitating electrons. *J. Geophys. Res.* 116, A12317. <http://dx.doi.org/10.1029/2011JA016982>, 2011.
- [2] Safaeinili, A., Kofman, W., Nouvel, J.F., Herique, A., Jordan, R.L., 2003. Impact of Mars ionosphere on orbital radar sounder operation and data processing. *Planet Space Sci.* 51 (7–8), 505–515.
- [3] Safaeinili, A. et al., 2007. Estimation of the total electron content of the martian ionosphere using radar sounder surface echoes. *Geophys. Res. Lett.* 34, L23204. <http://dx.doi.org/10.1029/2007GL032154>.
- [4] W.B. Hanson et al., The Martian Ionosphere as observed by the Viking retarding potential analyzers, *J. Geophys. Res.*, 82, 4351-4363, 1977.
- [5] C.E. Cook, M. Bernfeld, Radar Signals, Academic Press, New York, 1967
- [6] D.Biccari, G.Picardi, R.Seu , ADAPTIVE COMPENSATION OF MARS IONOSPHERE DISPERSION IN THE MARSIS EXPERIMENT IGARSS 2001
- [7] G.Picardi ELABORAZIONE DEL SEGNALE RADAR – Fd. Franco Angeli ‘88