Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

MNRAS **457,** 3084–3095 (2016) doi:10.1093/mnras/stw151

The Cepheids of NGC 1866: a precise benchmark for the extragalactic distance scale and stellar evolution from modern *UBVI* photometry

I. Musella,[1★] M. Marconi,[1★] P. B. Stetson,[2★] G. Raimondo,[3] E. Brocato,[4] R. Molinaro,[1] V. Ripepi,[1] R. Carini,[4] G. Coppola,[1] A. R. Walker[5] and D. L. Welch[6]

[1]*INAF – Osservatorio Astronomico di Capodimonte, Via Moiariello 16, I-80131 Napoli, Italy*
[2]*Dominion Astrophysical Observatory, NRC-Herzberg, 5071 West Saanich Road, Victoria, BC V9E 2E7, Canada*
[3]*INAF – Osservatorio Astronomico Collurania, via M. Maggini, I-64100 Teramo, Italy*
[4]*INAF – Osservatorio Astronomico di Roma, Via Frascati 33, I-00044 Monte Porzio Catone, Italy*
[5]*Cerro Tololo Inter-American Observatory, National Optical Astronomy Observatory, Casilla 603, La Serena, Chile*
[6]*Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, L8 S 4M1, Canada*

Accepted 2016 January 14. Received 2016 January 13; in original form 2015 December 2

ABSTRACT

We present the analysis of multiband time series data for a sample of 24 Cepheids in the field of the Large Magellanic Cloud cluster NGC 1866. Very accurate *BVI* Very Large Telescope photometry is combined with archival *UBVI* data, covering a large temporal window, to obtain precise mean magnitudes and periods with typical errors of 1–2 per cent and of 1 ppm, respectively. These results represent the first accurate and homogeneous data set for a substantial sample of Cepheid variables belonging to a cluster and hence sharing common distance, age and original chemical composition. Comparisons of the resulting multiband period-luminosity and Wesenheit relations to both empirical and theoretical results for the Large Magellanic Cloud are presented and discussed to derive the distance of the cluster and to constrain the mass–luminosity relation of the Cepheids. The adopted theoretical scenario is also tested by comparison with independent calibrations of the Cepheid Wesenheit zero-point based on trigonometric parallaxes and Baade–Wesselink techniques. Our analysis suggests that a mild overshooting and/or a moderate mass-loss can affect intermediate-mass stellar evolution in this cluster and gives a distance modulus of 18.50 ± 0.01 mag. The obtained *V,I* colour-magnitude diagram is also analysed and compared with both synthetic models and theoretical isochrones for a range of ages and metallicities and for different efficiencies of core overshooting. As a result, we find that the age of NGC 1866 is about 140 Myr, assuming $Z = 0.008$ and the mild efficiency of overshooting suggested by the comparison with the pulsation models.

Key words: stars: distances – stars: evolution – stars: variables: Cepheids – galaxies: star clusters: individual: NGC 1866.

1 INTRODUCTION

Classical Cepheids play a fundamental role in the calibration of the extragalactic distance scale thanks to their characteristic period–luminosity (PL) and period–luminosity–colour (PLC) relations. In particular, the application of a Large Magellanic Cloud (LMC)-based PL relation to external galaxies observed with the *Hubble Space Telescope* (*HST*) has led to the calibration of secondary distance indicators and in turn to an estimate of the Hubble constant (H_0; see Freedman et al. 2001; Saha et al. 2001; Sandage et al. 2006; Riess et al. 2011, and references therein).

The LMC distance has traditionally played a crucial role in the extragalactic distance determination, with values lower than 18.50 mag implying the so called short distance scale and values larger than 18.50 mag suggesting a 'long distance scale'.

One of the most important issues to be considered when dealing with the Cepheid PL relation is its dependence on the metallicity. The universality of the PL relation, and the possibility that its slope and/or zero-point might depend on the chemical composition, have been actively debated for two decades, on both observational and theoretical grounds, (see e.g. Gould 1994; Sasselov et al. 1997; Kennicutt et al. 1998; Fiorentino et al. 2002; Sakai et al. 2004; Marconi, Musella & Fiorentino 2005; Sandage et al. 2006; Bono et al. 2008; Romaniello et al. 2008; Marconi et al. 2010; Freedman & Madore 2011; Gerke et al. 2011; Fausnaugh et al. 2015), with controversial results. No general consensus has been

★ E-mail: ilaria.musella@oacn.inaf.it (IM); marcella.marconi@oacn.inaf.it (MM); Peter.Stetson@nrc-cnrc.gc.ca (PBS)

reached so far. Empirical calibrations of the LMC PL and PLC relations are generally based on field Cepheids, implying the presence of uncertainties due to the range in distance (depth effects; van der Marel & Cioni 2001), metallicity (see e.g. Luck et al. 1998; Davies et al. 2015, and references therein) and extinction (differential reddening, see e.g. Haschke, Grebel & Duffau 2011, and references therein).

On the other hand, any theoretical scenario for pulsation models aimed at providing robust support to empirical calibrations needs to be tested against observational constraints based on statistically significant samples of Cepheids with accurate light curves.

The populous blue LMC cluster NGC 1866 is already known to host an exceptionally rich sample of more than 20 Cepheids (Musella et al. 2006; Welch & Stetson 1993). One of these was identified by Musella et al. (2006) in a preliminary analysis of the proprietary *BVI* Very Large Telescope (VLT) data. It is unquestionable that such a unique sample of Cepheids – likely all members of the cluster and at the same distance, chemical composition and age – would constitute a milestone in our understanding of the Cepheid pulsational scenario. Indeed, it offers an unprecedented opportunity to investigate both empirical and theoretical estimates of the luminosity and colour of the pulsating structures and their relation with the observed periods. For this reason, many authors have studied the NGC 1866 Cepheids (Welch 1991; Welch & Stetson 1993; Gieren, Richtler & Hilker 1994; Walker 1995; Gieren et al. 2000; Storm et al. 2005; Testa et al. 2007) in both the optical and near-infrared bands, and have tested different methods to calibrate the PL relations in different filters. In Brocato et al. (2004), we already discussed the sample of the 23 known Cepheids in NGC 1866, concluding that unfortunately only 4–6 Cepheids had light curves accurate enough to allow a meaningful determination of their luminosities and colours. On the basis of such a tantalizing situation, we took advantage of assigned observing time at the ESO VLT to perform an accurate photometric investigation of the cluster field, with the aim of securing suitable data constraining the light curves of all the member Cepheids. Moreover, to get accurate information about radial velocities and chemical abundances of the stars in NGC 1866, we have performed FLAMES@VLT (Fibre Large Array Multi Element Spectrograph mounted on VLT) spectroscopic observations for 30 stars (19 belonging to the cluster and 11 to the LMC field), including three Cepheids (Mucciarelli et al. 2011; Molinaro et al. 2012). Mucciarelli et al. (2011) found that, as far as the chemical composition is concerned, the cluster stars are reasonably homogeneous. Indeed, they appear to share the same abundances within the uncertainties, and this property is independent of the evolutionary status. The average iron abundance is [Fe/H] = -0.43 ± 0.01 dex, with a dispersion $\sigma = 0.04$ dex. For the three spectroscopically investigated Cepheids Molinaro et al. (2012), adopting the same procedure used in Mucciarelli et al. (2011), found values fully consistent with the average iron content. Moreover, Molinaro et al. (2012) applied the CORS Baade–Wesselink (BW) method (Ripepi et al. 1997) to a sample of 11 Cepheids, using radial velocities obtained both from our FLAMES investigation and from literature data, and light curves based on a part of the *UBVI* data used in this paper complemented with *K* data by Testa et al. (2007). In this way, they obtained a direct estimate of the distance modulus of NGC 1866, $\mu_0 = 18.51 \pm 0.03$ mag (see Molinaro et al. 2012, for details).

In this paper, we analyse the properties of NGC 1866 Cepheids, relying on an extensive multifilter data set not only to derive information about the distance, but also to test pulsational and evolutionary theoretical models.

The paper is organized as follows. We present observations and data reduction in Section 2 and Cepheid properties in Section 3. The adopted pulsational models are briefly described in Section 4. To derive the distance of NGC 1866, we have applied in Section 5 the theoretical PL and Wesenheit relations, and in Section 6 two empirical calibrations based on trigonometric parallaxes and the BW method. In Section 7, a deep and accurate *V,V−I* colour–magnitude diagram (CMD) is presented and compared with theoretical isochrones to derive additional information on the cluster properties. The conclusions close the paper.

2 OBSERVATIONS AND DATA REDUCTION.

Our photometric data base comprises 1471 CCD images (see Table 1).

The median seeing of the images was 1.8 arcsec, but it is worth noting that for the *B*, *V*, and *I* filters, we also have a set of high-precision photometric proprietary data obtained with FORS1@VLT. For this data set, the detector was a 2048×2048 Tektronix CCD with 24µ × 24µ pixels. Projected on the sky, the scale is 0.2 arcsec pixel^{-1} for a total field of view of 6.8 arcmin × 6.8 arcmin. We have obtained time series photometry on one pointing centred on NGC 1866 in the *BVI* filters with exposures of 60 s in each band. The observations were efficiently carried out in service mode, with the constraint of seeing better than 0.7 arcsec because the target cluster is very crowded and many Cepheids are located near the cluster centre.

The reduction and calibration of all the photometric data was carried out with the DAOPHOT/ALLFRAME packages (Stetson 1987, 1994) – which combine excellent precision with a large degree of automation – and using standard methodologies as described in, for example, Stetson (2000) and Stetson, Catelan & Smith (2005). Special care has been taken in deriving an accurate point spread function for each image because of the high degree of crowding not only in the central regions of the cluster but also in its outskirts.

The calibration was based on local standards in NGC 1866. The median internal photometric precision based on these local standards is 0.01, 0.001, 0.0008 and 0.002 mag for *U*, *B*, *V* and *I* bands, respectively, using all the data sets, and 0.0006, 0.0004 and 0.0008 mag for the *B*, *V* and *I* bands, respectively, considering only the VLT data. Concerning the external accuracy of our photometry, the root-mean-square differences of our mean magnitudes for Landolt's standards (Landolt 1992), on a star-by-star basis, is of ∼0.029, ∼0.016, ∼0.013 and ∼0.016 mag in the *U*, *B*, *V* and *I* bands, respectively. These differences represent 180, 378, 400, and 250 individual Landolt standards, so our mean photometric system differs from Landolt's mean system by not less than ∼0.001 mag in *B*,*V* and *I*, and ∼0.002 mag in *U*. This is probably a fundamental limit for the external, absolute accuracy that can be obtained with CCD measurements, given that we use different filters and detectors than Landolt used.

3 NGC 1866 CEPHEIDS

The position and the light curves in the *UBVI* bands of the Cepheid identified in Musella et al. (2006) are reported in Figs 1 and 2, respectively. The coordinates of this Cepheid are α = 05:13:20.79 and δ = −65:24:57.8 (J2000) at about 3.5 arcmin from the cluster centre. The light curves of all other previously known variables are plotted in Fig. 3. In particular, grey dots are data from the MACHO project, red dots are VLT data and open circles are all the other archival data (see Table 1).

Table 1. Log of the observations, instrumental setup and number of images obtained in each filter.

Dates	Telescope	Detector	N_U	N_B	N_V	N_I
1986 December 18–23	CTIO 0.9 m	RCA	16	42	37	21
1987 November 22–23	CTIO 4.0 m	RCA1	3	17	18	–
1987 November 30–December 01	CTIO 0.9 m	TI1	1	1	1	–
1988 January 17–19	CTIO 4.0 m	TI1	2	2	–	–
1990 September 11	CTIO 4.0 m	TI1	5	5	5	5
1990 October 25–26	CTIO 0.9 m	TI3	6	6	6	6
1990 December 13	CTIO 4.0 m	Tek512	–	5	6	6
1990 December 16–17	CTIO 0.9 m	Thomson 1024	4	6	9	4
1991 January 22	CTIO 1.5 m	Tek1024	2	3	3	3
1991 February 01	CTIO 0.9 m	Tek1024	–	2	2	2
1991 March 03	CTIO 4.0 m	Thomson 1024	–	6	6	6
1993 January 14–2000 January 03*	Mt. Stromlo 50 in	CCD3 and CCD4	–	–	309	–
1993 November 02–05	CTIO 0.9 m	Tek1024	–	2	2	–
1994 November 24–28	CTIO 0.9 m	Tek2K_4	–	13	13	13
1999 December 01–04	CTIO 4.0 m	Mosaic2	–	44	44	46
2001 January 17–19	CTIO 4.0 m	Mosaic2	–	–	8	8
2001 March 23–30	MPI/ESO 2.2 m	WFI	–	24	16	24
2001 April 06–12	Mt. Stromlo 74 in	CCD17	–	29	38	34
2001 December 12–13	CTIO 1.5 m	Site2K_6	–	1	1	1
2003 October 03–December 25	ESO VLT 8.0 m	FORS1	–	69	90	62
2008 August 26–28	CTIO 4.0 m	Mosaic2	–	–	16	16

Note. *MACHO data.

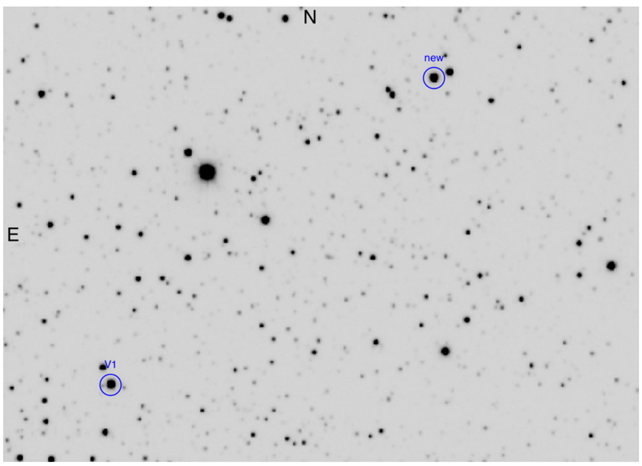

Figure 1. Identification map (a region of 2 arcmin × 1.5 arcmin) for the newly discovered Cepheid in the cluster NGC 1866.

The location in the extreme cluster periphery of the Cepheid identified in Musella et al. (2006) may raise serious doubts about its membership. Lupton et al. (1989) suggest that NGC 1866 is not tidally limited but is embedded in an unbound halo. Noyola & Gebhardt (2007), imposing the existence of a tidal radius, find an half-light radius of 49.7 arcsec: much less than the distance of this Cepheid (∼3.4 arcmin). However, in the following, we will demonstrate that both the location in the CMD and the pulsational properties of this variable appear in close agreement with the behaviour of the other cluster Cepheids. Obviously, to have a firmer conclusion about the membership of this distant Cepheid, the mean radial velocity should also be measured through accurate spectroscopic observations.

The obtained light curves show the superior accuracy of the VLT photometry when compared with the other data sets. In particular, for the MACHO data in the *V* band, it was necessary to reject several scattered phase points. We use all data sets, spanning a large time window, to determine accurate periods using PERIOD04 (Lenz & Breger 2005). This program allows us to determine also the error on the derived period via Monte Carlo simulations. On the other hand, to obtain mean (intensity-averaged) magnitudes and colours as accurately as possible, we fitted the light curves in all available bands with a smoothing spline obtained with a C code written by one of the authors (RM). In particular, for the *B*, *V* and *I* light curves we used only the VLT data, adopting the other data sets only for the *U* filter. All the derived periods, *UBVI* mean magnitudes and

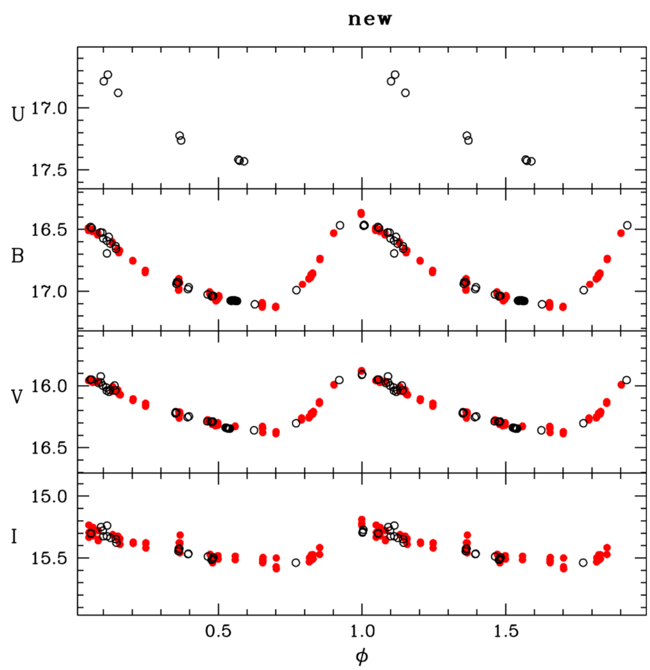

Figure 2. Light curves of the newly discovered Cepheid in NGC 1866. Marks are the same as used in Fig. 3.

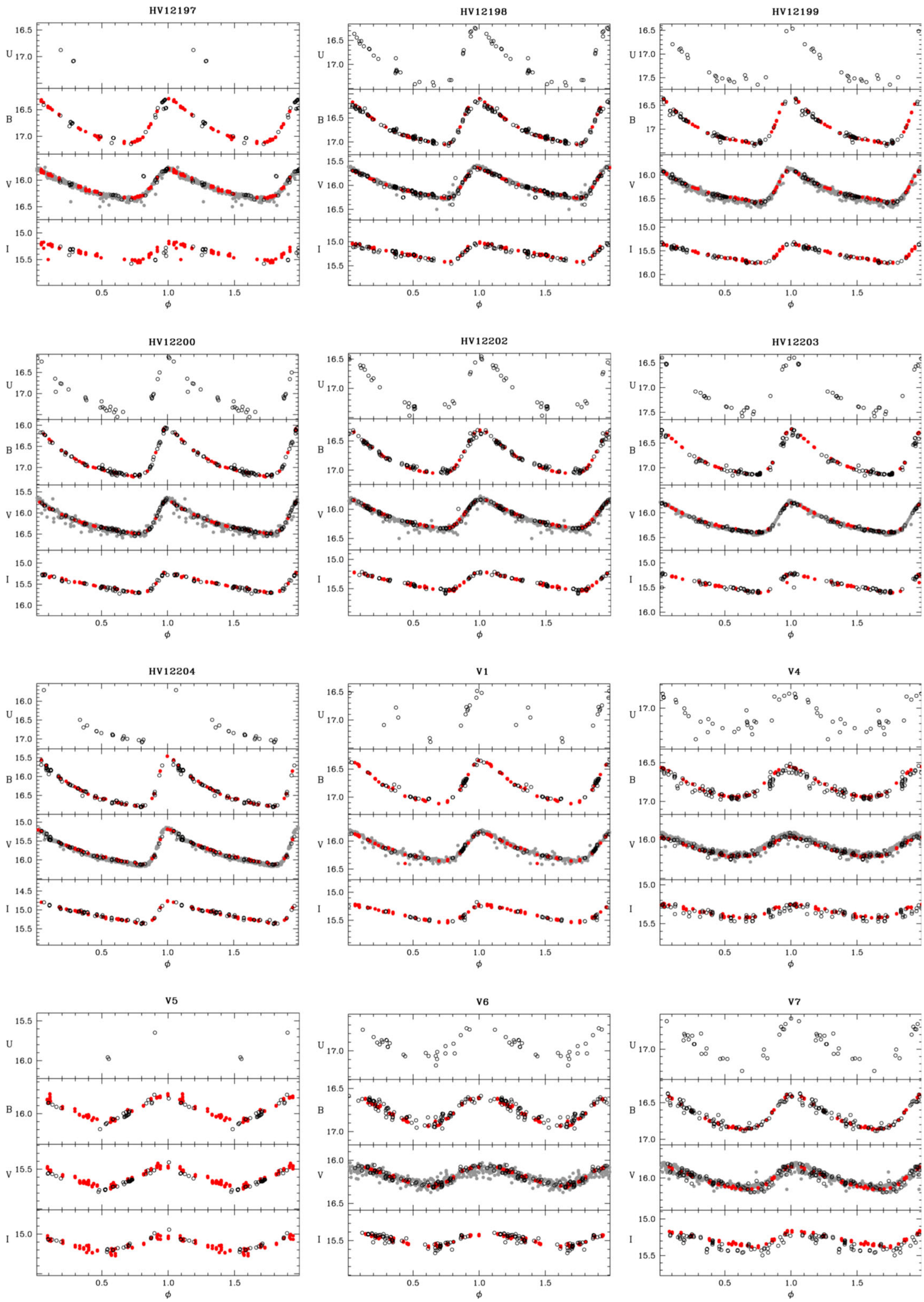

Figure 3. Light curves of NGC 1866 Cepheids. Grey dots are MACHO data, red dots are VLT data and open circles are the other archival data (see the text for details).

amplitudes A_λ (for each λ filter) are reported in Table 2, but we will not use the *U* band in the following analysis. As an estimate of the uncertainty in the calculated mean magnitudes, we report in Table 2 the rms of the residuals of the data around the fitted curves; of course, these residuals will not fully represent consistent photometric errors resulting from the blending conditions of individual stars. Table 2 also reports the complementary *K* band data obtained by Testa et al. (2007).

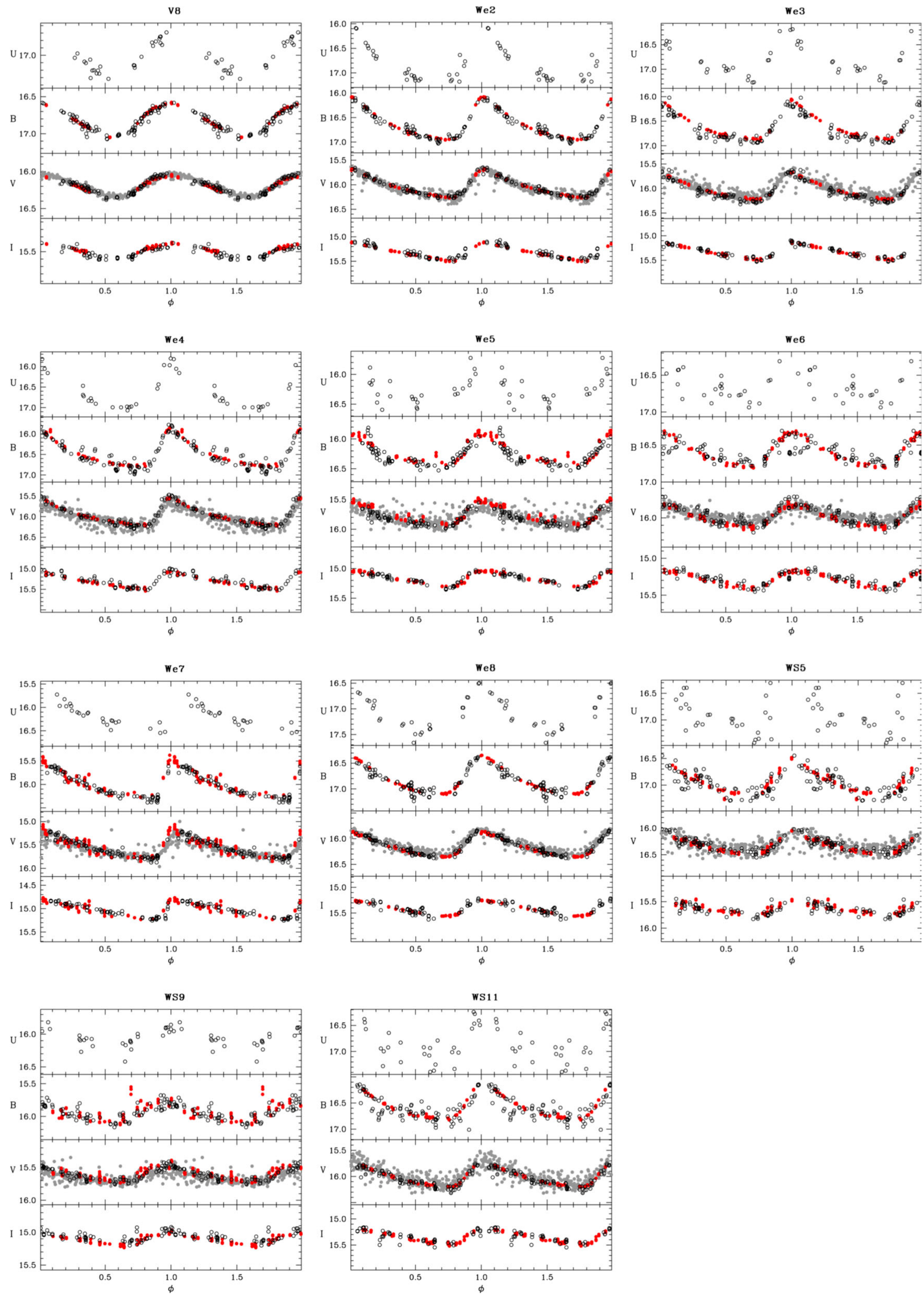

Figure 3 – *continued.*

Analysing the Cepheid light curves in Fig. 3, we note that We5, We7 and WS9 appear to be affected by more noise than the other objects, in particular in the *B* and *V* bands, and we can see similar but less evident scatter also for We4 and WS5. This is likely the result of blending due to the location of these variables in the central very crowded region, combined with varying seeing conditions. Moreover, a time series analysis for double-mode behaviour for such crowded stars is likely inconclusive given the long intervals between observational epochs and the relatively short periods with excellent seeing. The Cepheids in the external region have

Table 2. Properties of the Cepheids in NGC 1866.

ID	P (d)	U (rms) (mag)	B (rms) (mag)	V (rms) (mag)	I (rms) (mag)	K (mag)	A_U (mag)	A_B (mag)	A_V (mag)	A_I (mag)
V6	1.944 262 ± 0.000 001	16.90 (0.04)	16.76 (0.01)	16.18 (0.01)	15.49 (0.01)	14.60	0.40	0.34	0.25	0.15
V8	2.007 ± 0.003	16.94 (0.04)	16.80 (0.01)	16.19 (0.01)	15.47 (0.02)	14.57	0.60	0.46	0.26	0.20
V5**	2.039 071 ± 0.000 004	–	15.93 (0.02)	15.60 (0.02)	15.14 (0.02)	–	–	0.32	0.26	0.18
HV12199	2.639 16 ± 0.000 01	17.10 (0.10)	16.90 (0.03)	16.29 (0.02)	15.57 (0.02)	14.70	1.13	0.99	0.70	0.41
HV12200	2.724 98 ± 0.000 02	16.92 (0.06)	16.74 (0.02)	16.16 (0.02)	15.48 (0.02)	–	1.49	1.18	0.82	0.51
We4*	2.860 36 ± 0.000 02	16.52 (0.09)	16.44 (0.04)	15.95 (0.04)	15.31 (0.02)	–	1.17	0.95	0.67	0.43
WS5*	2.897 80 ± 0.000 03	16.71 (0.06)	16.89 (0.03)	16.32 (0.02)	15.63 (0.03)	–	1.11	0.70	0.42	0.30
New	2.942 93 ± 0.000 02	–	16.80 (0.02)	16.17 (0.02)	15.42 (0.03)	–	–	0.72	0.47	0.27
HV12203	2.954 11 ± 0.000 02	17.03 (0.02)	16.77 (0.03)	16.14 (0.02)	15.42 (0.02)	14.58	1.13	0.88	0.61	0.39
We8	3.039 849 ± 0.000 001	17.04 (0.09)	16.79 (0.04)	16.14 (0.02)	15.41 (0.03)	14.52	0.92	0.74	0.50	0.33
We3	3.049 04 ± 0.000 02	16.74 (0.06)	16.54 (0.01)	15.99 (0.01)	15.32 (0.01)	–	1.06	0.81	0.59	0.36
WS11	3.053 30 ± 0.000 02	16.91 (0.13)	16.54 (0.02)	16.00 (0.01)	15.34 (0.02)	–	1.06	0.62	0.42	0.29
We2	3.054 85 ± 0.000 02	16.77 (0.20)	16.59 (0.01)	16.01 (0.01)	15.31 (0.01)	14.41	1.03	0.87	0.59	0.39
WS9*	3.069 45 ± 0.000 02	16.04 (0.03)	15.93 (0.06)	15.59 (0.04)	15.12 (0.02)	–	0.51	0.31	0.27	0.18
V1	3.084 55 ± 0.000 01	–	16.77 (0.02)	16.13 (0.01)	15.38 (0.01)	–	–	0.74	0.50	0.37
HV12202	3.101 20 ± 0.000 03	16.99 (0.12)	16.74 (0.03)	16.10 (0.01)	15.37 (0.01)	14.40	1.06	0.73	0.52	0.34
HV12197	3.143 71 ± 0.000 03	–	16.76 (0.02)	16.11 (0.02)	15.37 (0.02)	14.47	–	0.79	0.52	0.35
We5*	3.1745 ± 0.0001	16.19 (0.09)	16.22 (0.04)	15.75 (0.04)	15.17 (0.02)	–	0.88	0.55	0.42	0.27
We7*	3.232 27 ± 0.000 02	16.11 (0.10)	15.95 (0.07)	15.54 (0.07)	15.03 (0.05)	–	0.91	0.79	0.65	0.41
We6	3.289 94 ± 0.000 02	16.56 (0.06)	16.59 (0.02)	15.99 (0.02)	15.29 (0.02)	–	0.54	0.48	0.32	0.22
V4	3.318 ± 0.001	17.06 (0.06)	16.78 (0.03)	16.10 (0.01)	15.34 (0.01)	14.39	0.44	0.39	0.25	0.17
HV12204**	3.438 82 ± 0.000 02	–	16.23 (0.02)	15.72 (0.02)	15.08 (0.02)	14.26	1.32	0.92	0.67	0.60
V7	3.452 07 ± 0.000 01	16.91 (0.06)	16.64 (0.02)	16.00 (0.01)	15.27 (0.02)	14.32	0.69	0.47	0.32	0.22
HV12198	3.522 800 ± 0.000 006	16.88 (0.07)	16.64 (0.03)	15.98 (0.01)	15.23 (0.03)	14.32	1.26	0.92	0.63	0.36

Notes. *These Cepheids have scattered light curves and are not considered in the following analysis.
**The membership to NGC 1866 of these Cepheids was ruled out by Welch (1991) and they are not considered in the following analysis.

well-defined light curves. V5, V6 and V8 have periods and light curves typical of first overtone (FO) Cepheids. In particular, V5 is the brightest Cepheid and has a very blue colour but, as pointed out by Welch (1991), its membership seems to be excluded according to its mean radial velocity and large distance from the centre of the cluster. Welch (1991), on the basis of the radial velocity, also ruled out membership for HV12204.

4 PULSATION MODELS

For the interpretation of the observed properties of the variable stars in NGC 1866, we adopt the theoretical pulsation scenario developed by our team (see e.g. Bono, Marconi & Stellingwerf 1999; Marconi et al. 2005; Marconi et al. 2010, 2013, and references therein) and based on non-linear convective pulsation models for different assumptions on the metallicity. For each assumed fixed chemical composition, a wide range of model masses is explored and the luminosities are selected according to evolutionary predictions for the mass-luminosity relation of central helium-burning intermediate-mass stars. In the 'canonical' scenario, both mass-loss and convective overshooting are neglected, whereas in the 'non-canonical' assumption, an overluminosity of 0.25 dex at fixed mass is expected as a result of mild core overshooting and/or moderate mass-loss (see e.g. Chiosi, Wood & Capitanio 1993; Bono et al. 1999, for details). According to this theoretical scenario, the effect of variations in both the metallicity and the helium content on the predicted coefficients of the PL and PLC relations is not negligible (Bono et al. 1999; Caputo, Marconi & Musella 2000; Marconi et al. 2005). On the other hand, we can adopt the Wesenheit (WPL) relations (Madore 1982), defined as $WPL(\lambda_1, \lambda_2) = \lambda_1 - R_{\lambda_1\lambda_2} \times (\lambda_1 - \lambda_2)$, where λ_i are the bands used and $R_{\lambda_1\lambda_2} = A_{\lambda_1}/E(\lambda_1 - \lambda_2)$ is the ratio of total to selective absorption (see e.g. Cardelli, Clayton & Mathis 1989); this represents a reddening-free formulation of the PL relation with a reduced intrinsic scatter. In particular, Caputo et al. (2000) showed that the intrinsic scatter and the dependence on the metallicity of the WPL(*V,I*) relation is almost negligible.

The coefficients of the theoretical PL and WPL relations for the chemical composition of NGC 1866 ($Z = 0.008$, $Y = 0.25$), derived in the period range from $\log P = 0.4$ (corresponding to the minimum fundamental period in NGC 1866[1]) to $\log P = 1.5$ (corresponding to the maximum period in the OGLE[2] sample), are those by Fiorentino et al. (2002) and Fiorentino et al. (2007), respectively. In particular, the WPL relation was also derived with an explicit dependence on both the metallicity and the excess luminosity, $\log L/L_c$, where L_c is the canonical luminosity level. This allows us to check the theoretical models both for canonical ($\log L/L_c = 0$) and non-canonical ($\log L/L_c = 0.25$ dex) assumptions (see table 3 in Fiorentino et al. 2007). For the *V,I* filter combination, moving from the canonical to the non-canonical assumption, the WPL(*V,I*) relation gets fainter by about 0.21 mag.

5 THE PL AND PERIOD–WESENHEIT RELATIONS

Fig. 4 shows the *BVI* PL relations for the Cepheids identified here (black filled circles are the reliable fundamental Cepheids, asterisks are We5, We7 and WS9, crosses are We4 and WS5 and open star is HV12204). The red symbols represent the FO Cepheids (the

[1] We did not include shorter periods due to the expected deviation from linearity of the instability strip topology for models with mass lower than $\sim 4\,M_{\odot}$ (see Bono et al. 2001, for details)
[2] Optical Gravitational Lensing Experiment (see e.g. Soszynski et al. 2008 and references therein).

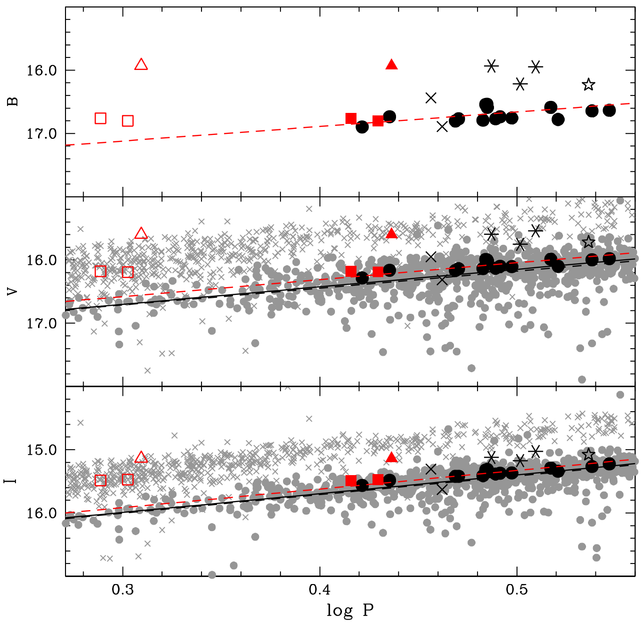

Figure 4. PL relation for NGC 1866 Cepheids in the *B*, *V* and *I* bands. In the *V* and *I* bands, our sample is compared with the OGLE one. See the text for symbols.

triangle is V5 and squares are V6 and V8, the two reliable FO Cepheids), plotted adopting both their own (open red symbols) and their fundamentalized periods[3] (filled red symbols). It is worth noting that at fixed period, even taking into account only the reliable Cepheids, the scatter in magnitude is significant (0.12, 0.09 and 0.07 mag in the *B*, *V*, and *I* bands, respectively). Given that our targets share the same age and chemical composition, this can only be ascribed to (i) the finite width of the instability strip (IS; see Bono et al. 1999; Caputo et al. 2000, for a detailed discussion), and ii) a possible mass-loss effect. For the *V* and *I* PL relations (middle and bottom panel of Fig. 4), we compared our sample with the OGLE one (grey crosses are FO Cepheids and grey filled circles the fundamental ones). The black straight lines represent the mean OGLE PL relations. It is evident that the NGC 1866 sample appears to be brighter than the OGLE III one (Soszynski et al. 2008). Indeed, when we overlay the OGLE III *V* and *I* PL slopes on the Cepheid population of NGC 1866,[4] we obtain a shift of $\Delta\mu_V = -0.12 \pm 0.02$ mag and $\Delta\mu_I = -0.08 \pm 0.02$ mag.

The black dotted lines and the red dashed lines in Fig. 4 represent the linear regression of the OGLE data and of our Cepheid sample, respectively, adopting the theoretical slopes for $Z = 0.008$ in the canonical assumption and in the same period range covered by OGLE ($0.4 < \log P < 1.5$) reported in table 6 of Fiorentino et al. (2002):

$$\overline{M_B} = -2.44 \log P - 0.93$$

$$\overline{M_V} = -2.75 \log P - 1.37$$

$$\overline{M_I} = -2.98 \log P - 1.95.$$

[3] The period the variable would have if it were a fundamental one, obtained as $\log P_{\rm fun} = \log P + 0.127$.

[4] We notice that in all our fits of the NGC 1866 sample, we include only the reliable fundamental Cepheids and the two FO V6 and V8, adopting their fundamentalized periods

Adopting these theoretical slopes we find for the OGLE sample $\mu_V^{\rm OGLE} = 18.904 \pm 0.007$ mag, $\mu_I^{\rm OGLE} = 18.826 \pm 0.005$ mag and for the NGC 1866 sample (only reliable Cepheids) $\mu_B^{\rm NGC\,1866} = 18.79 \pm 0.03$ mag, $\mu_V^{\rm NGC\,1866} = 18.77 \pm 0.02$ mag, $\mu_I^{\rm NGC\,1866} = 18.74 \pm 0.02$ mag (the adopted errors are the standard deviation of the mean). To derive the absolute distance moduli, we need to know the colour excess. One of the most used values for the colour excess of NGC 1866 and LMC is $E(B - V) = 0.06$ mag (see e.g. Schlegel, Finkbeiner & Davis 1998; Storm et al. 2005). However, we can also derive this value from the difference of the apparent moduli in two different bands. In particular, $E(B - V) = \mu_B - \mu_V$ and $E(V - I) = \mu_V - \mu_I$. Adopting the inferred apparent distance moduli, we obtain $E(V - I)_{\rm OGLE} = 0.078 \pm 0.009$ mag – a value that, combined with the extinction law by Cardelli et al. (1989), gives $E(B - V)_{\rm OGLE} = 0.064 \pm 0.007$ mag. From the apparent distance moduli of NGC 1866, we obtain $E(B - V)_{\rm NGC\,1866} = 0.02 \pm 0.04$ mag and $E(V - I)_{\rm NGC\,1866} = 0.03 \pm 0.03$ mag. The estimate for the LMC agrees quite well with the aforementioned value $E(B - V) = 0.06$ mag. However, the values that we obtain for NGC 1866 are significantly lower than the expected ones, but the distribution of the Cepheid individual colour excesses is not Gaussian and if we compute the medians, we obtain $E(B - V)_{\rm NGC\,1866} = 0.04 \pm 0.03$ mag and $E(V - I)_{\rm NGC\,1866} = 0.04 \pm 0.02$ mag. Moreover, it is worth remembering that all the other NGC 1866 stars that are bright enough to be significant when contaminating the Cepheids' photometry (either as binaries or due to coincidence along the line of sight) are going to be bluer than the Cepheids, thus affecting *B* more than *V* and *V* more than *I*. Perhaps only the very outer stars could be contaminated by redder field stars. For this reason, the reddening values determined by the apparent moduli differences are systematically underestimated. Due to the strength of the literature colour excess determination, we adopt for both the samples the value $E(B - V) = 0.06$ mag together with a ratio between A_V and $E(B - V)$ given by $R_V = 3.1$ (Cardelli et al. 1989), obtaining $\mu_0^{\rm OGLE} = 18.718$ mag and $\mu_0^{\rm NGC\,1866} = 18.58$ mag.

To reduce the problems due to the determination of the colour excess, we adopt the reddening free WPL relations. It is worth noting that the definition of the WPL relation is not unique: it depends on the adopted reddening law from which we derive the colour term. As theoretical WPL relations, we adopt those, dependent on metallicity *Z* and luminosity level $\log L/L_\odot$, obtained by Fiorentino et al. (2007) (see their table 3) using the absorption to reddening ratio from Dean, Warren & Cousins (1978) and Laney & Stobie (1993). For the *V* and *I* bands, $WPL(V, I) = I - 1.52(V - I) = -2.67 - 3.30\log P + 0.84\log L/L_\odot + 0.08\log Z$. On the other hand, Soszynski et al. (2008) have derived an empirical OGLE-based relation adopting $WPL(V, I) = I - 1.55(V - I)$. For this reason, we cannot perform a direct comparison. In Fig. 5, in the upper panel, we plot WPL(*V*,*I*) with 1.52 as colour term and compare OGLE and NGC 1866 data with the theoretical relation in the canonical assumption. In the bottom panel, we adopt the WPL relation with 1.55 as colour term and compare the data with the empirical relation by OGLE. Symbols are the same as in Fig. 4. If we adopt the OGLE WPL(*V*,*I*) relation, the difference between the distance moduli of NGC 1866 and LMC is $\Delta\mu = -0.018 \pm 0.014$ mag. Moreover, using the slope of the plotted canonical WPL(*V*,*I*) relation, we obtain $\mu_0^{\rm OGLE} = 18.717 \pm 0.003$ mag and $\mu_0^{\rm NGC\,1866} = 18.71 \pm 0.01$ mag. This result implies that removing the reddening problem, the inferred LMC and NGC 1866 moduli are in good agreement. The values based on the predicted canonical WPL relation seem to favour the 'long distance scale' for the LMC. On the other hand, as noted above, in the non-canonical assumption

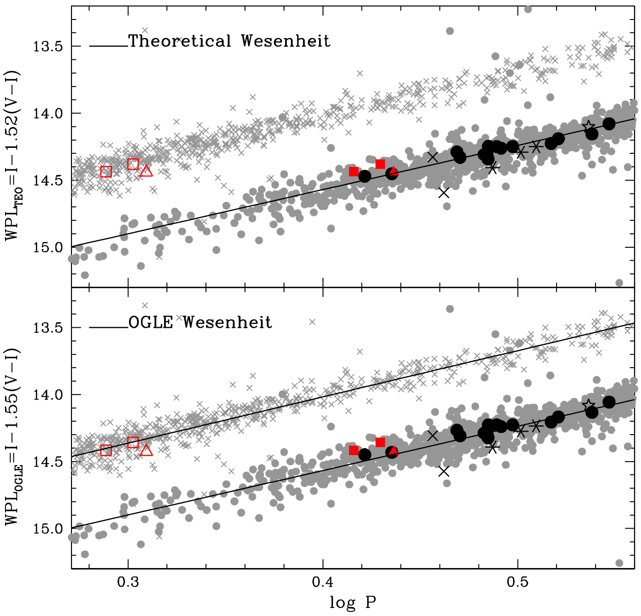

Figure 5. WPL(V, I) relation of NGC 1866 Cepheid sample compared with the OGLE one. In the upper panel, we plot the WPL relation with 1.52 as the colour term to use the theoretical slope by Fiorentino et al. (2007). In the bottom panel, we adopt the OGLE WPL relation with 1.55 as the colour term coefficient.

($\log L/L_c = 0.25$ dex), the predicted Wesenheit function is fainter by about 0.21 mag and in turn the inferred distance moduli are $\mu_0^{\rm OGLE} = 18.507 \pm 0.003$ mag and $\mu_0^{\rm NGC\ 1866} = 18.50 \pm 0.01$ mag.

6 THE EMPIRICAL ROUTE TO THE DISTANCE OF NGC 1866

To test the results obtained with the theoretical relation, we discuss two empirical calibrations of Cepheid distances based on two direct methods: (i) the trigonometric parallaxes available for a sample of Galactic Cepheids and obtained with the *Hipparcos* satellite (van Leeuwen et al. 2007) and the *HST* (Benedict et al. 2007; Riess et al. 2014; Casertano et al. 2015); (ii) the BW method applied to 70 Galactic (Storm et al. 2011a) and 36 LMC Cepheids (Storm et al. 2011b).

Following the same procedure adopted in Ripepi et al. (2012), their sample of MW Cepheids with distances from trigonometric parallaxes by van Leeuwen et al. (2007) and Benedict et al. (2007) has been selected (see their table 6) retaining only the variables with $\delta\pi/\pi \le 0.2$. For variables present in both the *Hipparcos* and the *HST* sets, Ripepi et al. (2012) have used a weighted average of the two parallaxes. Moreover, we add two Cepheids, SY Aur and SS CMa, with very accurate parallaxes obtained by Riess et al. (2014) and Casertano et al. (2015), respectively. In Table 3, we report the parallax, the V and I magnitudes (taken from table A1 in van Leeuwen et al. 2007) and the Lutz–Kelker (LK) correction (calculated according to Benedict et al. 2007) for the Galactic Cepheids used in our analysis.

In the upper panel of Fig. 6, we show WPL(V,I) (with the theoretical colour coefficient of 1.52) for these Galactic Cepheids (green symbols) and for our sample (black and red symbols). The green line represents the relation derived from a weighted fit of the Galactic Cepheids absolute magnitudes,

Table 3. Galactic Cepheids with known parallaxes used to test the calibration of Cepheid distances obtained by means of the theoretical WPL(V, I) relation.

ID	*logP* [d]	π [mas]	σ_π [mas]	V [mag]	I [mag]	LK [mag]
β Dor	0.993	3.26	0.14	3.757	2.929	−0.0184
RT Aur	0.572	2.40	0.19	5.448	4.811	−0.0627
ζ Gem	1.006	2.74	0.12	3.915	3.070	−0.0192
ℓ Car	1.551	2.03	0.16	3.698	2.522	−0.0621
X Sgr	0.846	3.17	0.14	4.564	3.635	−0.0195
W Sgr	0.880	2.30	0.19	4.670	3.834	−0.0682
FF Aql	0.650	2.64	0.16	5.373	4.513	−0.0367
δ Cep	0.730	3.71	0.12	3.953	3.200	−0.0105
SS CMa	1.092	0.428	0.054	9.925	8.470	−0.1192
SY Aur	1.006	0.348	0.038	9.066	7.854	−0.1592

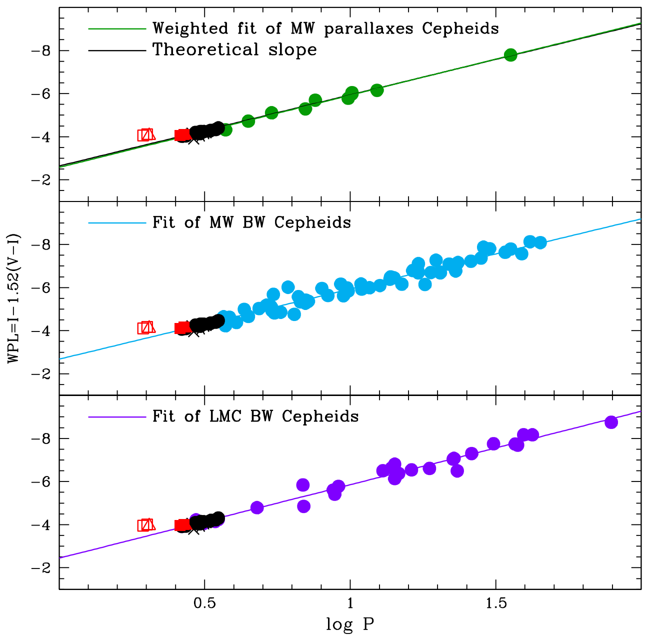

Figure 6. WPL(V,I) relation for NGC 1866 Cepheids (symbols used are the same of Fig. 4) compared with different samples in literature. In particular, in the upper panel, we adopt Galactic Cepheids (green symbols) with distances obtained from parallaxes (Benedict et al. 2007; van Leeuwen et al. 2007; Riess et al. 2014; Casertano et al. 2015) and in the middle and bottom panels, we show Cepheids with distances obtained from BW in the Galaxy (cyan symbols; Storm et al. 2011a) and in LMC (violet symbols; Storm et al. 2011b), respectively (see the text for details).

$WPL_{\rm par}(V, I) = -3.35\log P - 2.57$ ($\sigma = 0.05$), whereas the black line is the relation obtained assuming the slope of the theoretical WPL(V,I) relation for $Z = 0.008$ discussed in the previous section. From the former relation, we obtain $\mu_0^{\rm NGC\ 1866} = 18.49 \pm 0.05$ mag, whereas the theoretical relation provides $\mu_0^{\rm NGC\ 1866} = 18.73 \pm 0.04$ mag in the canonical assumption and 18.52 ± 0.04 mag in the non-canonical one. We note that in spite of the difference between the slope obtained for the Galactic Cepheids (−3.35) and the theoretical one (−3.30), when the theoretical slope is applied to the Galactic Cepheids, the zero-point obtained (-2.62 ± 0.04) is in good agreement with the theoretical non-canonical one (−2.63).

The middle and bottom panels of Fig. 6 show our NGC 1866 data (black and red symbols) together with MW (cyan symbols)

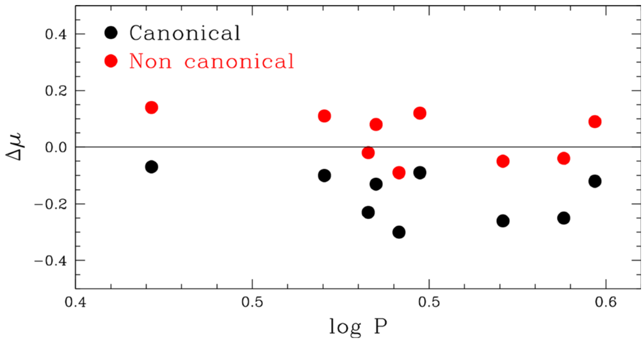

Figure 7. The differences between the individual moduli obtained by Molinaro et al. (2012) for a sample of 9 fundamental NGC 1866 Cepheids (see their table 5) and those obtained for the same variables, adopting the theoretical slope and zero-point of the WPL for $Z = 0.008$, both in the canonical (black symbols) and in the non-canonical (red symbols) assumption. The black line corresponds to $\Delta\mu = 0$.

and LMC (violet symbols) Cepheid samples with BW (BW) distance estimates by Storm et al. (2011a) and Storm et al. (2011b), respectively. In these papers, the authors adopt 1.54 as colour term in the WPL(*V,I*). For the Galactic sample, Storm et al. (2011a) give a WPL slope of −3.26 and a zero-point of −2.7. Adopting 1.52 as colour term, we obtain the same slope and a slightly different zero-point of −2.68 (instead of the values −3.38 and −2.54 obtained above from the weighted fit of the MW Cepheids with parallaxes). The BW slope (−3.26) and the theoretical one (−3.30) are very similar and when applied to the same NGC 1866 sample, adopting the zero-point by Storm et al. (2011a, recomputed using 1.52 as colour term coefficient), provide distance moduli of 18.55 ± 0.02 mag and 18.53 ± 0.04 mag, respectively.

For the LMC BW Cepheids, Storm et al. (2011a) give a WPL slope of −3.41 and a zero-point of −2.46. Adopting a colour term of 1.52, we obtain the same slope and −2.44 as zero-point. Applying this relation, we obtain a distance modulus of 18.39 ± 0.03 mag for NGC 1866. The shorter distance scale obtained from the BW calibration supports previous determinations based on this method in the literature (see, e.g. the discussion in Molinaro et al. 2012, and references therein). On the other hand, the application of the CORS version of the BW method by Molinaro et al. (2012) to the NGC 1866 Cepheids used in their work, gives a distance modulus of 18.51 ± 0.03 mag. The uncertainties on the distance estimates based on BW techniques depend on the controversial issue of the adopted projection factor which converts the measured radial velocity into the pulsational one (see e.g. Barnes 2009; Storm et al. 2011b; Molinaro et al. 2012, and references therein). In Fig. 7, we plot the differences between the individual moduli obtained by Molinaro et al. (2012) for a sample of nine fundamental NGC 1866 Cepheids and those obtained for the same variables adopting the theoretical slope and zero-point of the WPL for $Z = 0.008$, both in the canonical (black symbols) and in the non-canonical (red symbols) assumption. In this figure, we report also a black line for $\Delta\mu = 0$ mag corresponding to perfect agreement.

Inspection of these values and of Fig. 7 indicates that the adopted theoretical scenario in the non-canonical assumption provides better agreement with the most recent and widely assumed distance estimates for NGC 1866. Indeed, for the LMC, being near to our Galaxy and hosting several stellar distance indicators, we have hundreds of different and independent distance measurements. For example, a recent interesting measurement was obtained by the Carnegie Hubble Program (Monson et al. 2012) to calibrate the extragalactic distance scale, with the final goal to obtain H_0 with an accuracy of 2 per cent. For this purpose, they observed Cepheids in the Milky Way and nearby galaxies in the mid-infrared bands and found for the LMC a modulus of 18.48 ± 0.04 mag. The same value (with an error of ±0.05 mag) was obtained by Walker (2012) by the analysis of LMC distances based on different stellar distance indicators (Cepheids, RR Lyrae, red variables, red clump stars and eclipsing variables). Finally, it is worth noting that Pietrzyński et al. (2013) give a very accurate (within 2 per cent) LMC distance measurement based on the observations of eight late-type eclipsing binaries in this galaxy, finding a value of 18.493 ± 0.008(statistical)±0.047(systematic) mag.

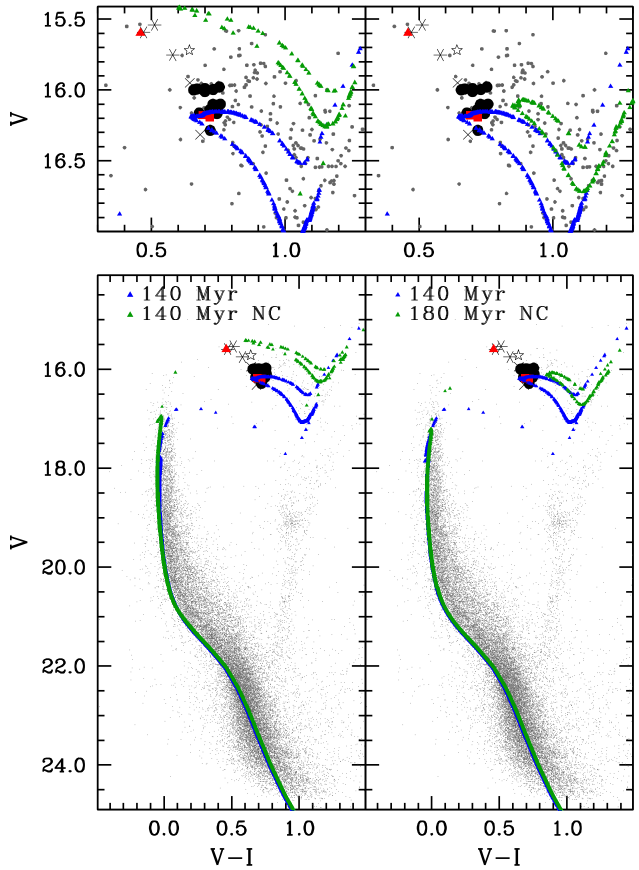

Figure 8. In this figure we show, in the bottom panels, the *V,V−I* CMD of NGC 1866 obtained using only the very precise VLT data compared with the synthetic CMDs computed with the stellar population synthesis code SPoT by Brocato et al. (2003). The symbols used for NGC 1866 Cepheids are the same of Fig. 4. The comparison with canonical (blue dots) and non-canonical (green dots and labelled as NC) synthetic CMDs are shown. The age adopted for the canonical synthetic CMD is 140 Myr, and that adopted for the non-canonical one is 140 Myr in the left-hand panel and 180 Myr in the right one. In the top panels, there is a zoom of the region around the Cepheid IS.

This conclusion seems to favour the hypothesis of a moderate overluminosity of NGC 1866 Cepheids at fixed mass, possibly due to a combination of core overshooting and mass-loss effects, in agreement with the results by Brocato et al. (2004).

7 THE COLOUR–MAGNITUDE DIAGRAM

As a by-product of the accurate time series multiband observations, we are able to build new colour magnitude diagrams (CMDs). Figs 8

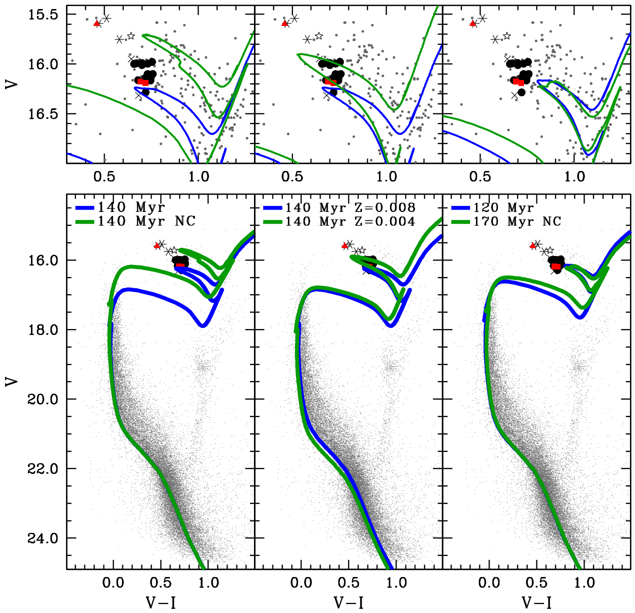

Figure 9. In the three bottom panels of this figure, we show the same observational CMD of Fig. 8 compared with the isochrones from the BaSTI data base (Pietrinferni et al. 2004, 2006) for different assumptions of overshooting efficiency, metallicities and ages (see the text for details). In the top panels, there is a zoom of the region around the Cepheid IS.

and 9 show in the bottom panels the $V,V-I$ CMD of NGC 1866 and in the top panels, a zoom of the region around the Cepheid IS. To derive the mean magnitudes for the stars identified in the observed field, we use only the VLT data that, as described in Section 2, are much more precise than the other ones. Thanks to the robustness of estimated mean magnitudes and colours, most of the reliable fundamental Cepheids (black filled circles) are located in a very restricted region of the diagram, corresponding to the tip of the blue loop of He-burning giants.

To compare the observed CMD and the Cepheid location with evolutionary prescriptions, in Fig. 8, we considered the synthetic CMDs computed with the stellar population synthesis code SPoT[5] by Brocato et al. (2003), based on two sets of stellar evolutionary tracks calculated for this purpose with the Pisa evolutionary code (Castellani et al. 2003), with canonical (no overshooting) and non-canonical (moderate overshooting, corresponding to an overluminosity of 0.25 dex) assumptions. In particular, we considered $Z = 0.007$ and ages of 140 and 180 Myr for comparison with the results by Brocato et al. (2003, 2004) (see those papers for more details). Moreover, we assumed a distance modulus of 18.5 mag and a colour excess $E(B-V) = 0.06$ mag. The left-hand panel of Fig. 8 shows the comparison between canonical and non-canonical synthetic CMDs for an age of 140 Myr with the observed data. We note that the location of the Cepheids is intermediate between the blue loops obtained in the two assumptions. This suggests a possibly lower overshooting efficiency compared to that adopted, or a slightly lower cluster metallicity. In the right-hand panel, we use the same canonical synthetic CMD, but increase the age of the non-canonical one to 180 Myr. We note that, for the higher age, the luminosity level predicted in the non-canonical assumption seems to reproduce the observed one, but the blue loop extension is too short (see also discussion below).

We also adopted the isochrones from the Bag of Stellar Tracks and Isochrones (BaSTI) data base (Pietrinferni et al. 2004, 2006), which also allows us to compare models based on the canonical assumptions and models including core overshooting at different chemical compositions. In the left-hand panel of Fig. 9, at fixed metallicity $Z = 0.008$ and age of 140 Myr, we compare the canonical isochrone with the non-canonical one. As expected, the canonical blue loop is fainter than the observed Cepheids, whereas the non-canonical one is too bright. We note that the overshooting efficiency adopted in the BaSTI data base is almost twice that adopted in the pulsational scenario, so that the result obtained seems to confirm our finding based on the pulsational analysis that Cepheids in NGC 1866 might be affected by mild overshooting of 0.2 dex and/or a small amount of mass-loss. At fixed age, the loop luminosity is also affected by metallicity; thus in the central panel, we compare the canonical isochrone at 140 Myr for $Z = 0.008$ with the one for $Z = 0.004$. The latter isochrone seems to better match the observed Cepheid location, but it does not properly reproduce the main sequence (MS). However, we cannot exclude an intermediate metallicity close to $Z = 0.006$. Finally, in the right-hand panel, we try to reproduce the luminosity of the observed Cepheids by varying the isochrone age in the two scenarios, assuming a metallicity of $Z = 0.008$. We find that in the canonical assumption, the predicted age is close to 120 Myr and in the non-canonical one it is about 170 Myr. The overshooting efficiency in the non-canonical isochrone is higher than expected according to the pulsational results. In the case of mild overshooting, an intermediate age of about 140 Myr is predicted. We note that the theoretical blue loops for $Z = 0.008$ appear too short at least for some ages and this discrepancy can be due to different numerical and physical assumptions in the theoretical scenario (Castellani, Chieffi & Straniero 1990; Bono et al. 2000; Valle et al. 2009; Walmswell, Tout & Eldridge 2015), even if – as noted above – a slightly lower metal content cannot be excluded.

8 CONCLUSIONS

We have investigated the properties of a sample of Cepheids in NGC 1866 on the basis of both archival and very precise VLT data.

As a result of the time series analysis, accurate light curves are obtained for 21 fundamental and three FO pulsators. We also derive the mean magnitudes and colours for each pulsator and compare their behaviour with both empirical (OGLE) and predicted LMC PL and WPL(*V,I*) relations. The results from the PL are affected both by the intrinsic dispersion and by reddening uncertainties, whereas the WPL(*V,I*) relation gives more reliable results. Adopting the slope of the theoretical canonical WPL(*V,I*) relation, we obtain $\mu_0^{\rm OGLE} = 18.717 \pm 0.003$ mag and $\mu_0^{\rm NGC\,1866} = 18.71 \pm 0.01$ mag, for the OGLE LMC field sample and our NGC 1866 data, respectively. On the other hand in the non-canonical assumption ($\log L/L_{\rm c} = 0.25$ dex), the inferred distance moduli are $\mu_0^{\rm OGLE} = 18.57 \pm 0.003$ and $\mu_0^{\rm NGC\,1866} = 18.50 \pm 0.01$ mag.

As a test of our theoretical approach, we also considered two empirical calibrations of Cepheid distances based on trigonometric parallaxes and BW measurements. Even if the slope of the WPL(*V,I*) relation obtained for the Galactic Cepheids with parallaxes is steeper by 0.06 than the theoretical one, its application to the NGC 1866 sample gives a distance modulus of 18.49 ± 0.07 mag, in very good agreement with that obtained applying the theoretical non-canonical slope (18.52 ± 0.04 mag).

[5] For the details of the numerical procedures and physical assumptions of the SPoT code, see Brocato et al. (2000), Raimondo et al. (2005) and Raimondo (2009).

On the other hand, using Galactic Cepheids with BW measurements, the slope of the WPL(*V,I*) relation is very similar to the theoretical one. Both the slopes give the same distance modulus of 18.54 mag for NGC 1866, adopting the corresponding zero-point. Using the LMC BW sample and adopting the Storm et al. (2011b) WPL(*V,I*) relation (recomputed using 1.52 as colour term), we obtain for NGC 1866 a distance of 18.39 ± 0.03 mag – significantly shorter than the values derived above. This shorter distance scale is in agreement with previous determinations based on the BW method and its deviation from most of the other recent results for the LMC has already been debated in the literature (see e.g. Molinaro et al. 2012).

Therefore, our investigation seems to favour a value close to 18.5 mag for the distance modulus of NGC 1866, as also obtained by the theoretical non-canonical WPL(*V,I*) relation, thus suggesting that mild overshooting and/or moderate mass-loss can affect intermediate-mass stellar evolution in this cluster.

To conclude, we have compared the *V,V−I* CMD of NGC 1866 with the predictions of evolutionary and synthetic models, adopting $\mu_0 = 18.5$ mag, $E(B-V) = 0.06$ mag. The main conclusions of this comparison are:

(i) for the typical metallicity adopted for NGC 1866 ($Z = 0.007/0.008$), an age close to 120 Myr (170 Myr) is suggested by the comparison with canonical (non-canonical) isochrones. As the non-canonical isochrones are computed for an overshooting efficiency almost twice that assumed in the pulsational analysis, we expect an age around 140 Myr in the mild overshooting and/or moderate mass-loss scenario;

(ii) decreasing the metallicity of the canonical isochrone at 140 Myr from $Z = 0.008$ to 0.004, we find better agreement with the Cepheid distribution, but a worse fit for the MS phase. On this basis, an intermediate metallicity close to $Z = 0.006$ cannot be ruled out.

Finally, at $Z = 0.008$, taking into account the moderate overluminosity required by pulsational models to find a distance modulus in agreement with most empirical calibrations, we can conclude that our age determination for NGC 1866 is close to 140 Myr.

ACKNOWLEDGEMENTS

This paper is based on observations made with the ESO/VLT. This work has made use of BaSTI web tools and was supported by PRIN-INAF 2014 "EXCALIBURS: EXtragalactic distance scale CALIBration Using first- Rank Standard candles." (PI: G. Clementini). We thank our anonymous referee for her/his valuable comments.

REFERENCES

Barnes T. G., 2009, AIP Conf. Proc. Vol. 1170, in Guzik J. A., Bradley P. A., eds, The Cepheid Distance Scale: Recent Progress in Fundamental Techniques. Am. Inst. Phys., New York, p. 3
Benedict G. F. et al., 2007, AJ, 133, 1810
Bono G., Caputo F., Castellani V., Marconi M., 1999, ApJ, 512, 711
Bono G., Marconi M., Stellingwerf R. F., 1999, ApJS, 122, 167
Bono G., Caputo F., Cassisi S., Marconi M., Piersanti L., Tornambèè A., 2000, ApJ, 543, 955
Bono G., Gieren W. P., Marconi M., Fouqué P., Caputo F., 2001, ApJ, 563, 319
Bono G., Caputo F., Fiorentino G., Marconi M., Musella I., 2008, ApJ, 684, 102
Brocato E., Castellani V., Poli F. M., Raimondo G., 2000, A&AS, 146, 91
Brocato E., Castellani V., Di Carlo E., Raimondo G., Walker A. R., 2003, AJ, 125, 3111
Brocato E., Caputo F., Castellani V., Marconi M., Musella I., 2004, AJ, 128, 1597
Caputo F., Marconi M., Musella I., 2000, A&A, 354, 610
Cardelli J. A., Clayton G. C., Mathis J. S., 1989, ApJ, 345, 245
Casertano S. et al., 2015, preprint (arXiv:1512.09371)
Castellani V., Chieffi A., Straniero O., 1990, ApJS, 74, 463
Castellani V., Degl'Innocenti S., Marconi M., Prada Moroni P. G., Sestito P., 2003, A&A, 404, 645
Chiosi C., Wood P. R., Capitanio N., 1993, ApJS, 86, 541
Davies B., Kudritzki R.-P., Gazak Z., Plez B., Bergemann M., Evans C., Patrick L., 2015, ApJ, 806, 21
Dean J. F., Warren P. R., Cousins A. W. J., 1978, MNRAS, 183, 569
Fausnaugh M. M., Kochanek C. S., Gerke J. R., Macri L. M., Riess A. G., Stanek K. Z., 2015, MNRAS, 450, 3597
Fiorentino G., Caputo F., Marconi M., Musella I., 2002, ApJ, 576, 402
Fiorentino G., Marconi M., Musella I., Caputo F., 2007, A&A, 476, 863
Freedman W. L., Madore B. F., 2011, ApJ, 734, 46
Freedman W. L. et al., 2001, ApJ, 553, 47
Gerke J. R., Kochanek C. S., Prieto J. L., Stanek K. Z., Macri L. M., 2011, ApJ, 743, 176
Gieren W. P., Richtler T., Hilker M., 1994, ApJ, 433, L73
Gieren W. P., Gómez M., Storm J., Moffett T. J., Infante L., Barnes T. G., III, Geisler D., Fouqué P., 2000, ApJS, 129, 111
Gould A., 1994, ApJ, 426, 542
Haschke R., Grebel E. K., Duffau S., 2011, AJ, 141, 158
Kennicutt R. C., Jr, et al., 1998, ApJ, 498, 181
Landolt A. U., 1992, AJ, 104, 340
Laney C. D., Stobie R. S., 1993, MNRAS, 263, 921
Lenz P., Breger M., 2005, Commun. Asteroseismology, 146, 53
Luck R. E., Moffett T. J., Barnes T. G., III, Gieren W. P., 1998, AJ, 115, 605
Lupton R. H., Fall S. M., Freeman K. C., Elson R. A. W., 1989, ApJ, 347, 201
Madore B. F., 1982, ApJ, 253, 575
Marconi M., Musella I., Fiorentino G., 2005, ApJ, 632, 590
Marconi M. et al., 2010, ApJ, 713, 615
Marconi M. et al., 2013, ApJ, 768, L6
Molinaro R. et al., 2012, ApJ, 748, 69
Monson A. J., Freedman W. L., Madore B. F., Persson S. E., Scowcroft V., Seibert M., Rigby J. R., 2012, ApJ, 759, 146
Mucciarelli A. et al., 2011, MNRAS, 413, 837
Musella I. et al., 2006, Mem. Soc. Astron. Ital., 77, 291
Noyola E., Gebhardt K., 2007, AJ, 134, 912
Pietrinferni A., Cassisi S., Salaris M., Castelli F., 2004, ApJ, 612, 168
Pietrinferni A., Cassisi S., Salaris M., Castelli F., 2006, ApJ, 642, 797
Pietrzyński G. et al., 2013, Nature, 495, 76
Raimondo G., 2009, ApJ, 700, 1247
Raimondo G., Brocato E., Cantiello M., Capaccioli M., 2005, AJ, 130, 2625
Riess A. G. et al., 2011, ApJ, 730, 119
Riess A. G., Casertano S., Anderson J., MacKenty J., Filippenko A. V., 2014, ApJ, 785, 161
Ripepi V., Barone F., Milano L., Russo G., 1997, A&A, 318, 797
Ripepi V. et al., 2012, MNRAS, 424, 1807
Romaniello M. et al., 2008, A&A, 488, 731
Saha A., Sandage A., Thim F., Labhardt L., Tammann G. A., Christensen J., Panagia N., Macchetto F. D., 2001, ApJ, 551, 973
Sakai S., Ferrarese L., Kennicutt R. C., Jr, Saha A., 2004, ApJ, 608, 42
Sandage A., Tammann G. A., Saha A., Reindl B., Macchetto F. D., Panagia N., 2006, ApJ, 653, 843
Sasselov D. D. et al., 1997, A&A, 324, 471
Schlegel D. J., Finkbeiner D. P., Davis M., 1998, ApJ, 500, 525
Soszynski I. et al., 2008, Acta Astron., 58, 163
Stetson P. B., 1987, PASP, 99, 191
Stetson P. B., 1994, PASP, 106, 250
Stetson P. B., 2000, PASP, 112, 925
Stetson P. B., Catelan M., Smith H. A., 2005, PASP, 117, 1325

Storm J., Gieren W. P., Fouqué P., Barnes T. G., III, Gómez M., 2005, A&A, 440, 487
Storm J., Gieren W., Fouqué P. et al., 2011b, A&A, 534, A94
Storm J., Gieren W., Fouqué P., Barnes T. G., Soszyński I., Pietrzyński G., Nardetto N., Queloz D., 2011a, A&A, 534, A95
Testa V. et al., 2007, A&A, 462, 599
Valle G., Marconi M., Degl'Innocenti S., Prada Moroni P. G., 2009, A&A, 507, 1541
van der Marel R. P., Cioni M.-R. L., 2001, AJ, 122, 1807
van Leeuwen F., Feast M. W., Whitelock P. A., Laney C. D., 2007, MNRAS, 379, 723
Walker A. R., 1995, AJ, 110, 638
Walker A. R., 2012, Ap&SS, 341, 43
Walmswell J. J., Tout C. A., Eldridge J. J., 2015, MNRAS, 447, 2951
Welch D. L., 1991, AJ, 101, 538
Welch D. L., Stetson P. B., 1993, AJ, 105, 1813

This paper has been typeset from a TEX/LATEX file prepared by the author.